Multi-objective approaches in a single-objective optimization environment

Shinya Watanabe
College of Information Science & Engineering,
Ritsumeikan Univ.
1-1-1 Nojihigashi, Kusatsu
Shiga 525-8577, Japan
sin@sys.ci.ritsumei.ac.jp

Kazutoshi Sakakibara
College of Information Science & Engineering,
Ritsumeikan Univ.
1-1-1 Nojihigashi, Kusatsu
Shiga 525-8577, Japan
sakaki@sys.ci.ritsumei.ac.jp

Abstract- This paper presents two new approaches for transforming a single-objective problem into a multi-objective problem. These approaches add new objectives to a problem to make it multi-objective and use a multi-objective optimization approach to solve the newly defined problem. The first approach is based on relaxation of the constraints of the problem and the other is based on the addition of noise to the objective value or decision variable. Intuitively, these approaches provide more freedom to explore and a reduced likelihood of becoming trapped in local optima.

We investigated the characteristics and effectiveness of the proposed approaches by comparing the performance on single-objective problems and multi-objective versions of those same problems. Through numerical examples, we showed that the multi-objective versions produced by relaxing constraints can provide good results and that using the addition of noise can obtain better solutions when the function is multimodal and separable.

1 Introduction

Recently, there has been a great deal of progress in the application of evolutionary computation to multi-objective optimization (EMO)[1, 2].

In recent years, several new algorithms have been developed that can find good Pareto optimal solutions with small calculation costs[1]. There has also been a great deal of research into the application of these algorithms to real-world multi-objective optimization problems (MOOP) [2].

On the other hand, there have been a few reports concerning multi-objectivizing (multi-objectivization)“ [1]. The multi-objectivization approach translates single-objective optimization problems (SOOP) into MOOP and then applies EMO to the translated problem.

Previous studies of multi-objectivization can be divided roughly into two categories as follows:

- Addition of new objectives to a problem.
- Decomposing a problem into sub-problems.

These multi-objectivizations have a number of effects, such as the reduction of the effect of local optima, making the problem easier, or increasing search paths to the global optimum. In this paper, we propose two new multi-objectivization approaches based on the addition of new objectives as follows:

- Relaxing the constraints of the problem.
- Adding noise to the objective value or decision variables.

The former approach uses the concept of constraint relaxation, while the latter is based on escape from a local optimum. These approaches aim to increase the paths to the global optimum that are not available under the original SOOP, and maintain diversity of the population.

Here, we investigated the characteristics and effectiveness of the proposed approaches by comparing the performance on the original SOOP and multi-objectivized versions. In numerical experiments, we used two types of GA, a single-objective GA and a multi-objective GA. These GAs were ”ga2k“ [4][2] as single-objective GA and NSGA-II[5] as a multi-objective GA. These algorithms have been shown to be more efficient than other GA methods[5, 4].

2 Single-objective and Multi- objective Optimization

A single-objective optimization problem (SOOP) has the objective function ($f(\vec{x})$), which must be minimized or maximized and a number of constraints ($g(\vec{x})$). Equation (1) shows the formula of the SOOP in its general form.

$$\begin{cases} \text{minimize } f(\vec{x}) & \\ \text{s.t.} \qquad g_j(\vec{x}) \geq 0 \quad (j = 1, \ldots, m) \\ \qquad\qquad \vec{x} \in X \subset R^n \end{cases} \tag{1}$$

where $\vec{x}$ is a vector of n decision variables, $\vec{x} = (x_1, x_2, \ldots, x_n)^T$, and X represents a feasible region.

Similarly, multi-objective optimization problems (MOOP) with a number of objective functions ($\mathbf{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x}))^T$) can be stated as follows:

$$\begin{cases} \text{minimize } \mathbf{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x}))^T \\ \text{s.t.} \qquad g_j(\vec{x}) \geq 0 \quad (j = 1, \ldots, m) \\ \qquad\qquad \vec{x} \in X \subset R^n \end{cases} \tag{2}$$

[1] This term was used previously by Knowles et al.[3].

[2] ”ga2k“ was written by T. Hiroyasu et al.[4] and is available at (`http://mikilab.doshisha.ac.jp/dia/research/pdga/archive/index.html`).

2.1 multi-objectivization

When we multi-objectivize a problem, the global optimum of the original SOOP must be one of the Pareto optima in the multi-objectivization problem. We state this relation between solutions in the original SOOP and multi-objectivization MOOP as follows:

$$\forall x^{\text{opt}} \in X^* \tag{3}$$

where x^{opt} is a global optimum of the original SOOP, and X^* represents Pareto optima of the multi-objectivization problem. $\forall x^{\text{opt}}$ indicates multiple optima in the SOOP.

3 Description of the new multi-objectivization approach

The methods of multi-objectivization can be roughly classified as follows:

- Adding new objectives in addition to the original objective.
- Replacing the original objective with a set of new objectives.

As an example of the former approach, Coello et al. [6] proposed a technique that treats constraints as objectives, and showed that their proposed technique was more efficient than more traditional penalty techniques. This technique makes it possible to handle constraints without the use of a penalty function, since all the individuals are feasible. On the other hand, Runarsson and X.Yao [7] reported that the multi-objective approach to constraint handling is not so effective in some cases, because most of the time is spent on searching infeasible regions. Their results indicate that the effectiveness of this technique depend heavily on a feature of problem.

As an example of the latter approach, Knowles et al.[3] reported a technique in which the original SOOP is decomposed into sub-problems. This technique treats the original problem as a combination of sub-objective problems. As each sub-objective problem is easier to solve than the original problem, better solutions can be found by using this decomposing approach. However, there are few problems that can be completely decomposed into sub-problems and the optimality of solutions obtained by decomposition must be verified.

In this paper, we propose two new approaches to add another objective as follows:

- Relaxation of the constraints of the problem.
- Addition of noise to the original objective value or decision variables.

Both approaches are based on multi-objectivization, which adds additional objectives to the original problem. The aims of these approaches are to increase paths to the global optimum that are difficult to obtain under the original SOOP and to maintain the diversity of the population.

These approaches have a low risk of providing solutions far from the optimal solutions as these approaches always deal with the original SOOP objective. In addition, these approaches can treat many types of problems and hardly produce new tasks such as decomposition of a problem into sub-problems.

The former approach is based on the concept of constraint relaxation. In this approach, a trade-off between the original and the relaxed objectives can be brought by differences in two constrains. Therefore, a search of EMO can be concentrated around the constrains of a problem. This approach can be expected to search effectively for the global optimum along the boundary of the feasible regions settled by the original constraints.

The latter approach takes advantage of escaping from local optima. In this approach, the trade-off relation is introduced by the difference between the original objective and the new objective with noise. This approach will be useful for escaping from local optima using trade-off regions.

The details of the proposed approaches are described below using practical problems.

3.1 Relaxation of constraints

In this paper, we treat a 0/1 knapsack problem[8] as a test problem. Generally, the formula for the knapsack problem is as follows:

$$\begin{cases} \text{maximize } f(\vec{x}) = \sum_{j=1}^{m} p_j \cdot x_j \\ \text{s.t.} \\ \qquad\qquad g(\vec{x}) = \sum_{j=1}^{m} w_j \cdot x_j \leq c \end{cases} \tag{4}$$

where p_j and w_j are the profit and weight of item j, respectively, and c is the knapsack capacity.

In this paper, we propose the following multi-objectivization that makes use of the relaxation of constraints.

$$\begin{cases} \text{maximize} \;\; f_1(\vec{x}) \;= \sum_{j=1}^{m} p_j \cdot x_j - \alpha \cdot \text{penalty} \\ \text{maximize} \;\; f_2(\vec{x}) \;= \sum_{j=1}^{m} p_j \cdot x_j \\ \text{s.t.} \\ \qquad \text{penalty} = -\min(0, c - \sum_{j=1}^{m} w_j \cdot x_j) \\ \qquad g(\vec{x}) \;= \sum_{j=1}^{m} w_j \cdot x_j \leq c' \;\; (c' \geq c) \end{cases} \tag{5}$$

where $c'(c' \geq c)$ is the relaxation capacity of knapsack for an additional objective($f_2(\vec{x})$), and α is the penalty value for violation of c.

In Equation (5), the feasible regions are settled by relaxing constraint. In addition, $f_1(\vec{x})$ indicates optimization of the original problem and $f_2(\vec{x})$ represents optimization of the problem based on the relaxed constraints.

In Equation (5), the trade-off relationship between $f_1(\vec{x})$ and $f_2(\vec{x})$ is obtained only when a solution($\vec{x}$) does not satisfy the original constraint. In general, the global optima of many problems lie on the boundaries of the feasible regions. As the EMO search tends to concentrate in the trade-off regions, this approach can be expected to perform an effective search. Figure 1 shows the concept of this approach in a maximum problem.

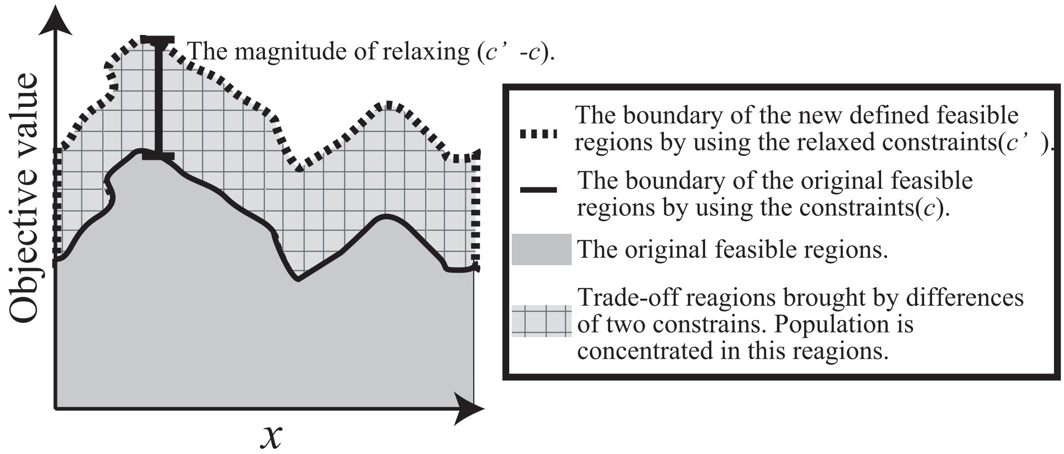


Figure 1: The concept figure of the multi-objectivization of relaxing a constraint(in a maximum problem).

In the case in which the problem has more than one constraint, a user should decide which constraint to relax[3]. It is most effective to relax only the constraints near the global optima.

The research of Runarsson and X.Yao [7] shows that the multi-objective approach using Equation (5) is less likely to converge feasible solutions. Therefore, our approach is assumed to be suitable for the problem in which a repair method have already designed to make infeasible solutions feasible.

3.2 Addition of noise

In this section, we describe the multi-objectivization approach in which noise is added to the original objective value or decision variables. We expect that a population will be able to escape from local optima by adding noise to the original problem because the diversity of the population is increased as compared to the original SOOP.

This approach can be applied to SOOPs with the following formula: Equation (1). In the case of addition of noise to the original problem, the formula can be stated as follows:

$$\begin{cases} \text{minimize } f_1(\vec{x}) = F(\vec{x}) \\ \text{minimize } f_2(\vec{x}) = f_1(\vec{x}) + D \cdot \text{Gauss}(0,1) \\ \text{s.t.} \quad g_j(\vec{x}) \geq 0 \quad (j = 1, \dots, m) \\ \quad \vec{x} \in \ X \subset R^n \end{cases} \tag{6}$$

where $F(\vec{x})$ is the original SOOP function and D is the parameter for adjusting the magnitude of the noise.

On the other hand, the approach of adding noise to decision variables can be presented as the following formula:

$$\begin{cases} \text{minimize } f_1(\vec{x}) = F(\vec{x}) \\ \text{minimize } f_2(\vec{x}) = f_1(\vec{x} + D \cdot \text{Gauss}(0,1)) \\ \text{s.t.} \quad g_j(\vec{x}) \geq 0 \quad (j = 1, \dots, m) \\ \quad \vec{x} \in \ X \subset R^n \end{cases} \tag{7}$$

[3] User can select plural constraints to relax, and use different relaxation values for each constraint

In Equation (7), if the value of $\vec{x} + D \cdot \text{Gauss}(0,1)$ exceeds the upper bound of $\vec{x}$, let this value be the upper bound. Conversely, if the value is below the lower bound, let this value be the lower bound.

In the case in which noise is added to the original objective, the magnitude of the noise set as Equation (8) is used.

$$D = (f^{\max}(\vec{x}) - f^{\min}(\vec{x})) \cdot \alpha \tag{8}$$

where D is the magnitude of the noise and f_{max} and f_{min} are the maximum and minimum values of $f(\vec{x})$, respectively. Therefore, D is calculated dynamically at each generation in this case.

In the case in which the noise is added to decision variables, the magnitude of the noise (D) is calculated by multiplying the range of decision variables by α. Since the range of decision variables are fixed, D is static in this case.

These multi-objectivizations with addition of noise create trade-off relations between $f_1(\vec{x})$ and $f_2(\vec{x})$ in the neighbourhood of the local optima. Therefore, we can expect to reduce the effect of local optima and to increase the diversity of the population using this multi-objectivization.

4 Numerical Examples

In this paper, we describe application of the proposed approaches to two types of numerical experiment. To verify the effectiveness of multi-objectivization of relaxing the constraints, the 0/1 knapsack problem with 750 items was used. In addition, to confirm the effectiveness of multi-objectivization with addition of noise, typical test functions (Rastrigin, Schwefel, etc.) were used.

In implementing our proposed approach, we used two types of GA:

- ”ga2k“[4] as a single-objective GA.
- ”NSGA-II“[5] as a multi-objective GA.

ga2k is based on the island GA model. The difference between ga2k and a traditional GA is nothing except its island model. A prototype implementation has been written in C++ and can be downloaded from [4].

Table 1: GA Parameters.

population size	200
crossover rate	1.0
mutation rate	1/bit length
terminal criterion	200 generation
number of trial	30

4.1 Implementation of GA

In these experiments, GAs applied to the two types of experiment used bit coding. Similarly, two-point crossover and bit flip were implemented as for crossover and mutation. Table 1 shows the GA parameters used (ga2k and NSGA-II use the same parameters described in Table 1). We performed 30 trials and all results are shown as averages of 30 trials. In addition, the terminal condition of all experiments was 200 generations.

To apply single-objective GA (ga2k) to MOOPs, we used the weight-based method that multiplies each objective function by a weight$\omega(0 \leq \omega \leq 1)$ and sums all weighted objective functions. The fitness for a single-objective GA is calculated as follows:

$$F(\vec{x}) = (1 - w) \cdot f_1(\vec{x}) + w \cdot f_2(\vec{x}) \tag{9}$$

By applying Equation (9) to Equation (5) or Equation (6), the two multi-objectivizations becomes to the penalty or noise of the original single-objective problems. Therefore, single-objective GA based on the weight-based method would not gain the effectiveness of multi-objectivation as described in Section 3.

4.2 Multi-objectivization with relaxation of constraints

We used the 0/1 knapsack problem with 750 items and multi-objectivized the problem using Equation (5). This multi-objectivization problem was a maximization problem. The length of the chromosome was 750 bits, equivalent to the number of items, and the simulation was terminated when the generation number reached over 200.

This example had the constraint of knapsack capacity. As many codings lead to infeasible solutions, we should implement a repair method as a constraint handling technique. We used the repair method proposed by Zitler et al.[8]. In this technique, the items are deleted in increasing order of $v(j) = \{p_j/w_j\}(j = 1, 2, \ldots, n)$, which is the ratio of benefit to weight of item j, until satisfying the capacity constraint. This technique to make infeasible solutions feasible appears to be most appropriate for various kinds of knapsack capacities.

The computational cost of using this technique is very low. Because the order of $v(j)$ is static (fixed by the definition of the problem), and it can easily calculate which items should be deleted from the violation magnitude of weight and the order of $v(j)$.

The problem of Equation (5) has two knapsack capacities. We used the repair method when the codings violated the relaxed capacity c'. In addition, we used $\alpha = 3.0$ (Equation (5)) based on the results of our pilot study.

Table 2: The four types experiments of NSGA-II

Type	f_1	f_2
Multi-objective type	$f_1^{Eq.5}$	$f_2^{Eq.5}$
zero relaxing magnitude type	$f^{Eq.4}$	$f^{Eq.4}$
F_1 Single-objective type	$f_1^{Eq.5}$	$f_1^{Eq.5}$
F_2 Single-objective type	$f_2^{Eq.5}$	$f_2^{Eq.5}$

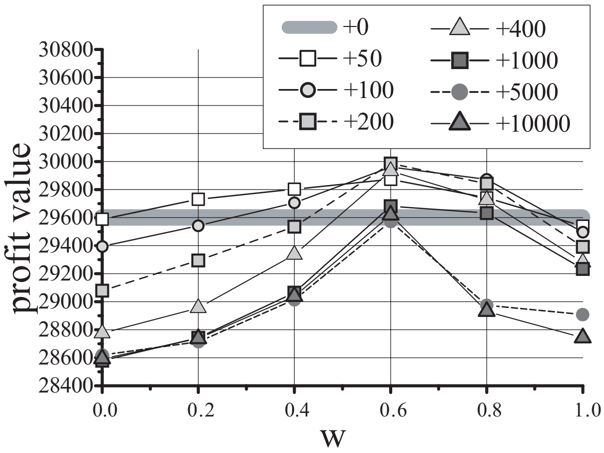


Figure 2: The results of ga2k (knapsack problem)

4.2.1 Results

The results of ga2k are shown in Figure 2 and those of NSGA-II are shown in Figure 3. The horizontal axis in Figure 2 indicates the value of the weight parameter ω(in 9) and that in Figure 3 indicates the magnitude of relaxation of the constraints settled by the difference between the original capacity c and relaxed c'. Figure 2 shows the results of 7 experiments based on the magnitude of relaxation: 50, 100, 200,400, 1000, 5000, 10000 (the original capacity was $c = 20351.5$). Figure 3 shows the results of 4 experiments based on the implementation of objectives (f_1 and f_2). Table 2 shows the 4 experiments.

The grey bands in both Figure 2 and 3 indicate the results of the original SOOP (Equation (4)) obtained by ga2k and NSGA-II. Therefore, by investigating whether the results of multi-objectivization were higher or lower than the grey band, it was possible to determine the useful of multi-objectivization for this problem.

In this experiment, the final solutions that could not satisfy the capacity c (Equation (5)) were repaired using the above repair method. Therefore, all solutions of Figure 2 and 3 satisfied the original capacity c.

The results of ga2k (Figure 2) indicated that the case of weight $\omega = 0.6$ was better than larger or smaller values in all magnitude cases of relaxation.

In addition, we found that the solutions of $\omega = 0.6$ were better than those of the original SOOPs (grey band), and the results of the weight-based methods where ω was close to 0.0 or 1.0 become worse.

On the other hand, Figure 3 indicates that solutions of

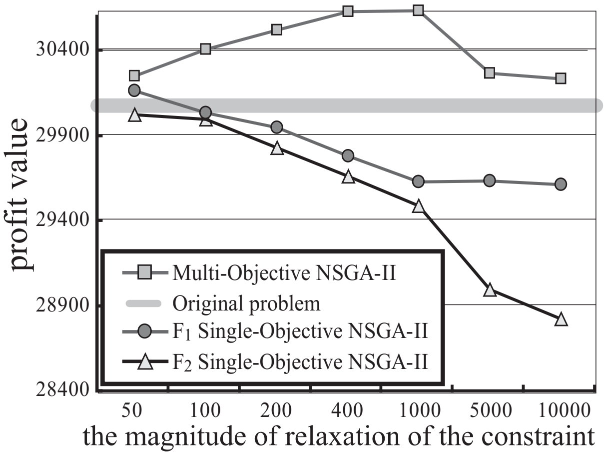


Figure 3: The results of NSGA-II (knapsack problem)

NSGA-II in all cases of multi-objectivization were better than those of the original SOOPs. The magnitudes of constrain relaxation of 400 and 1000 yielded good solutions, and the quality of the obtained solutions decreased at magnitudes deviating from these values. Therefore, we guessed that there was an optimum relaxation magnitude for f_2 in Equation (5). The single optimization of F_1 or F_2 only could not obtain better solutions than the original SOOPs, and the quality of the obtained solutions decreased with greater ω, in the same way as described above for multi-objectivization.

In both resutls (Figure 2 and Figure 3), the constrain relaxation of nearly 400 seemed to provid the better relsuts for both algorithms. Therefore, we guessed that the optimum ratio of relaxation magnitude to the original capacity is near 2 % (400/20351.5 $\simeq$ 0.02) in knapsack problem.

4.3 Multi-objectivization with addition of noise

We examined the effectiveness of multi-objectivization by adding noise according to the formula Equation (6) using typical test functions (Rastrigin, Schwefel, etc.). In these experiments, all problems were minimization problems.

4.3.1 Test functions

In this example, we used functions from the perspective of the modality (unimodal or multimodal) of function and epistasis among decision variables (separable or non-separable). We used 5 types of function: Rastrigin, Schwefel, Ridge, Rotated Rastrigin and Rotated Schwefel functions. Two of the former functions had epistasis, while the others did not. In addition, only the Ridge function is a unimodal function, while the others are multimodal. We treated all functions as having 10 decision variables in this example. The details of the functions used are given below:

Rastrigin

$$F_{Rastrigin}(\vec{x}) = 10n + \sum_{i=1}^{n}\left(x_i^2 - 10\cos(2\pi x_i)\right) \quad (-5.12 \le x_i < 5.12) \tag{10}$$

where n is the number of decision variables.

The optimum of the Rastrigin function is $F_{Rastrigin}(0,0,\ldots,0) = 0$. This function has high-quality local optima that tend to centre around the global optimum. This function is multimodal and does not have epistasis.

Schwefel

$$F_{Schwefel}(\vec{x}) = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{|x_i|}\right) + 418.98288727 \cdot n \quad (-512 \le x_i < 512) \tag{11}$$

The optimum of the Schwefel function is $F_{Schwefel}$ (420.968746, ..., 420.968746) = 0. This function is multimodal and does not have epistasis. This function is characterized by a second-best minimum, which is far away from the global optimum. Therefore, it is necessary to perform a global search at the beginning of the search.

Ridge

$$F_{Ridge}(\vec{x}) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_j\right)^2 \quad (-64 \le x_i < 64) \tag{12}$$

The optimum of the Ridge function is $F_{Ridge}(0,0,\ldots,0) = 0$. It is a unimodal function, but has strong epistasis among decision variables.

Rotated Rastrigin

The Rotated Rastrigin function is formed by randomly rotating the Rastrigin function (Equation (10)) around the origin. This is a multimodal function with epistasis among parameters.

Rotated Schwefel

Similarly, the Rotated Schwefel function is formed by randomly rotating the Schwefel function around the origin. This is a multimodal function with epistasis among parameters.

In this example, the grey coding is used and the length of the chromosome is 20 bits per decision variable. The simulation of all functions is terminated when the generation reaches over 200.

4.3.2 Results

The performances of the 5 functions are shown in Figure 4, 5, 7 and 8, respectively.

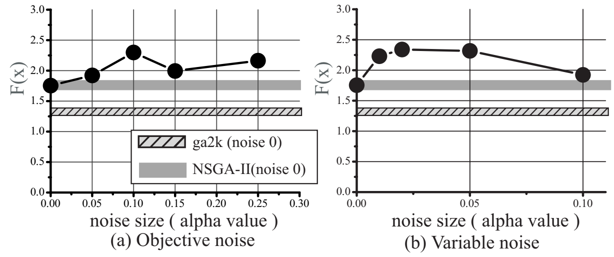


Figure 4: The result of Rastrigin

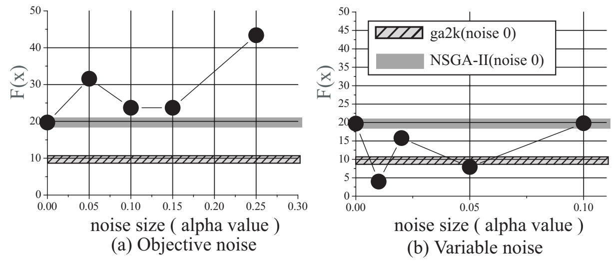


Figure 5: The result of Schwefel

In these figures, the horizontal axes show the values of α (in Equation (6)), which is the parameter for adjusting the magnitude of the noise, and the vertical axes show the values of the original functions. The left (a) and right (b) figures show the results of adding noise to the objective value and to the decision variables, respectively. In addition, the grey bands in the figures indicate the results of the original function value (i.e., the magnitude of noise is zero) obtained by ga2k and NSGA-II, the bands marked with diagonal lines show the results of ga2k and the other bands show those of NSGA-II. [4]

As shown in Figure 4, all cases with the addition of noise were worse than those of the original SOOP in the Rastrigin function. On the other hand, in the Schwefel function, which is multimodal and does not have epistasis, similarly to the Rastrigin function, the addition of noise to the decision variables yielded better results than with the original SOOP. Figure 6 shows the number of runs in which the algorithm succeeded in finding the global optimum in the

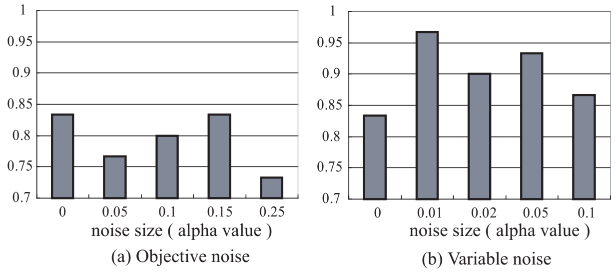


Figure 6: The number of runs in which the algorithm succeeded in finding the global optimum (Schwefel)

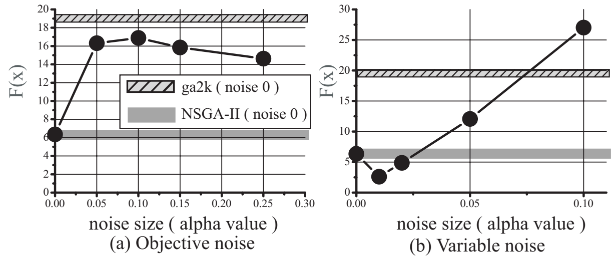


Figure 7: The result of Ridge

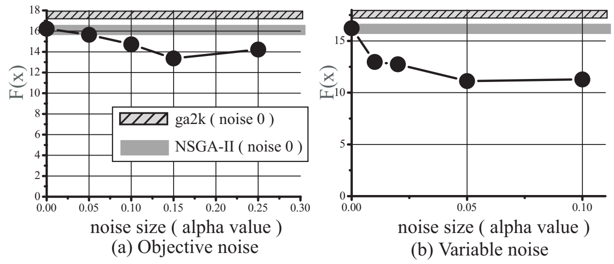


Figure 8: The result of Rotated Rastrigin

Schwefel function. The results indicated that the addition of noise to decision variables yielded a stronger probability of finding the global optimum than the original SOOP. In addition, ga2k obtained get better solutions than NSGA-II in both Rastrigin and Schwefel functions. We postulated that this was due to the distribution effect of the island GA.

The Schwefel function is different from the Rastrigin function in that it has local optima around the global optimum. Therefore, it is important that the population does not converge at the beginning of the search. The results with the Schwefel function indicated that the addition of noise to decision variables maintains the diversity of the population. However, in the Rastrigin function, the addition of noise did not yield good results because the ability to perform a local search failed by multi-objectivization.

Consider the results with epistasis among decision variables shown in Figure 7, 8 and 9. With the Ridge function (Figure 7), addition of noise did not yield good results in almost all cases. However, with the Rotated Rastrigin and Rotated Schwefel functions (Figure 8), addition of noise yielded better solutions than the original SOOP in almost all cases. Especially, the addition of noise to decision variables facilitated an effective search. As shown in Figure 9(b), the Rotated Schwefel function yielded a V-shaped plot, indicating that there was an optimal magnitude of noise addition

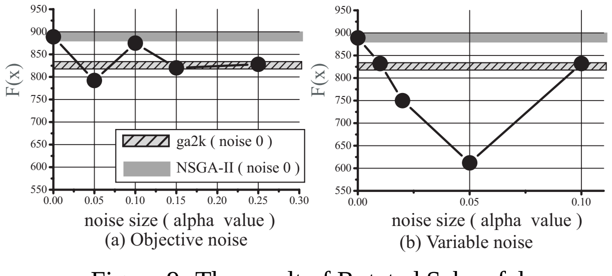
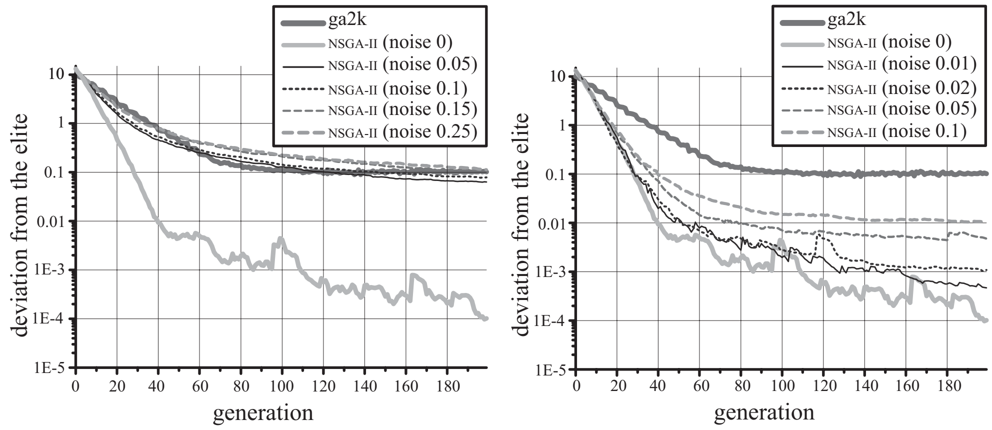


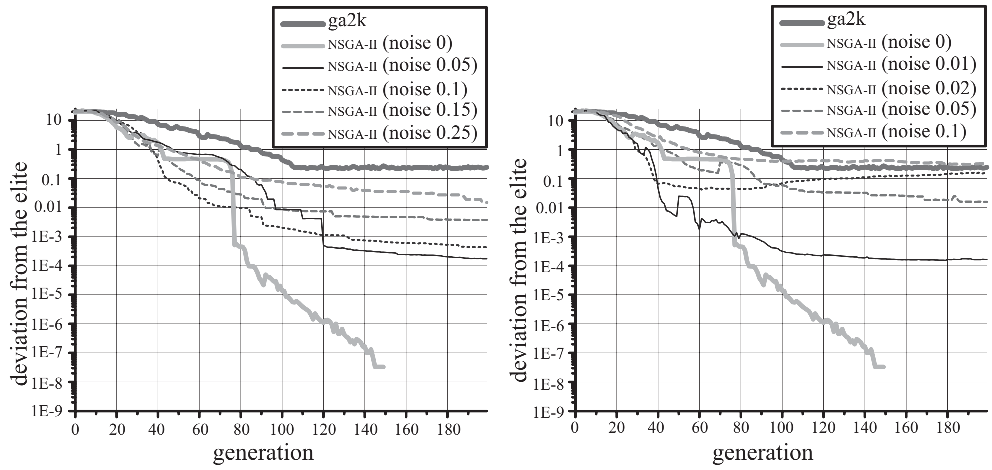
Figure 9: The result of Rotated Schwefel

[4] In the preliminary experiment, all results of ga2k in multi-objectivization problems were worse than in the original SOOP (i.e., the magnitude of noise is zero). Therefore, these results have been omitted in this paper.



(a) Objective noise (b) Variable noise

Figure 10: The transition of diversitiy (Ridge).



(a) Objective noise (b) Variable noise

Figure 11: The transition of diversitiy (Rotated Schwefel).

(in this case, $\alpha = 0.05$).

The results obtained with ga2k and NSGA-II in functions with epistasis were compared, and NSGA-II was found to yield better solutions than ga2k in all problems. This was because multi-objective algorithms have a tendency not to concentrate the population in a particular pattern. On the other hand, ga2k obtained better solutions in functions without epistasis as ga2k has a higher capacity to perform local searches than NSGA-II.

Analysis of the diversity of the population

Here, we describe the transition of diversity in the population to investigate the effectiveness of multi-objectivization by adding noise. As diversity in a population, we used deviation from elite individuals. The formula of this deviation is as follows:

$$s_N^{\text{elite}} = \sqrt{\frac{1}{N} \cdot \sum_{i=1}^{N} (\overrightarrow{x_i} - \overrightarrow{x^{\text{elite}}})^2} \tag{13}$$

where N is the total number of individuals in the population and $\overrightarrow{x^{\text{elite}}}$ is the elite individual in the generation.

The results of diversity transition in Ridge and Rotated Schwefel functions are shown in Figure 10 and fgreffig::diversity-ro-schwefel, respectively. In these figures, the horizontal axes show generation and the vertical axes show The value of deviation from the elite (s_N^{elite}) on a log scale. From the value of s_N^{elite}, we can determine the diversity of the population[5].

The results shown in Figure 10 and 11 indicate that the diversity of the population in all functions was increased by adding noise, and was increased with increasing magnitude of noise. The diversity of ga2k based on the island GA model can maintain a high value, while that of NSGA-II(noise 0) was decreased by a large generation number[6].

Consider the relation between the results of the original functions and diversity. In the Ridge function, the results (Figure 7 and 10) indicated that less diversity can yield better results. This was because a population that has a strong capacity for local searches has less diversity. As the Ridge

[5] If the deviation from the elite (s_N^{elite}) is high, the population has a high degree of diversity.

[6] In Figure 11, the line of NSGA-II(noise 0) is not shown because s_N^{elite} was 0 for more than 150 generations.

function is unimodal, the local search ability is more important than the diversity of the population.

On the other hand, the results of the Rotated Schwefel function (shown in Figure 9 and 11) indicated different tendencies in the relations between the result function and diversity with the two types of noise addition, although it was a common feature of both types that the diversity of the population was increased by increasing the magnitude of the noise. With the addition of noise to the original objective, the magnitude of the noise showed little influence on the results of the original function. However, with the addition of noise to decision variables, the results were affected by the magnitude of the noise.

In the Rotated Schwefel function, the influence of a wide-ranging search on the results was very strong as this function is multimodal and has epistasis among parameters. Therefore, we concluded that the multi-objectivization approach with the addition of noise to the original objective showed little effect in extending the range of the search, but the approach involving addition of noise to decision variables allowed a wider-ranging search than the original single-object problem.

From the above results, it is apparent that the multi-objectivization approach using the addition of noise to decision variables is very effective for multimodal functions with epistasis, such as the Rotated Rastrigin and Rotated Schwefel functions. In addition, the relation between the results of the original functions and diversity indicated that the approach using addition of noise to decision variables allowed wide-ranging searches using a diverse population.

5 Conclusions

In this paper, we proposed two new approaches for multi-objectivization as follows:

- Relaxation of the constraints of the problem.
- Addition of noise to the objective value or decision variables.

The proposed approaches are based on the addition of new objectives. The former approach uses the concept of constraint relaxation, while the latter is based on escape from local optima. We investigated the characteristics and effectiveness of the proposed approaches by comparing the performance of the original SOOP and multi-objectivized versions. Through numerical examples, the following points were clarified:

1. Multi-objectivization with relaxation of constraints
   In this approach, an additional objective is defined by relaxing the constraints of a problem. This approach aims to achieve an effective search of the global optimum along the boundary of the feasible regions settled by the original constraints. The experimental results confirmed that this multi-objectivization approach using a multi-objective GA is effective for the 0/1 knapsack problem. In addition, we found that there is an optimum magnitude of relaxation for an additional objective.
2. Multi-objectivization with addition of noise
   In this approach, an additional objective is defined by adding noise to the original objective value or decision variables. These multi-objectivizations make trade-off relations in the neighbourhood of local optima. This approach aims at escaping from local optima by using trade-off regions. We examined the effectiveness of this approach using 5 typical test functions: i.e., Rastrigin, Schwefel, Ridge, Rotated Rastrigin and Rotated Schwefel functions. The experimental results confirmed that multi-objectivization using addition of noise to decision variables is very effective in problems that have epistasis and multimodality, such as the Rotated Rastrigin and Rotated Schwefel functions.

   In addition, the relation between the results of the original functions and diversity confirmed that the approach using the addition of noise to decision variables facilitates a wide-ranging search using a diverse population. On the other hand, the approach using addition of noise to the original objective seemed not to extend the range of the search.

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