Comparison of Multiobjective Evolutionary Algorithms on Test Functions of Different Difficulty

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Evolutionary algorithms (EAs) have become established as *the* method at hand to explore the Pareto-optimal front in multiobjective optimization problems. This is not only because there are hardly any alternatives in the field of multiobjective optimization; due to their inherent parallelism and their capability to exploit similarities of solutions by crossover, they are able to capture several Pareto-optimal solutions in a single optimization run. The numerous applications and the rapidly growing interest in the area of multiobjective EAs take this fact into account.

After the first pioneering studies on evolutionary multiobjective optimization appeared in the mid-1980s (Schaffer 1985; Fourman 1985), a couple of different EA implementations were proposed in the years 1991–1994 (Kursawe 1991; Hajela and Lin 1992; Fonseca and Fleming 1993; Horn, Nafpliotis, and Goldberg 1994; Srinivas and Deb 1994). Later, these approaches (and variations of them) were successfully applied to various multiobjective optimization problems. In recent years, some researchers have investigated particular topics of evolutionary multiobjective search, such as convergence to the Pareto-optimal front, niching, and elitism, while others have concentrated on developing new evolutionary techniques.

In spite of this variety, there is a lack of studies which compare the performance and different aspects of the several approaches. Consequently, the question arises, which implementations are suited to which sort of problem and what are the specific advantages and drawbacks, respectively, of different techniques.

First steps in this direction have been made in both theory and practice. On the theoretical side, Fonseca and Fleming (1995) discussed the influence of different fitness assignment strategies on the selection process. On the practical side, Zitzler and Thiele (1998b, 1999) used a NP-hard 0/1 knapsack problem to compare several multiobjective EAs.

In this study[1], we provide a systematic comparison of six multiobjective EAs, including a random search strategy as well as a single-objective EA using objective aggregation. The basis of this empirical study is formed by a set of well-defined, domain-independent test functions which allow the investigation of independent problem features. We thereby draw upon results presented in (Deb 1998), where problem features that may make convergence of EAs to the Pareto-optimal front difficult are identified and, furthermore, methods of constructing appropriate test functions are suggested. The functions considered here cover the range of convexity, non-convexity, discrete Pareto fronts, multimodality, deception, and biased search spaces. Hence, we are able to systematically compare the approaches based on different kinds of difficulty and to determine more exactly where certain techniques are advantageous or have trouble. In this context, we also examine further factors such as population size and elitism.

Major results of this study are:

- The suggested test functions provide suffi-

[1]This work is described in more detail with additional extensions in (Zitzler, Deb, and Thiele 1999).

cient complexity to compare different multiobjective optimizers. Multimodality and deception seem to cause the most difficulty for evolutionary approaches. However, nonconvexity is also a problem feature which mainly weighted-sum based algorithms appear to have problems with.

- A clear hierarchy of algorithms emerges regarding the distance to the Pareto-optimal front in descending order of merit:

  1. SPEA (Zitzler and Thiele 1998a; Zitzler and Thiele 1999).
  2. NSGA (Srinivas and Deb 1994).
  3. VEGA (Schaffer 1985).
  4. HLGA (Hajela and Lin 1992)
  5. NPGA (Horn, Nafpliotis, and Goldberg 1994).
  6. FFGA (Fonseca and Fleming 1993).

  While there is a clear performance gap between SPEA and NSGA as well as between NSGA and the remaining algorithms, the fronts achieved by VEGA, HLGA, NPGA, and FFGA are rather close together. However, the results indicate that VEGA might be slightly superior to the other three EAs, while NPGA achieves fronts closer to the global optimum as FFGA. Moreover, it seems that VEGA and HLGA have difficulties evolving well-distributed trade-off fronts on the non-convex function.

- Elitism is an important factor in evolutionary multiobjective optimization. On the one hand, this statement is supported by the fact that SPEA i) clearly outperforms all algorithms on five of the six test functions and ii) is the only method among the ones under consideration which incorporates elitism as a central part of the algorithm. On the other hand, the performance of the other algorithms improved significantly when SPEA's elitist strategy was included. Preliminary results indicate that NSGA with elitism equals the performance of SPEA.

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