MODE-LD+SS: A Novel Differential Evolution Algorithm Incorporating Local Dominance and Scalar Selection Mechanisms for Multi-Objective Optimization

Alfredo Arias Montaño Carlos A. Coello Coello Efrén Mezura-Montes

***Abstract*— In this paper, we present a novel Multi-Objective Evolutionary Algorithm (MOEA) called MODE-LD+SS, which combines Differential Evolution with local dominance and a scalar selection mechanism for improving both its convergence rate and its distribution of solutions along the Pareto front. In order to assess the performance of the proposed approach, we use a set of standard test functions and performance measures taken from the specialized literature. Results are compared with respect to three MOEAs representative of the state-of-the-art in the area: NSGA-II, SPEA2, and MOEA/D.**

I. Introduction

Many real-world optimization problems require the simultaneous optimization of two or more objective functions. Such problems are called Multi-Objective Optimization Problems (MOPs). In contrast with single-objective optimization problems, MOPs do not have a single solution, but a set of them, which correspond to the best possible trade-offs among the objectives (i.e., no further improvement in one objective is possible without worsening another one). These solutions are contained in the so-called *Pareto optimal set* (the vectors of the solutions contained in the Pareto optimal set are called *nondominated*) and their corresponding objective function values are called the *Pareto front*. MOPs have been a subject of study within Operations Research for several years [19], but the limitations of mathematical programming techniques have motivated the use of evolutionary algorithms to solve them. Multi-objective evolutionary algorithms (MOEAs) have gained popularity mainly because of their generality (i.e., they require little problem-specific information), ease of use and effectivity. A wide variety of MOEAs are currently available, although few of them have become popular [5]. MOEAs aim to find solutions that are as close as possible to the true Pareto front but that, at the same time, are as diverse as possible, so that the entire Pareto front can be covered. These two goals turn out to be quite difficult in some cases, which has motivated a significant amount of research. Here, we present a MOEA called MODE-LD+SS, which is based on the use of Differential Evolution (DE) [20] as its global search engine. Our main motivation to use DE was that MOEAs based on this search engine have been found to be very effective, outperforming those based on genetic algorithms [23]. Our proposed approach incorporates two additional mechanisms. The first (local dominance) is used to improve the convergence rate towards the Pareto front, while the second (a selection mechanism based on a scalarization function) is used to find nondominated solutions covering the entire Pareto front. To assess the performance of the proposed algorithm, we adopt nine test functions (5 with two objectives and 4 with three objectives), and two performance measures taken from the specialized literature. Our results are compared with respect to the NSGA-II [6], SPEA2 [28], and MOEA/D [25], which are three MOEAs representative of the state-of-the-art in the area.

The remainder of the paper is organized as follows: In Section II some basic multiobjective optimization concepts are introduced. In Section III some previous related work is summarized. Section IV is devoted to describe the proposed approach. Then, the experimental setup is presented in Section V. In Section VI the obtained results are presented and discussed. Finally, in Section VII we provide our conclusions and some possible lines of future work.

II. Basic Concepts

A Multi-Objective Optimization Problem (MOP) can be mathematically defined as[1]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$, $j = 1, \ldots, p$ are the constraint functions of the problem.

The set of constraints of the problem defines the feasible region in the search space of the problem. Any vector of variables $\vec{x}$ which satisfies all the constraints is considered a feasible solution.

Regarding optimal solutions in MOPs, the following definitions are relevant:

Alfredo Arias Montaño and Carlos A. Coello Coello are with CINVESTAV-IPN, Departamento de Computación, Av. IPN No. 2508, Col. San Pedro Zacatenco, México, D.F., 07360, MEXICO. Carlos A. Coello Coello is also affiliated to the UMI LAFMIA 3175 CNRS at CINVESTAV-IPN. (email: aarias@computacion.cs.cinvestav.mx, ccoello@cs.cinvestav.mx).

Efrén Mezura-Montes is with LANIA A.C. Rébsamen 80, Centro, Xalapa, Veracruz, 91000, MEXICO (email: emezura@lania.mx).

[1]Without loss of generality, minimization is assumed in the following definitions, since any maximization problem can be transformed into a minimization one.

Definition 1. A vector of decision variables $\vec{x} \in \mathbb{R}^n$ dominates another vector of decision variables $\vec{y} \in \mathbb{R}^n$, (denoted by $\vec{x} \prec \vec{y}$) if and only if $\vec{x}$ is partially less than $\vec{y}$, i.e. $\forall i \in \{1, \ldots, k\}, f_i(\vec{x}) \leq f_i(\vec{y}) \wedge \exists i \in \{1, \ldots, k\} : f_i(\vec{x}) < f_i(\vec{y})$.

Definition 2. A vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. A vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto optimal}\}$$

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

The goal when solving a MOP consists on determining the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3).

III. Previous related work

DE is a simple and powerful evolutionary algorithm that has been found to outperform genetic algorithms in a variety of numerical single-objective optimization problems [20]. DE encodes solutions as vectors and uses operations such as vector addition, scalar multiplication and exchange of components (crossover) to construct new solutions from the existing ones. DE operates as follows: a newly created solution, also called *candidate*, is compared to its parent. If the candidate is better than its parent, it replaces the parent in the population; otherwise, the candidate is discarded. Being a steady-state algorithm, it implicitly enforces *elitism*, i.e., no solution from the population can be deleted unless a better solution is created. DE was originally proposed to deal with real-numbers encoding.

A. Multi-Objective Differential Evolution

DE has been adopted to solve MOPs in several ways. In the earlier approaches (PDE [1] and GDE [14]), only the concept of Pareto dominance was used to compare individuals. The parent was replaced only if it was dominated by the candidate, it was discarded otherwise. Many subsequent approaches (PDEA [17], MODE [24], NSDE [10], GDE2 [12], DEMO [21], GDE3 [13] and NSDE-DCS [11]), use nondominated sorting and/or the crowding distance metric to evaluate the fitness of the individuals. Only recently, new algorithms that do not follow the environmental selection of NSGA-II were proposed, such as $\epsilon - MyDE$ [22], DEMORS [9], $\epsilon - ODEMO$ [4], and MOEA/D-(DE) [16]. The first three algorithms of this last group, make use of the ϵ-dominance concept, as used by Laumanns et al. [15]. ϵ-dominance is adopted for spreading the solutions in a uniform manner without resorting to any crowding distance metric. Finally the MOEA/D-(DE) algorithm is based on the MOEA/D of Zhang and Li [25], but using the differential evolution operators. A comprehensive review of some these multi-objective differential evolution approaches can be found in [18].

IV. Our Proposed Approach

Algorithm 1 MODE-LD+SS

INPUT:
$P[1, \ldots, N]$ = Population
N = Population Size
F = Scaling factor
CR = Crossover Rate
$\lambda[1, \ldots, N]$ = Weight vectors
NB = Neighborhood Size
$GMAX$ = Maximum number of generations
OUTPUT:
PF = Pareto front approximation
Begin
$g \leftarrow 0$
Randomly create P_i^g , $i = 1, \ldots, N$
Evaluate P_i^g , $i = 1, \ldots, N$
while $g < GMAX$ **do**
  $\{LND\} = \{\emptyset\}$
  for $i = 1$ to N **do**
    DetermineLocalDominance(P_i^g,NB)
    if P_i^g is locally nondominated **then**
      $\{LND\} \leftarrow \{LND\} \cup P_i^g$
    end if
  end for
  for $i = 1$ to N **do**
    Randomly select $\vec{u_1}$, $\vec{u_2}$, and $\vec{u_3}$ from $\{LND\}$
    $v \leftarrow$ *CreateMutantVector*(u_1, u_2, u_3)
    $P_i^{g+1} \leftarrow$ *Crossover*(P_i^g, v)
    Evaluate P_i^{g+1}
  end for
  $Q \leftarrow P^g \cup P^{g+1}$
  Determine $z*$ for Q
  for $i = 1$ to N **do**
    $P_i^{g+1} \leftarrow$ *MinimumTchebycheff*($Q, \lambda_i, z*$)
    $Q \leftarrow Q \backslash P_i^{g+1}$
  end for
end while
ReturnPF
End

The MOEA presented in this work (called MODE-LD+SS), adopts the evolutionary operators from differential evolution. In the basic DE algorithm, and during the offspring creation stage, for each current vector $P_i \in \{P\}$, three parents (mutually different among them) $\vec{u_1}, \vec{u_2}, \vec{u_3} \in \{P\}$ ($\vec{u_1} \neq \vec{u_2} \neq \vec{u_3} \neq P_i$) are randomly selected for creating a mutant vector $\vec{v}$ using the following mutation operation:

$$\vec{v} \leftarrow \vec{u_1} + F \cdot (\vec{u_2} - \vec{u_3}) \tag{4}$$

$F > 0$, is a real constant *scaling factor* which controls the amplification of the difference $(\vec{u_2} - \vec{u_3})$. Using this mutant vector, a new offspring $P_i^{'}$ (also called trial vector in DE) is created by crossing over the mutant vector $\vec{v}$ and the current solution P_i, in accordance to:

$$P_j^{'} = \begin{cases} v_j & \text{if } (rand_j(0,1) \leq CR \text{ or } j = j_{rand} \\ P_j & \text{otherwise} \end{cases} \tag{5}$$

In the above expression, the index j refers to the jth component of the decision variables vectors. CR is a positive

constant and j_{rand} is a randomly selected integer in the range $[1, \ldots, D]$ (where D is the dimension of the solution vectors) ensuring that the offspring is different at least in one component with respect to the current solution P_i. The above DE variant is known as $Rand/1/bin$, and is the version adopted in the present work. Additionally, the proposed algorithm incorporates two mechanisms for improving both the convergence towards the Pareto front and the uniform distribution of nondominated solutions along the Pareto front. These mechanisms correspond to the concept of local dominance and the use of an environmental selection based on a scalar function. Below, we explain these two mechanisms in more detail. Algorithm 1 shows the description of our proposed MODE-LD+SS.

In Algorithm 1, the solution vectors $\vec{u_1}, \vec{u_2}, \vec{u_3}$, required for creating the trial vector $\vec{v}$ (in equation (4)), are selected from the current population, only if they are locally nondominated in their neighborhood $\aleph$. Local dominance is defined as follows:

Definition 6. Pareto Local Dominance Let $\vec{x}$ be a feasible solution, $\aleph(\vec{x})$ be a neighborhood structure for $\vec{x}$ in the decision space, and $\vec{f}(\vec{x})$ a vector of objective functions.

- We say that a solution $\vec{x}$ is locally nondominated with respect to $\aleph(\vec{x})$ if and only if there is no $\vec{x}'$ in the neighborhood of $\vec{x}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$

The neighborhood structure is defined as the NB closest individuals to a particular solution. Closeness is measured by using the Euclidean distance between solutions. The major aim of using the local dominance concept, as defined above, is to exploit good individuals' genetic information in creating DE trial vectors, and the associated offspring, which might help to improve the MOEA convergence rate toward the Pareto front. From Algorithm 1, it can be noted that this mechanism has a stronger effect during the earlier generations, where the portion of nondominated individuals is low in the global population, and progressively weakens, as the number of nondominated individuals grows during the evolutionary process. This mechanism is automatically switched off, once all the individuals in the population become nondominated, and has the possibility to be switched on, as some individuals become dominated. Aditionally, the diversity of the created offspring can be controled by the local dominance neighborhood size NB. Low values of NB will increase the diversity of offspring, and viceversa.

The second mechanism that we introduced is called *selection based on a scalar function*, and is based on the Tchebycheff scalarization function given by:

$$g(\vec{x}|\lambda, z^*) = \max_{1 \leq i \leq m} \{\lambda_i |f_i(x) - z_i^*|\} \tag{6}$$

In the above equation, $\lambda_i, i = 1, \ldots, N$ represents the set of weight vectors used to distribute the solutions along the entire Pareto front (see Figure 1). In this work, this set is calculated using the procedure described in Zhang and Li [25]. z^* corresponds to a reference point, defined in objective space and determined with the minimum objective values of the combined population Q, consistent on the actual parents and the created offspring. This reference point is updated at each generation, as the evolution progresses. The procedure *MinimumTchebycheff*($Q, \lambda_i, z*$) finds, from the set Q, the solution vector that minimizes equation (6) for each weight vector λ_i and the reference point z^*.

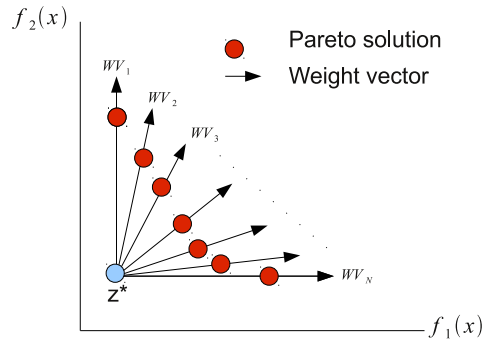

Fig. 1. Distribution of the weight vectors

V. Experimental setup

In order to validate the proposed approach, our results are compared with respect to those of NSGA-II [6], SPEA2 [28], and MOEA/D [25], which are three MOEAs representative of the state-of-the-art in evolutionary multiobjective optimization. Our approach was validated using nine test problems: five from the ZDT (Zitzler-Deb-Thiele) test suite [27] each with 2 objectives (ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6), and four more from the DTLZ (Deb-Thiele-Laumanns-Zitzler) test suite [8], each with 3 objectives (DTLZ1, DTLZ2, DTLZ3, and DTLZ4). The selected test functions comprise different difficulties such as convex, concave, and disconnected Pareto fronts, as well as problems with multiple fronts. The details of these test problems are omitted here due to space constraints, but can be found in [26], [7], [5].

Two performance measures were adopted in order to assess our results: *Hypervolume (Hv)* and *Two Set Coverage (C-Metric)*. A brief description of them is presented next.

A. Hypervolume (Hv):

Given a Pareto approximation set PF_{known}, and a reference point in objective space z_{ref}, this performance measure estimates the *Hypervolume* attained by it. Such hypervolume corresponds to the non-overlaping volume of all the hypercubes formed by the reference point (z_{ref}) and every vector in the Pareto set approximation. This is mathematically defined as:

$$HV = \{\cup_i vol_i | vec_i \in PF_{known}\}$$

vec_i is a nondominated vector from the Pareto set approximation, and vol_i is the volume for the hypercube formed by the reference point and the nondominated vector vec_i. Here, the reference point (z_{ref}) in objective space for the 2-objective MOPs was set to (1.05,1.05), for DTLZ1 was set to

(0.6,0.6,0.6), and to (1.05,1.05,1.05) for DTLZ2, DTLZ3 and DTLZ4. This performance measure is Pareto compliant [29], [30], and is used to assess both convergence and distribution of the solutions along the approximated Pareto front. High values indicate that the solutions are closer to the true Pareto front and that they cover a wider extension of it.

B. Two Set Coverage (C-Metric):

This performance measure is also Pareto compliant, and estimates the coverage proportion, in terms of percentage of dominated solutions, between two sets. Given the sets A and B, both containing only nondominated solutions, the C-Metric is mathematically defined as:

$$C(A,B) = \frac{|\{u \in B | \exists v \in A : v \; dominates \; u\}|}{|B|}$$

This metric indicates the portion of vectors in B being dominated by any vector in A. In the present work this measure is used in two different ways. In the first, the set A is the true Pareto front, which is knonw for all test functions used; therefore, the C-Metric can be considered as a measure for the ability of the algorithm to find solutions that are nondominated with respect to the Pareto optimal set (i.e., solutions that also belong to the Pareto optimal set). In the second way, sets A and B correspond to two different Pareto approximations, as obtained by two different algorithms. Therefore, the C-Metric is used for pairwise comparisons between the two algorithms used.

C. Parameters settings:

The parameters used in the experiments for the different algorithms adopted were set as follows. The common parameters for all algorithms comprise the population size N and maximum number of generations $GMAX$. These were set to N = 100 for all the bi-objectives MOPs and N = 300 for all the MOPs having three objectives. We adopted $GMAX$ = 150 for all MOPs, except for ZDT4 and DTLZ3, in which we used $GMAX$ = 200. As for specific parameters of each algorithm, for both, the NSGA-II and the MOEA/D algorithms, some common parameters used were: Crossover probability $p_c = 1.0$; mutation probability $p_m = 1/NVARS$; and, distribution index for mutation $\eta_m = 20$. For the NSGA-II, distribution index for crossover $\eta_c = 15$; while for the MOEA/D, distribution index for crossover $\eta_c = 20$. SPEA2 was taken from PISA [2], [3], and was used with the parameters defined therein:

individual_mutation_probability = 1.0;
individual_recombination_probability = 1.0;
variable_mutation_probability = 1/NVARS;
variable_swap_probability = 0.5;
variable_recombination_probability = 0.5;
distribution index for crossover $\eta_c = 15$;
distribution index for mutation $\eta_m = 20$;
use_symmetric_recombination = 0.

For our MODE-LD+SS, the associated parameters were the following: Scaling factor, F = 0.5 for all MOPs; crossover rate, CR = 0.5 for all MOPs, except for ZDT4 and DTLZ3, where we adopted CR = 0.3; Neighborhood size NB = 5 for all MOPs, except for ZDT4, where NB = 1 was used. The statistics presented for the Hypervolume (Hv) and the C-Metric, when measured with respect to the true Pareto front, were obtained as average values from 32 independent runs for each MOP and for each algorithm. In the case of the statistics for the C-Metric comparing pairs of algorithms (i.e. C-Metric(A,B)), they were obtained as average values of the comparison of all the independent runs from the first algorithm with respect to all the independent runs from the second algorithm.

VI. Results and Discussion

In this section, we present the results obtained by the proposed algorithm MODE-LD+SS, for the nine selected test functions. We also present the comparison with respect to the results attained by NSGA-II, SPEA2, and MOEA/D.

Table I shows the results obtained for the Hypervolume (Hv) measure for all MOPs, and for the four algorithms compared in this paper. From this table it can be observed that, with respect to the Hv performance measure, MODE-LD+SS outperforms NSGA-II and SPEA2, in all the bi-objective MOPs. It also outperforms MOEA/D (with respect to Hv as well) in four (ZDT1, ZDT2, ZDT3, and ZDT6) of five bi-objective MOPs. In the case of the 3-objective MOPs, SPEA2 attains the best results for the Hv measure in three MOPs (DTLZ1, DTLZ2, and DTLZ4), while MODE-LD+SS attains the best result in DTLZ3. However, our proposed MODE-LD+SS obtained values very close to those of SPEA2 in DTLZ1, DTLZ2 and DTLZ4 and better values in DTLZ2 and DTLZ3, as compared to those of MOEA/D. In all cases, our proposed approach outperforms NSGA-II by an ample margin.

Tables II to X show the comparison matrices for the C-Metric values obtained with the different algorithms and for all the MOPs used in the experiments. The diagonal values of each matrix correspond to the C-Metric for each algorithm, as evaluated with respect to the true Pareto front (i.e. C-Metric(PF_{true},Algorithm)); while the off-diagonal elements correspond to the comparisons between each pair of algorithms. From these tables, it can be observed that MODE-LD+SS significantly outperforms all other algorithms in terms of convergence. MODE-LD+SS is able to converge closer to the true Pareto front in all MOPs, and to generate Pareto front approximations, having fewer solutions (none in many cases) being dominated by those generaterd by the other algorithms. Additionally, our proposed MODE-LS+SS generated more solutions that dominate those generated by the other algorithms. It is also important to note that for ZDT6, our proposed MODE-LD+SS, was able to reach the true Pareto front in the 32 independent runs performed.

For the case of DTLZ1 and DTLZ2, and regarding the C-Metric values presented in Tables VII and VIII, it can be observed that MODE-LD+SS is able to converge very close to the true Pareto front as indicated by the corresponding C-Metric measure.

TABLE I

COMPARISON OF THE HYPERVOLUME METRIC (HV) FOR ALL THE ALGORITHMS

Test Function	ALGORITHM							
	NSGA		SPEA2		MOEA/D		MODE-LD+SS	
	Mean	σ	Mean	σ	Mean	σ	Mean	σ
ZDT1	0.757357	0.000928	0.761644	0.000556	0.749964	0.009777	**0.763442**	0.000112
ZDT2	0.422221	0.001263	0.321971	0.171286	0.387237	0.061361	**0.430358**	0.000141
ZDT3	0.611480	0.008038	0.615533	0.000416	0.608377	0.015638	**0.616381**	0.000150
ZDT4	0.217626	0.192914	0.287359	0.188726	**0.745887**	0.009983	0.741770	0.058697
ZDT6	0.345949	0.008772	0.392697	0.002336	0.397720	0.002886	**0.411054**	0.000003
DTLZ1	0.165918	0.026090	**0.191437**	0.000248	0.188726	0.000371	0.187445	0.000347
DTLZ2	0.571146	0.001942	**0.590833**	0.000900	0.578679	0.001460	0.581028	0.001193
DTLZ3	0.000000	0.000000	0.467163	0.148867	0.568895	0.007220	**0.581129**	0.003303
DTLZ4	0.572327	0.002537	**0.590942**	0.000978	0.579301	0.001550	0.578038	0.001840

TABLE II

C-METRIC(A,B) FOR ZDT1

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	0.968750 (0.013854)	0.000771 (0.003989)	0.033167 (0.034072)	0.000000 (0.000000)
SPEA2	0.378115 (0.115819)	0.895000 (0.036100)	0.106198 (0.067101)	0.000000 (0.000000)
MOEA/D	0.299833 (0.107821)	0.047767 (0.042724)	0.883930 (0.064323)	0.000000 (0.000000)
MODE-LD+SS	0.589893 (0.088597)	0.214844 (0.064899)	0.274901 (0.100480)	**0.374333** (0.0783750

TABLE III

C-METRIC(A,B) FOR ZDT2

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	1.000000 (0.000000)	0.000303 (0.001877)	0.026203 (0.035988)	0.000000 (0.000000)
SPEA2	0.362813 (0.227343)	0.985938 (0.037232)	0.041712 (0.045770)	0.004331 (0.007530)
MOEA/D	0.450922 (0.164440)	0.154067 (0.121784)	0.393976 (0.170768)	0.057031 (0.030938)
MODE-LD+SS	0.702266 (0.086673)	0.242832 (0.167042)	0.110288 (0.061098)	**0.381057** (0.064387)

TABLE IV

C-METRIC(A,B) FOR ZDT3

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	0.656875 (0.075666)	0.002246 (0.008387)	0.064717 (0.073280)	0.000000 (0.000000)
SPEA2	0.339297 (0.106685)	0.389375 (0.065102)	0.142818 (0.092675)	0.000067 (0.000958)
MOEA/D	0.221500 (0.070634)	0.082778 (0.037489)	0.389439 (0.106475)	0.023824 (0.025038)
MODE-LD+SS	0.377051 (0.073449)	0.171533 (0.046961)	0.299007 (0.112709)	**0.199554** (0.039211)

TABLE V

C-METRIC(A,B) FOR ZDT4

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	1.000000 (0.000000)	0.301200 (0.455773)	0.002571 (0.004654)	0.000166 (0.001994)
SPEA2	0.546084 (0.489198)	1.000000 (0.000000)	0.003022 (0.004910)	0.000566 (0.003887)
MOEA/D	0.977144 (0.019361)	0.938814 (0.176612)	0.952296 (0.083845)	0.164757 (0.368419)
MODE-LD+SS	0.988408 (0.106452)	0.976602 (0.151037)	0.689329 (0.359937)	**0.220064** (0.357158)

TABLE VI

C-METRIC(A,B) FOR ZDT6

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	0.986873 (0.004523)	0.000000 (0.000000)	0.001372 (0.003499)	0.000000 (0.000000)
SPEA2	1.000000 (0.000000)	0.990000 (0.000000)	0.040134 (0.076380)	0.000000 (0.000000)
MOEA/D	0.986999 (0.007570)	0.615444 (0.272926)	0.990552 (0.003241)	0.000000 (0.000000)
MODE-LD+SS	0.992119 (0.005944)	0.990000 (0.000000)	0.976816 (0.051151)	**0.000000** (0.000000)

TABLE VII

C-METRIC(A,B) FOR DTLZ1

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	0.655461 (0.143824)	0.001915 (0.003127)	0.000095 (0.001111)	0.000000 (0.000000)
SPEA2	0.707633 (0.234981)	0.258360 (0.100637)	0.012861 (0.027180)	0.000000 (0.000000)
MOEA/D	0.377986 (0.130387)	0.019929 (0.017216)	0.163130 (0.102328)	0.0005383 (0.001719)
MODE-LD+SS	0.611632 (0.243895)	0.045080 (0.019503)	0.263892 (0.110080)	**0.008116** (0.004630)

TABLE VIII

C-METRIC(A,B) FOR DTLZ2

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	0.354375 (0.031910)	0.027106 (0.009214)	0.000000 (0.000000)	0.000000 (0.000000)
SPEA2	0.044411 (0.012929)	0.806858 (0.029297)	0.000000 (0.000000)	0.000000 (0.000000)
MOEA/D	0.722926 (0.013865)	0.071016 (0.013078)	0.142447 (0.023822)	0.005816 (0.005185)
MODE-LD+SS	0.082272 (0.014202)	0.078098 (0.013666)	0.008309 (0.012255)	**0.074566** (0.017111)

TABLE IX

C-METRIC(A,B) FOR DTLZ3

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	1.000000 (0.000000)	0.000221 (0.000868)	0.000000 (0.000000)	0.000000 (0.000000)
SPEA2	0.877437 (0.163622)	0.798140 (0.107026)	0.010756 (0.042299)	0.001108 (0.008432)
MOEA/D	0.418284 (0.084607)	0.380625 (0.265565)	0.639418 (0.181815)	0.008286 (0.022365)
MODE-LD+SS	0.977820 (0.041731)	0.535140 (0.262711)	0.370058 (0.158495)	**0.339882** (0.128920)

TABLE X

C-METRIC(A,B) FOR DTLZ4

C-Metric(A,B)	NSGA-II Mean (σ)	SPEA2 Mean (σ)	MOEA/D Mean (σ)	MODE-LD+SS Mean (σ)
NSGA-II	0.361563 (0.038679)	0.026370 (0.010174)	0.000008 (0.000180)	0.000000 (0.000000)
SPEA2	0.043145 (0.014076)	0.746696 (0.020173)	0.000000 (0.000000)	0.000000 (0.000000)
MOEA/D	0.076018 (0.019680)	0.067166 (0.014841)	0.124613 (0.035191)	0.009343 (0.006096)
MODE-LD+SS	0.077891 (0.019727)	0.077581 (0.016563)	0.001519 (0.002431)	**0.107422** (0.006116)

These results contrast with the Hv measure obtained by SPEA2 for these same MOPs. The differences can be explained by the fact that SPEA2 obtained a better distribution of solutions. Thus, in this case, one algorithm provided better convergence (MODE-LD+SS), while the other provided better spread of solutions (SPEA2) (see Figures 7 and 8). Figures 2 to 10 show the comparison of the obtained Pareto fronts by the four MOEAs, for all the MOPs adopted in our study. It is worth noting that for the two-objective MOPs, and for comparison purposes, the Pareto front approximations are plotted with a horizontal and vertical shifts.

VII. CONCLUSIONS AND FUTURE WORK

We have introduced a new MOEA called MODE-LD+SS, which combines differential evolution with local dominance and scalar selection mechanisms. Local dominance aims to improve the convergence rate and the scalar selection mechanism intends to improve the distribution of solutions along the Pareto front. In order to assess the performance of our proposed approach, we adopted 9 test problems and two performance measures (Hypervolume and C-Metric) taken from the specialized literature. Our results were compared with respect to those produced by NSGA-II, SPEA2, and MOEA/D, which are elitist MOEAs representative of the state-of-the-art in the area.

Our comparative study showed that our proposed MODE-LD+SS outperforms NSGA-II, SPEA2 and MOEA/D in 5 of the 9 MOPs used with respect to the Hypervolume. Our approach was also found to be competitive with respect to SPEA2 and MOEA/D in those 3-objective MOPs in which it did not attain the best Hypervolume measure (DTLZ1, DTLZ2 and DTLZ4). Regarding the C-Metric, our proposed MODE-LD+SS outperformed NSGA-II, SPEA2 and MOEA/D in all the 9 MOPs adopted. Based on these results, we can conclude that our proposed approach has good convergence properties.

As part of our future work, we are interested in undertaking a thorough statistical analysis of the performance of our proposed approach, including an analysis of variance that allows us to determine its most suitable parameter values. We also intend to apply our proposed approach to real-world problems to see if its good convergence properties remain valid in practical applications.

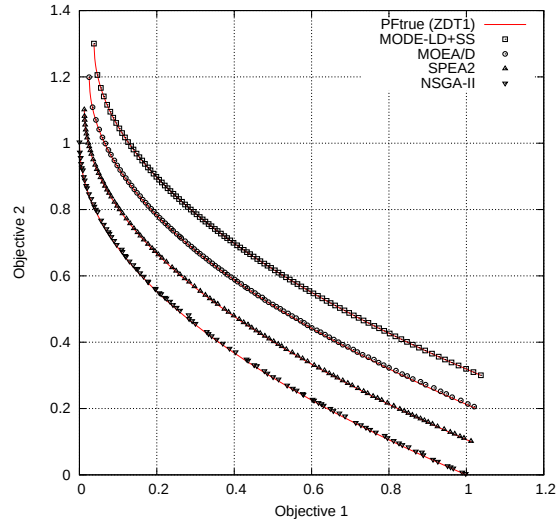

Fig. 2. Pareto front obtained by the different algorithms for ZDT1 MOP.

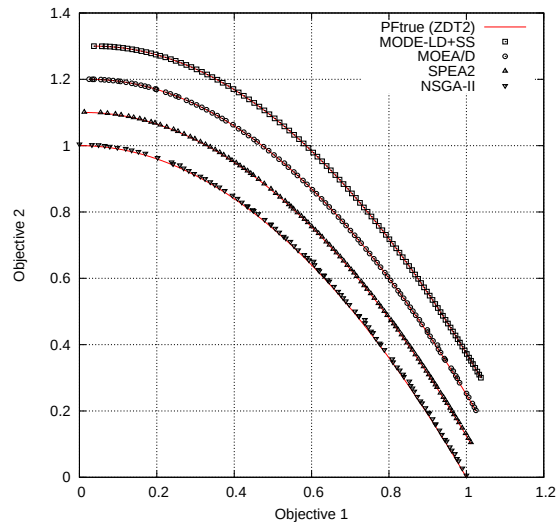

Fig. 3. Pareto front obtained by the different algorithms for ZDT2 MOP.

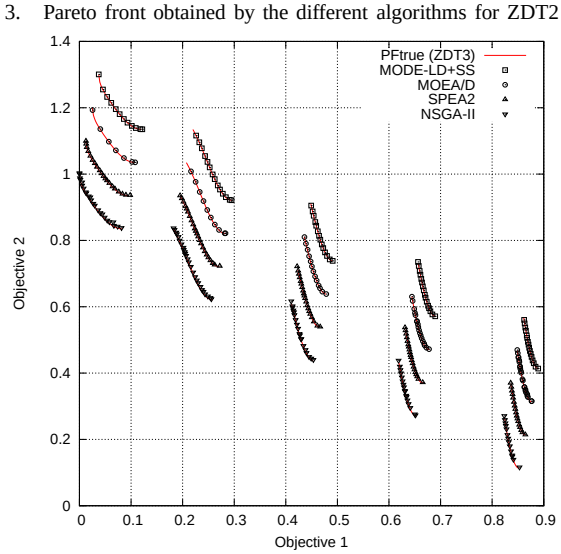

Fig. 4. Pareto front obtained by the different algorithms for ZDT3 MOP.

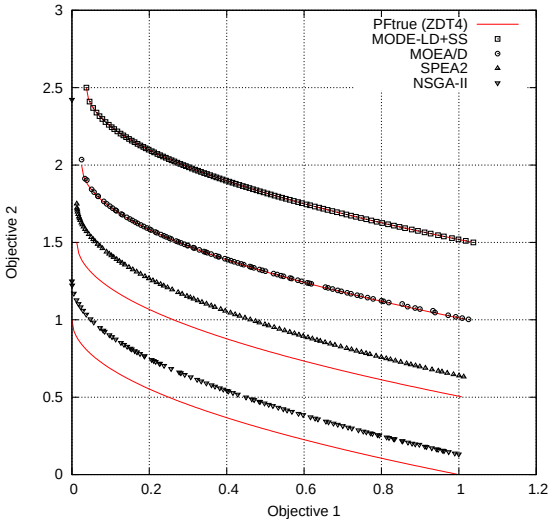

Fig. 5. Pareto front obtained by the different algorithms for ZDT4 MOP.

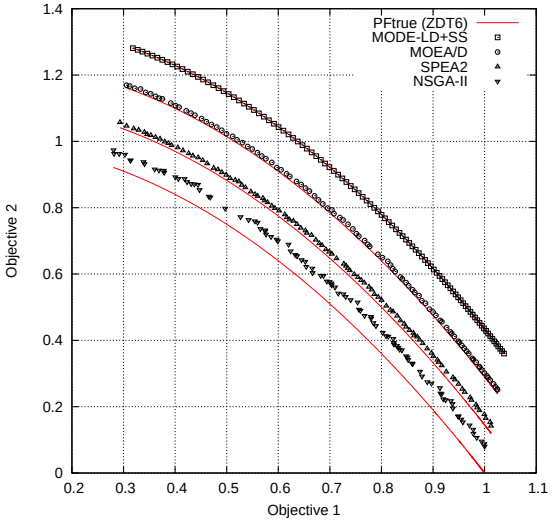

Fig. 6. Pareto front obtained by the different algorithms for ZDT6 MOP.

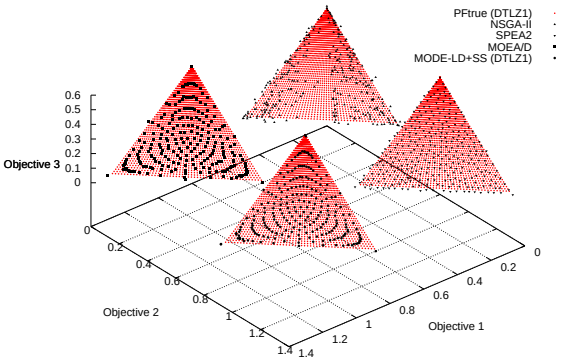

Fig. 7. Pareto front obtained by the different algorithms for DTLZ1 MOP.

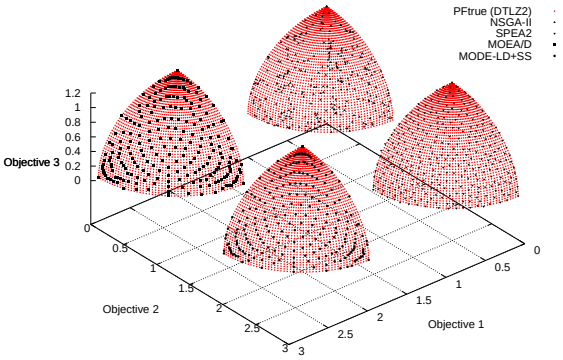

Fig. 8. Pareto front obtained by the different algorithms for DTLZ2 MOP.

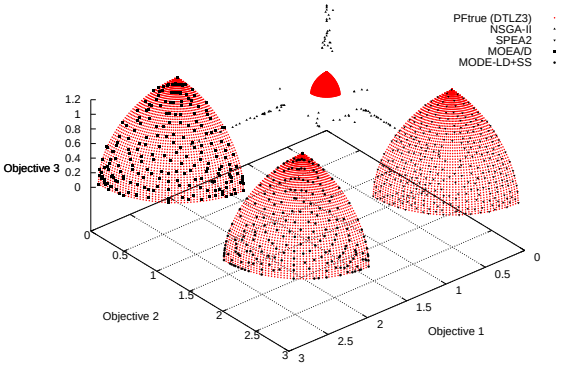

Fig. 9. Pareto front obtained by the different algorithms for DTLZ3 MOP.

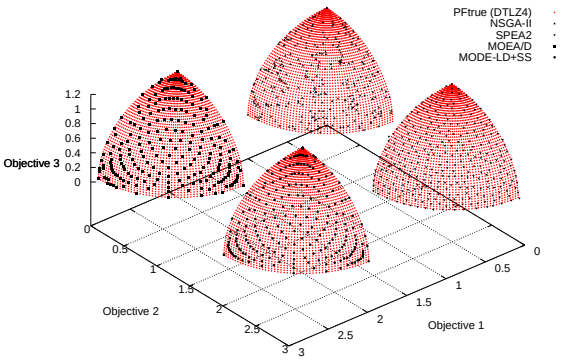

Fig. 10. Pareto front obtained by the different algorithms for DTLZ4 MOP.

ACKNOWLEDGMENTS

The first author acknowledges support from CONACyT and IPN to pursue graduate studies at the Computer Science Department of CINVESTAV-IPN. The second author acknowledges support from CONACyT project no. 103570.

REFERENCES

[1] H. A. Abbass, R. Sarker, and C. Newton. PDE: A Pareto-frontier Differential Evolution Approach for Multi-objective Optimization Problems. In *Proceedings of the Congress on Evolutionary Computation 2001 (CEC'2001)*, volume 2, pages 971–978, Piscataway, New Jersey, May 2001. IEEE Service Center.

[2] S. Bleuler, M. Laumanns, L. Thiele, and E. Zitzler. PISA — A Platform and Programming Language Independent Interface for Search Algorithms. TIK Report 154, Computer Engineering and Networks Laboratory (TIK), ETH Zurich, October 2002.

[3] S. Bleuler, M. Laumanns, L. Thiele, and E. Zitzler. PISA—A Platform and Programming Language Independent Interface for Search Algorithms. In C. M. Fonseca, P. J. Fleming, E. Zitzler, K. Deb, and L. Thiele, editors, *Evolutionary Multi-Criterion Optimization. Second International Conference, EMO 2003*, pages 494–508, Faro, Portugal, April 2003. Springer. Lecture Notes in Computer Science. Volume 2632.

[4] Z. Cai, W. Gong, and Y. Huang. A Novel Differential Evolution Algorithm Based on ϵ-Domination and Orthogonal Design Method for Multiobjective Optimization. In S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, and T. Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 286–301, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.

[5] C. A. Coello Coello, G. B. Lamont, and D. A. Van Veldhuizen. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Springer, New York, second edition, September 2007. ISBN 978-0-387-33254-3.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA–II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, April 2002.

[7] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable Multi-Objective Optimization Test Problems. In *Congress on Evolutionary Computation (CEC'2002)*, volume 1, pages 825–830, Piscataway, New Jersey, May 2002. IEEE Service Center.

[8] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable Test Problems for Evolutionary Multiobjective Optimization. In A. Abraham, L. Jain, and R. Goldberg, editors, *Evolutionary Multiobjective Optimization. Theoretical Advances and Applications*, pages 105–145. Springer, USA, 2005.

[9] A. G. Hernández-Díaz, L. V. Santana-Quintero, C. Coello Coello, R. Caballero, and J. Molina. A New Proposal for Multi-Objective Optimization using Differential Evolution and Rough Sets Theory. In M. K. et al., editor, *2006 Genetic and Evolutionary Computation Conference (GECCO'2006)*, volume 1, pages 675–682, Seattle, Washington, USA, July 2006. ACM Press. ISBN 1-59593-186-4.

[10] A. W. Iorio and X. Li. Solving rotated multi-objective optimization problems using differential evolution. In *AI 2004: Advances in Artificial Intelligence, Proceedings*, pages 861–872. Springer-Verlag, Lecture Notes in Artificial Intelligence Vol. 3339, 2004.

[11] A. W. Iorio and X. Li. Incorporating Directional Information within a Differential Evolution Algorithm for Multi-objective Optimization. In M. K. et al., editor, *2006 Genetic and Evolutionary Computation Conference (GECCO'2006)*, volume 1, pages 691–697, Seattle, Washington, USA, July 2006. ACM Press. ISBN 1-59593-186-4.

[12] S. Kukkonen and J. Lampinen. An Extension of Generalized Differential Evolution for Multi-objective Optimization with Constraints. In *Parallel Problem Solving from Nature - PPSN VIII*, pages 752–761, Birmingham, UK, September 2004. Springer-Verlag. Lecture Notes in Computer Science Vol. 3242.

[13] S. Kukkonen and J. Lampinen. GDE3: The third Evolution Step of Generalized Differential Evolution. In *2005 IEEE Congress on Evolutionary Computation (CEC'2005)*, volume 1, pages 443–450, Edinburgh, Scotland, September 2005. IEEE Service Center.

[14] J. Lampinen. De's selection rule for multiobjective optimization. Technical report, Lappeenranta University of Technology, 2001.

[15] M. Laumanns, L. Thiele, E. Zitzler, and K. Deb Archiving with Guaranteed Convergence and Diversity in Multi-Objective Optimization. In W.B. Langdon et al., editors, *2002 Genetic and Evolutionary Computation Conference (GECCO'2002)*, pages 439–447, San Francisco, California, USA, July 2002.

[16] H. Li and Q. Zhang. Multiobjective Optimization Problems With Complicated Pareto Sets, MOEA/D and NSGA-II. *IEEE Transactions on Evolutionary Computation*, 13(2):284–302, April 2009.

[17] N. K. Madavan. Multiobjective Optimization Using a Pareto Differential Evolution Approach. In *Congress on Evolutionary Computation (CEC'2002)*, volume 2, pages 1145–1150, Piscataway, New Jersey, May 2002. IEEE Service Center.

[18] E. Mezura-Montes, M. Reyes-Sierra, and C. A. Coello Coello. Multi-Objective Optimization using Differential Evolution: A Survey of the State-of-the-Art. In U. K. Chakraborty, editor, *Advances in Differential Evolution*, pages 173–196. Springer, Berlin, 2008. ISBN 978-3-540-68827-3.

[19] K. Miettinen. *Nonlinear Multiobjective Optimization*. Kluwer Academic Publishers, Boston, Massachuisetts, 1999.

[20] K. V. Price, R. M. Storn, and J. A. Lampinen. *Differential Evolution. A Practical Approach to Global Optimization*. Springer, Berlin, 2005. ISBN 3-540-20950-6.

[21] T. Robič and B. Filipič. DEMO: Differential Evolution for Multiobjective Optimization. In C. A. Coello Coello, A. Hernández Aguirre, and E. Zitzler, editors, *Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005*, pages 520–533, Guanajuato, México, March 2005. Springer. Lecture Notes in Computer Science Vol. 3410.

[22] L. V. Santana-Quintero and C. A. Coello Coello. An Algorithm Based on Differential Evolution for Multiobjective Problems. In C. H. Dagli, A. L. Buczak, D. L. Enke, M. J. Embrechts, and O. Ersoy, editors, *Smart Engineering System Design: Neural Networks, Evolutionary Programming and Artificial Life*, volume 15, pages 211–220, St. Louis, Missouri, USA, November 2005. ASME Press.

[23] T. Tušar and B. Filipič. Differential Evolution Versus Genetic Algorithms in Multiobjective Optimization. In S. Obayashi, K. Deb, C. Poloni, T. Hiroyasu, and T. Murata, editors, *Evolutionary Multi-Criterion Optimization, 4th International Conference, EMO 2007*, pages 257–271, Matshushima, Japan, March 2007. Springer. Lecture Notes in Computer Science Vol. 4403.

[24] F. Xue, A. C. Sanderson, and R. J. Graves. Pareto-based Multi-Objective Differential Evolution. In *Proceedings of the 2003 Congress on Evolutionary Computation (CEC'2003)*, volume 2, pages 862–869, Canberra, Australia, December 2003. IEEE Press.

[25] Q. Zhang and H. Li. MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition. *IEEE Transactions on Evolutionary Computation*, 11(6):712–731, December 2007.

[26] E. Zitzler, K. Deb, and L. Thiele. Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. Technical Report 70, Computer Engineering and Networks Laboratory (TIK), Swiss Federal Institute of Technology (ETH) Zurich, Gloriastrasse 35, CH-8092 Zurich, Switzerland, December 1999.

[27] E. Zitzler, K. Deb, and L. Thiele. Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evolutionary Computation*, 8(2):173–195, Summer 2000.

[28] E. Zitzler, M. Laumanns, and L. Thiele. SPEA2: Improving the Strength Pareto Evolutionary Algorithm. In K. Giannakoglou, D. Tsahalis, J. Periaux, P. Papailou, and T. Fogarty, editors, *EUROGEN 2001. Evolutionary Methods for Design, Optimization and Control with Applications to Industrial Problems*, pages 95–100, Athens, Greece, 2002.

[29] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. *IEEE Transactions on Evolutionary Computation*, 7(2):117–132, April 2003.

[30] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. Grunert da Fonseca. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. Technical Report 139, Computer Engineering and Networks Laboratory, ETH Zurich, June 2002.