

# Effective Ranking + Speciation = Many-Objective Optimization

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**Abstract**—Multiobjective optimization problems have been widely addressed using evolutionary computation techniques. However, when dealing with more than three conflicting objectives (the so-called many-objective problems), the performance of such approaches is known to deteriorate. The problem lies in the inability of Pareto dominance to provide an effective discrimination. Alternative ranking methods have been successfully used to cope with this issue. Nevertheless, the high selection pressure associated with these approaches usually leads to diversity loss. In this study, we focus on parallel genetic algorithms, where multiple partially isolated subpopulations are evolved concurrently. As in nature, isolation leads to speciation, the process by which new species arise. Thus, evolving multiple subpopulations can be seen as a potential source of diversity and it is known to improve the search performance of genetic algorithms. Such a behavior, integrated with an effective ranking, is expected to be suitable for many-objective optimization.

## I. INTRODUCTION

Multiobjective optimization problems arise in many scientific and engineering applications, where multiple conflicting goals are required to be simultaneously satisfied. Rather than searching for a single optimal solution, the task in multi-objective optimization is to find a set of trade-offs among the competing objectives. Evolutionary algorithms (EAs) have demonstrated to be very successful approaches to face such problems. The population-based nature of EAs allows them to simultaneously explore different regions of the search space and to generate several elements of the Pareto optimal set within a single execution.

Nevertheless, when dealing with more than three objectives, the so-called *many-objective optimization problems* [1], the performance of even the most popular multiobjective EAs (MOEAs) is known to deteriorate [2], [3], [4]. Such a scalability problem can be explained through the fact that *Pareto dominance* (PD) [5] loses its discrimination potential as the number of objective functions increases. As a consequence, no preferences can be set among individuals (potential solutions) for selection purposes, leading PD-based MOEAs to perform an almost random search.

PD's drawbacks have motivated researchers to use alternative discrimination mechanisms to enhance the performance of

MOEAs when solving many-objective problems [6], [7], [8], [9], [10], [11], [12]. In our previous work [10], [11], [12], we performed a series of comparative experiments to investigate the advantages of using several of such alternative approaches.

One of our main findings was that an effective ranking of solutions allows to improve convergence in many-objective optimization. An effective ranking is that providing a fine-grained discrimination. It should be taken into account how better solutions are from each other in each objective. Discarding this information can lead to wrong discrimination decisions and can negatively affect the convergence behavior of MOEAs.

Nevertheless, a fine-grained discrimination entails a high selection pressure, which tends to sacrifice genetic diversity. Diversity loss not only has a detrimental impact on the exploratory capabilities of MOEAs, but also prevents their convergence towards a representative approximation of the Pareto optimal surface. Satisfying both the convergence and diversity requirements for many-objective optimizers is known to be, by itself, a multiobjective problem. A better diversity is commonly associated with a poorer proximity [13]. However, convergence is usually prioritized over diversity. In fact, a poorly spread set of solutions which are close to the Pareto front would be rather preferable than a well-spread set of points which are far from it [8].

In this study, we investigate the suitability of a class of parallel genetic algorithms (PGAs) for many-objective optimization. In such PGAs, individuals are organized into multiple subpopulations which evolve in isolation most of the time. As in nature, isolated subpopulations are expected to evolve in different directions, allowing new species to arise. This process is known as *speciation*. Thus, evolving multiple (partially isolated) subpopulations can be seen as a potential source of diversity and it is known to improve the way genetic algorithms explore search spaces. In our approach, subpopulations are evolved by means of a genetic algorithm which is driven by a fine-grained ranking to promote a convergent behavior. In this study, the aim of using PGAs is not to improve computational efficiency. Instead, we focus on the impact that the subpopulations scheme could have on the outcome of the optimization process.

The remainder of this paper is organized as follows. Background concepts are provided in Section II. Section III describes the implemented PGA. Our experimental results are discussed in Section IV. Finally, Section V provides our conclusions as well as some possible directions for future research.

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## II. BACKGROUND

### A. Multiobjective optimization

A *multiobjective optimization problem* (MOP) can be formally stated as follows:<sup>1</sup>

$$\begin{aligned} &\text{Minimize} && \mathbf{F}(\mathbf{X}) = [f_1(\mathbf{X}), f_2(\mathbf{X}), \dots, f_M(\mathbf{X})]^T \\ &\text{subject to} && \mathbf{X} \in \mathcal{F} \end{aligned} \quad (1)$$

where  $\mathbf{X}$  is a *decision vector* with  $n$  *decision variables*,  $\mathbf{F}(\mathbf{X})$  is the  $M$ -dimensional *objective vector* ( $M \geq 2$ ),  $f_m(\mathbf{X})$  is the  $m$ -th objective function ( $f_m : \mathbb{R}^n \rightarrow \mathbb{R}$ ) and  $\mathcal{F}$  is the feasible region. Here, we are interested in solving *many-objective optimization problems*; that is, the subset of MOPs involving  $M > 3$  objectives [1].

In multiobjective optimization we wish to determine, from among all  $\mathbf{X} \in \mathcal{F}$ , the particular  $\mathbf{X}^*$  which yields the optimum values for all the objective functions. However, it is unusual that there is a single solution simultaneously optimizing all the (conflicting) objectives. Instead, we are interested in finding a set of *trade-off solutions*. The most commonly adopted notion of optimality is the so-called *Pareto optimality* [5].

Let us first define the *Pareto dominance* (PD) relation. Given two solutions  $\mathbf{X}, \mathbf{Y} \in \mathcal{F}$ , we say that  $\mathbf{X}$  *Pareto-dominates*  $\mathbf{Y}$ , denoted by  $\mathbf{X} \prec \mathbf{Y}$ , if and only if:

$$\begin{aligned} &\forall m \in \{1, 2, \dots, M\} : f_m(\mathbf{X}) \leq f_m(\mathbf{Y}) \quad \wedge \\ &\exists m \in \{1, 2, \dots, M\} : f_m(\mathbf{X}) < f_m(\mathbf{Y}) \end{aligned} \quad (2)$$

otherwise, we say that  $\mathbf{Y}$  is *nondominated* with respect to  $\mathbf{X}$ . We say that a point  $\mathbf{X}^* \in \mathcal{F}$  is *Pareto optimal* if there is no  $\mathbf{X} \in \mathcal{F}$  such that  $\mathbf{X} \prec \mathbf{X}^*$ . The set of all  $\mathbf{X}^* \in \mathcal{F}$  satisfying this condition constitutes the *Pareto optimal set*, whose image in objective space is called *Pareto front* or *trade-off surface*.

### B. Pareto dominance in many-objective optimization

Pareto dominance (PD) is known to become ineffective as the number of optimization criteria raises. Figure 1 shows how the proportion of nondominated solutions in the population behaves with respect to the number of objectives and as the search progresses. We adopted a well-known scalable test

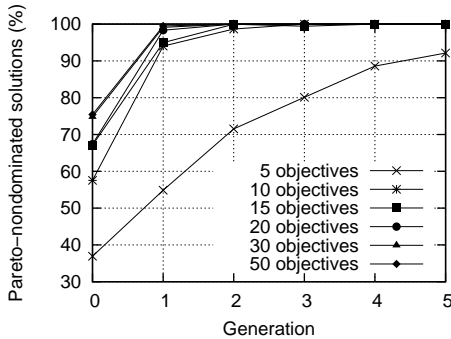


Fig. 1. Proportion of Pareto-nondominated solutions.

<sup>1</sup>In this study, we assume that all objectives are equally important and, without loss of generality, we will refer only to minimization problems.

problem, DTLZ1 [14], and a basic MOEA (described later in Section III-B) with a population of  $N = 100$  individuals. 31 independent executions were performed.

From Figure 1, we can clearly see that the increment in the number of objectives raises the proportion of nondominated individuals. Even in the case of the initial population (at generation zero), which is randomly generated, a high percentage of the population corresponds to nondominated solutions. After a few generations, the population became completely nondominated in all cases. Thus, no preferences can be set among individuals for selection purposes, leading the algorithm to perform practically a random search. Recently, alternative ranking approaches have been adopted to cope with this issue [11], [11], [12], [6], [7], [8], [9].

### C. Parallel genetic algorithms

There are different approaches to parallelize genetic algorithms (GAs) [15], [16]. On the one hand, the evaluation of the population's individuals as well as the application of the genetic operators may be explicitly distributed among multiple processing units. This is the so-called *global parallelization*, where computational efficiency can be improved while the behavior of the algorithm remains unchanged.

On the other hand, some other parallelization strategies introduce fundamental changes in the way GAs explore search spaces. This is the case of *coarse-grained parallel GAs*, also referred to as *island model GAs*, *GAs based on punctuated equilibria*, or just *parallel GAs* (PGAs). The GA's population is partitioned into multiple *subpopulations*, or *demes*, which evolve in isolation for a period known as an *epoch*. At the end of each epoch, some individuals are copied from one deme to another through a process known as *migration*.

Just as occurs in nature due to geographic isolation, each independently evolving deme is expected to follow a different search trajectory, leading to *speciation*. In population genetics, speciation refers to the process through which new species arise. This can be seen as a potential diversity promotion mechanism. Such a behavior may allow the efficient exploration of large search spaces and it has been successfully exploited to deal with, for example, linearly separable [17], multimodal [18] and even multiobjective optimization problems [19].

The performance of PGAs depends upon several decisions such as the number and size of the demes and the migration policies; *i.e.*, which and how many individuals to migrate, how often (epoch length) and between what demes can individuals be exchanged (*interconnection topology*).

## III. A PARALLEL GENETIC ALGORITHM FOR MANY-OBJECTIVE OPTIMIZATION

We implemented a coarse-grained parallel genetic algorithm (PGA). In PGAs (see Section II-C), the population is divided into multiple subpopulations or demes which evolve isolated most of the time. From time to time, individuals are allowed to migrate from one deme to another.

Two main modules integrate the implemented PGA. On the one hand, the high-level processing module invokes the evolution of the subpopulations and manages the migration process. On the other hand, the low-level module is concerned with the local processing within each deme. These modules are to be separately described below.

#### A. High-level processing: demes and migration management

The high-level processing module implements the subpopulations scheme. At the beginning, the initial individuals are generated at random and arbitrarily organized into demes. Then, the evolution of each deme is invoked and the migration process is performed according to the given time intervals. The workflow of this module is shown in Figure 2.

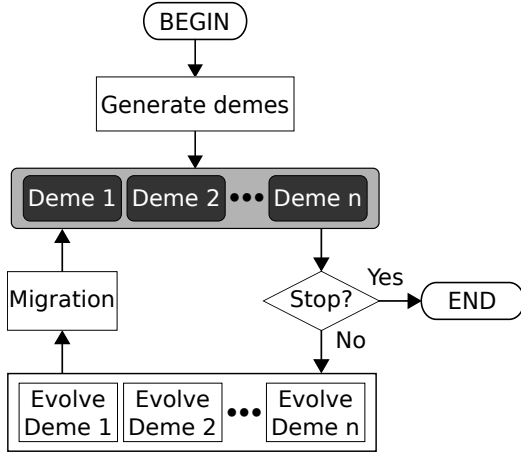


Fig. 2. High-level processing: subpopulations scheme.

Migration is done by replacing the worst individual of each deme with a copy of the best individual from another. As our interconnection topology, we adopted a unidirectional ring, *i.e.*, individuals from deme  $D_i$  can only migrate to deme  $D_{i+1}$ , for  $1 \leq i \leq d-1$ , and deme  $D_1$  can only receive migrants from deme  $D_d$ , where  $d$  denotes the number of demes.

#### B. Low-level processing: elitist genetic algorithm

Low-level processing refers to the isolated evolution of each subpopulation. In our approach, such a task is based on the elitist genetic algorithm illustrated in Figure 3.

Individuals in the concerned deme constitute the initial parent population. Parents are ranked and the fittest are selected for mating (*selection-for-variation*). Then, children are generated by applying the variation operators to the selected parents. Finally, parent and children populations are combined and the best individuals are selected to survive in order to become the new parent population (*selection-for-survival*). This process is performed during a given number of generations (an epoch).

Individuals are ranked by means of the weighted sum method. Such a simple aggregative approach has demonstrated to provide an effective discrimination mechanism, suitable for many-objective optimization [10], [12]. The weighted sum for

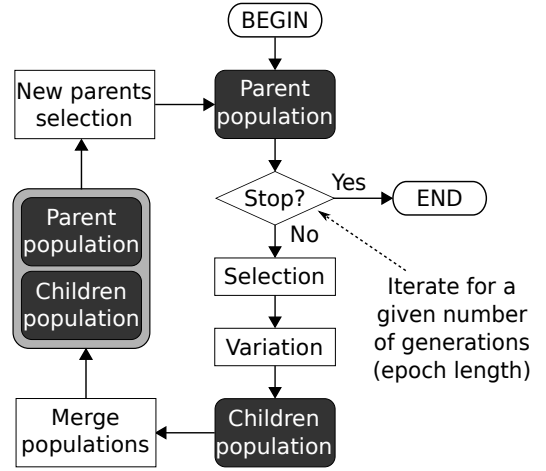


Fig. 3. The elitist genetic algorithm adopted for low-level processing.

a given solution  $\mathbf{X}$  is computed as follows:

$$wsum(\mathbf{X}) = \sum_{m=1}^M \omega_m f_m(\mathbf{X}) \quad (3)$$

where  $\omega_m$  is the weighting coefficient which denotes the relative importance of the  $m$ -th objective. Since we assume that all objectives are equally important, the weighting coefficients in equation (3) were simply omitted for this study.

The implemented operators are: *binary tournament selection*. *Simulated binary crossover* ( $\eta_c = 15$ ) with probability of 1, *Polynomial mutation* ( $\eta_m = 20$ ) with probability of  $1/n$ , where  $n$  is the number of decision variables. In order to investigate the intrinsic ability of PGAs to favor diversity, no additional diversity promotion mechanisms were implemented.

## IV. EXPERIMENTAL RESULTS

#### A. Experimental setup

Problems DTLZ1, DTLZ3 and DTLZ6 [14] were adopted for our experimental study. These problems can be scaled to any number of objectives and decision variables. The total number of variables in these problems is  $n = M + k - 1$ , where  $M$  denotes the number of objectives and  $k$  is a difficulty parameter. We used  $k = 5$  for DTLZ1 and  $k = 10$  for the remaining problems. Experiments were performed for instances with  $M = \{5, 10, 15, 20, 30, 50\}$  objectives for the three test problems adopted.

As a convergence measure, we computed the average distance from Pareto-nondominated solutions in the obtained approximation set to the true Pareto front [20]. Since equations defining the true Pareto front are known for all the test problems adopted, this measure was analytically determined. Additionally, we adopted the *Inverted Generational Distance* (IGD), which allows to measure both convergence and diversity. IGD is a variation of the *Generational Distance* indicator [21] and it is defined by  $IGD = \left( \sqrt{\sum_{i=1}^{|P^*|} d_i^2} \right) / |P^*|$ , where  $P^*$  is a reference set of points in the true Pareto front and  $d_i$  is the Euclidean distance between the  $i$ -th solution in  $P^*$  and

the nearest point in the obtained approximation set. The two adopted performance measures are to be minimized.

For all the approaches compared, we used a total population size of  $N = 100$  individuals, 300 generations and we performed 100 independent executions of each experiment.

### B. Settings for the studied parallel genetic algorithm

As stated in Section II-C, the performance of PGAs is sensitive to the number and size of the demes as well as to migration policies. In this study, a major concern is to analyze the impact that the adjustment of these parameters has on the search performance of the approach.

In order to keep constant the final number of objective functions evaluations, the total number of individuals was fixed to  $N = 100$ . Therefore, the more the demes, the smaller their size. The adopted settings for these parameters are as follows:  $1 \times 100$  (i.e., 1 deme of 100 individuals) consistent with the conventional model,  $2 \times 50$ ,  $4 \times 25$  and  $5 \times 20$ .

Given that the maximum number of demes we adopted is relatively low (i.e., 5), a change in the interconnection topology is not expected to have a major impact on performance.<sup>2</sup> Thus, migration is to be performed in the way described in Section III-A. Nevertheless, an important aspect to be considered in this study is the frequency of migrations. We tested with migration intervals of  $\{10, 20, 30, 50, 75, N\}$  generations, where  $N$  refers to no migration.

The combination of the above described settings leads to 19 different PGA configurations. These configurations are to be referred to by acronyms such as “4-25-N”, which denotes 4 demes of 25 individuals with no migration.

### C. State-of-the-art approaches

We considered four state-of-the-art MOEAs as a reference:

- *Nondominated Sorting Genetic Algorithm II* (NSGA-II) [22]. NSGA-II implements the *Nondominated Sorting*, which is a ranking method based on Pareto dominance. An explicit diversity promotion mechanism, the *Crowding Distance*, is used as a secondary criterion to discriminate among equally ranked solutions.
- *Diversity Management Operator* (DMO) [23]. According to its authors, diversity promotion can be harmful in many-objective optimization, since it tends to prefer solutions with a poor convergence and, therefore, to guide the search away from the Pareto front. DMO is an adaptive strategy: diversity promotion is performed only when it is required. This approach was implemented on NSGA-II.
- *Hypervolume Estimation Algorithm* (HypE) [24]. HypE is a many-objective optimizer which uses the hypervolume metric to guide the search process. Since the calculation of this metric becomes computationally expensive with the increase in the number of objectives, HypE approximates it by using a Monte Carlo simulation.

<sup>2</sup>Unidirectional and bidirectional rings, as well as the fully connected topology were explored with similar results. However, due to space restrictions, only results for the unidirectional ring topology are reported in this paper.

- *Multiple Single Objective Pareto Sampling* (MSOPS) [25]. MSOPS is a many-objective optimizer which uses a set of weighted vectors to guide the search process in multiple directions, simultaneously.

For a more detailed description of these approaches, the reader is referred to their original publications.

### D. Convergence metric results

Tables I, II and III present the obtained results for problems DTLZ1, DTLZ3 and DTLZ6, respectively, with respect to the convergence metric. These tables show the mean and standard deviation of the 100 independent executions performed for each experiment. The best results (lowest mean and standard deviation) for each problem size have been highlighted.

Regarding problem DTLZ1, Table I shows that most instances were better solved by using some PGA configurations (5-20-10, 2-50-10 and 2-50-20). Only for the 20-objectives instance the best results were obtained by the conventional approach (1-100-N). Problem DTLZ3 (Table II) imposed higher convergence difficulties. As for DTLZ1, the 20-objectives instance was the only case where the single population model allowed the best performance. For all other instances, the 2-50-10 and 2-50-20 configurations performed the best. Due to the hardness of this problem, larger subpopulations and frequent migration were essential to improve convergence. From Table III, it is clear that the subpopulations scheme outperformed the conventional model. For all instances of problem DTLZ6, some PGA configurations reported the best convergence (5-20-20, 5-20-10, 4-25-20, 4-25-10 and 2-50-50).

For all the three adopted problems, the worst performance of PGA was consistently shown by the 5-20-N configuration. However, note that the worst behavior of PGA is even better than that of the state-of-the-art approaches in most cases (MSOPS achieved better results than 5-20-N for the largest instances of problems DTLZ1 and DTLZ3).

In general, it is possible to note that migration becomes more important as the size of the subpopulations decreases. This can be better explained by analyzing the behavior of PGA as illustrated in Figure 4. This figure shows the average performance of each PGA configuration for the 5-objectives instance of DTLZ1. The convergence measure is to be minimized. Thus, the three higher peaks exhibit the poor performance of configurations where no migration is applied; the smaller the subpopulations, the poorer the performance. On the other hand, as the frequency of migrations was increased, the convergence of the approach was gradually improved in all cases. In fact, it is not easy to imagine a small population providing, by itself, an acceptable convergence. However, migration breaks isolation, allowing subpopulations to collaborate with each other to perform a more effective search.

Regarding state-of-the-art approaches, NSGA-II provided the worst behavior for most instances of this experiment. Even when DMO is also based on Pareto dominance, this approach performed better than NSGA-II, and even better than HypE in most cases. Among the considered state-of-the-art MOEAs, the MSOPS algorithm performed the best in most cases.

TABLE I  
MEAN AND STANDARD DEVIATION OF THE CONVERGENCE METRIC. DTLZ1 PROBLEM.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
1-100-N	0.000145 ± 0.0002	0.003313 ± 0.0021	0.000247 ± 0.0003	<b>0.000226 ± 0.0003</b>	0.008524 ± 0.0261	0.022512 ± 0.0411
2-50-N	0.003311 ± 0.0156	0.005667 ± 0.0175	0.031743 ± 0.0488	0.040447 ± 0.0453	0.061652 ± 0.0739	0.124940 ± 0.1036
2-50-75	0.000319 ± 0.0004	0.000193 ± 0.0002	0.001032 ± 0.0067	0.004736 ± 0.0165	0.008281 ± 0.0227	0.022515 ± 0.0336
2-50-50	0.000222 ± 0.0003	0.001716 ± 0.0157	0.000192 ± 0.0002	0.001338 ± 0.0111	0.003236 ± 0.0124	0.021468 ± 0.0327
2-50-30	0.000177 ± 0.0002	0.000176 ± 0.0002	0.000176 ± 0.0002	0.002538 ± 0.0138	0.004960 ± 0.0202	0.020931 ± 0.0489
2-50-20	0.000178 ± 0.0002	0.000174 ± 0.0003	<b>0.000139 ± 0.0001</b>	0.000639 ± 0.0045	0.004745 ± 0.0186	<b>0.011906 ± 0.0263</b>
2-50-10	0.000148 ± 0.0002	<b>0.000133 ± 0.0002</b>	0.000208 ± 0.0003	0.001263 ± 0.0111	0.006708 ± 0.0231	0.022585 ± 0.0432
4-25-N	0.100139 ± 0.0911	0.168522 ± 0.1159	0.214087 ± 0.1412	0.293740 ± 0.1666	0.373407 ± 0.1998	0.469142 ± 0.2121
4-25-75	0.006748 ± 0.0164	0.015775 ± 0.0242	0.028567 ± 0.0380	0.039554 ± 0.0464	0.060781 ± 0.0573	0.084687 ± 0.0629
4-25-50	0.002580 ± 0.0168	0.001742 ± 0.0057	0.008708 ± 0.0200	0.014518 ± 0.0301	0.026244 ± 0.0397	0.044886 ± 0.0472
4-25-30	0.000303 ± 0.0003	0.000668 ± 0.0039	0.001796 ± 0.0079	0.003516 ± 0.0137	0.013236 ± 0.0320	0.026595 ± 0.0436
4-25-20	0.000209 ± 0.0002	0.000157 ± 0.0002	0.001927 ± 0.0134	0.003750 ± 0.0197	0.008396 ± 0.0283	0.015368 ± 0.0292
4-25-10	0.000144 ± 0.0002	0.000144 ± 0.0002	0.000200 ± 0.0003	0.001341 ± 0.0111	0.010213 ± 0.0278	0.013562 ± 0.0289
5-20-N	0.170176 ± 0.1393	0.291416 ± 0.1863	0.373880 ± 0.2329	0.469041 ± 0.2427	0.554432 ± 0.2414	0.678818 ± 0.2887
5-20-75	0.030281 ± 0.0511	0.048407 ± 0.0473	0.066023 ± 0.0653	0.075825 ± 0.0563	0.128956 ± 0.0859	0.164711 ± 0.1109
5-20-50	0.006492 ± 0.0151	0.011727 ± 0.0228	0.022221 ± 0.0329	0.035367 ± 0.0548	0.059192 ± 0.0583	0.078205 ± 0.0655
5-20-30	0.001085 ± 0.0047	0.002173 ± 0.0093	0.006259 ± 0.0201	0.005936 ± 0.0146	0.021291 ± 0.0431	0.042569 ± 0.0493
5-20-20	0.000643 ± 0.0041	0.001039 ± 0.0064	0.002437 ± 0.0146	0.003484 ± 0.0160	0.010060 ± 0.0242	0.025029 ± 0.0386
5-20-10	<b>0.000128 ± 0.0002</b>	0.000149 ± 0.0002	0.000189 ± 0.0003	0.002492 ± 0.0156	<b>0.002130 ± 0.0104</b>	0.016853 ± 0.0308
NSGAI1	60.40604 ± 10.742	121.0549 ± 6.7985	110.9575 ± 4.8384	98.27711 ± 3.7724	81.04219 ± 3.5620	62.76571 ± 2.6549
DMO	1.978099 ± 3.1316	17.63203 ± 13.963	8.515378 ± 9.3491	4.663428 ± 5.0863	4.231022 ± 4.1588	2.59132 ± 2.4951
HypE	11.11295 ± 5.5457	19.09323 ± 7.2913	17.81295 ± 7.1807	14.99726 ± 6.1881	13.35879 ± 4.5939	11.44772 ± 3.7236
MSOPS	0.915619 ± 1.1626	1.051954 ± 1.2674	0.607193 ± 1.0100	0.461847 ± 0.7541	0.247411 ± 0.3449	0.319473 ± 0.3524

TABLE II  
MEAN AND STANDARD DEVIATION OF THE CONVERGENCE METRIC. DTLZ3 PROBLEM.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
1-100-N	0.110874 ± 0.3251	0.114468 ± 0.2993	0.244882 ± 0.4922	<b>0.303538 ± 0.5213</b>	0.807442 ± 0.8343	1.404978 ± 1.5611
2-50-N	1.242278 ± 1.0319	1.550547 ± 1.4800	2.443372 ± 1.9784	2.697353 ± 1.8758	4.253623 ± 2.7378	7.225648 ± 4.2487
2-50-75	0.326534 ± 0.5202	0.392058 ± 0.4758	0.592496 ± 0.6838	0.732805 ± 0.7039	1.195928 ± 0.9733	2.539687 ± 1.9921
2-50-50	0.200406 ± 0.3695	0.240207 ± 0.4471	0.354875 ± 0.5079	0.525874 ± 0.8143	0.902689 ± 1.2055	2.056778 ± 1.9583
2-50-30	0.065390 ± 0.2023	0.142951 ± 0.3152	0.348322 ± 0.5882	0.386375 ± 0.7860	0.677882 ± 0.8612	1.675690 ± 1.5302
2-50-20	0.140189 ± 0.3263	0.155142 ± 0.4052	0.294635 ± 0.5262	0.386002 ± 0.6218	0.672328 ± 0.9607	<b>1.310794 ± 1.2162</b>
2-50-10	<b>0.033189 ± 0.1399</b>	<b>0.103401 ± 0.2817</b>	<b>0.176882 ± 0.3739</b>	0.352088 ± 0.6453	<b>0.637230 ± 0.7538</b>	1.361328 ± 1.2547
4-25-N	6.467268 ± 3.0169	7.773832 ± 3.7339	10.24902 ± 4.9429	12.40833 ± 5.0433	16.54271 ± 5.5799	22.82575 ± 7.5528
4-25-75	2.546508 ± 1.7014	2.632010 ± 1.5242	3.636015 ± 2.3712	4.041165 ± 2.0070	5.480490 ± 2.5995	7.966121 ± 3.8573
4-25-50	1.438815 ± 1.0206	1.526694 ± 1.1181	2.329290 ± 1.3138	2.464536 ± 1.4908	3.103310 ± 1.6123	4.537799 ± 2.5431
4-25-30	0.560115 ± 0.6733	0.524491 ± 0.6846	0.887705 ± 0.7548	0.949683 ± 0.8750	1.760112 ± 1.3398	2.792737 ± 1.9777
4-25-20	0.336762 ± 0.4414	0.315477 ± 0.5778	0.470433 ± 0.6365	0.641609 ± 0.6948	0.950593 ± 1.0969	1.773685 ± 1.7147
4-25-10	0.176214 ± 0.4177	0.170499 ± 0.4201	0.293718 ± 0.5005	0.484682 ± 0.6008	0.801266 ± 1.5318	1.331595 ± 1.2657
5-20-N	10.55198 ± 4.4161	12.99393 ± 5.2355	15.47916 ± 5.4506	17.12666 ± 5.6819	23.23956 ± 5.9798	28.77152 ± 7.4553
5-20-75	4.068486 ± 2.1421	5.191120 ± 2.5615	6.209245 ± 2.7835	6.864043 ± 3.0956	8.926646 ± 4.1732	11.49357 ± 4.4097
5-20-50	2.312997 ± 1.6080	2.816194 ± 1.7063	3.801178 ± 2.4154	3.939140 ± 2.1048	5.516923 ± 3.1359	6.839001 ± 3.8715
5-20-30	1.064449 ± 0.8902	1.245251 ± 0.9402	1.475467 ± 1.1115	1.593372 ± 1.0486	2.594352 ± 2.0557	3.674352 ± 2.4132
5-20-20	0.415442 ± 0.5658	0.681066 ± 0.8293	0.714349 ± 0.7016	0.795948 ± 0.9176	1.607120 ± 1.4149	2.428715 ± 1.9395
5-20-10	0.266878 ± 0.4964	0.176141 ± 0.3963	0.413167 ± 0.6988	0.346827 ± 0.5655	0.812771 ± 0.9366	1.568172 ± 1.4730
NSGAI1	428.4210 ± 47.707	1397.412 ± 69.816	1640.337 ± 66.624	1720.620 ± 58.120	1767.131 ± 52.259	1791.930 ± 49.096
DMO	250.0566 ± 54.623	640.3385 ± 119.88	596.3158 ± 133.57	557.2623 ± 149.70	498.9534 ± 149.04	418.0680 ± 166.37
HypE	375.4823 ± 57.670	582.6380 ± 69.824	600.9199 ± 56.229	605.6658 ± 68.860	626.9526 ± 66.416	636.3059 ± 68.991
MSOPS	41.80983 ± 20.642	46.70703 ± 30.752	41.36390 ± 45.132	23.58651 ± 30.388	21.86103 ± 20.731	27.78660 ± 19.844

TABLE III  
MEAN AND STANDARD DEVIATION OF THE CONVERGENCE METRIC. DTLZ6 PROBLEM.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
1-100-N	0.079495 ± 0.0281	0.079909 ± 0.0318	0.086295 ± 0.0318	0.089318 ± 0.0310	0.087839 ± 0.0332	0.095208 ± 0.0329
2-50-N	0.087421 ± 0.0262	0.094208 ± 0.0263	0.095022 ± 0.0295	0.085724 ± 0.0301	0.091331 ± 0.0312	0.107071 ± 0.0342
2-50-75	0.073760 ± 0.0264	0.074113 ± 0.0261	0.079949 ± 0.0261	0.082266 ± 0.0283	0.085210 ± 0.0330	0.096093 ± 0.0322
2-50-50	0.075583 ± 0.0256	0.076201 ± 0.0289	0.079577 ± 0.0289	0.081772 ± 0.0261	0.084561 ± 0.0261	<b>0.086108 ± 0.0323</b>
2-50-30	0.077382 ± 0.0269	0.074343 ± 0.0297	0.079258 ± 0.0241	0.081285 ± 0.0327	0.081934 ± 0.0313	0.092608 ± 0.0294
2-50-20	0.074907 ± 0.0291	0.079655 ± 0.0290	0.078048 ± 0.0263	0.079929 ± 0.0295	0.088990 ± 0.0296	0.091934 ± 0.0316
2-50-10	0.075832 ± 0.0283	0.073103 ± 0.0281	0.085615 ± 0.0303	0.083467 ± 0.0310	0.089384 ± 0.0306	0.093351 ± 0.0278
4-25-N	0.093818 ± 0.0495	0.109898 ± 0.0435	0.155537 ± 0.0857	0.261416 ± 0.1566	0.440901 ± 0.2199	0.929475 ± 0.3000
4-25-75	0.087551 ± 0.0257	0.085145 ± 0.0243	0.093805 ± 0.0285	0.109032 ± 0.0364	0.167796 ± 0.0406	0.331447 ± 0.0603
4-25-50	0.076224 ± 0.0242	0.076422 ± 0.0292	0.084713 ± 0.0227	0.091756 ± 0.0290	0.135116 ± 0.0333	0.254987 ± 0.0509
4-25-30	0.068405 ± 0.0261	0.071149 ± 0.0251	0.075853 ± 0.0274	0.077860 ± 0.0265	0.097237 ± 0.0245	0.162564 ± 0.0364
4-25-20	0.066187 ± 0.0258	0.068359 ± 0.0255	<b>0.070586 ± 0.0293</b>	0.074712 ± 0.0274	0.079098 ± 0.0240	0.123273 ± 0.0288
4-25-10	0.072989 ± 0.0266	0.076893 ± 0.0277	0.071319 ± 0.0312	0.076511 ± 0.0273	<b>0.075024 ± 0.0255</b>	0.093491 ± 0.0352
5-20-N	0.142931 ± 0.0833	0.234765 ± 0.1502	0.352164 ± 0.1883	0.488690 ± 0.2320	0.717407 ± 0.2311	1.345676 ± 0.2995
5-20-75	0.086591 ± 0.0242	0.102812 ± 0.0319	0.132354 ± 0.0389	0.191205 ± 0.0616	0.290661 ± 0.0815	0.559770 ± 0.1433
5-20-50	0.077181 ± 0.0255	0.085892 ± 0.0285	0.104217 ± 0.0299	0.134469 ± 0.0318	0.216849 ± 0.0479	0.411344 ± 0.0674
5-20-30	0.069882 ± 0.0245	0.078661 ± 0.0257	0.080054 ± 0.0253	0.092980 ± 0.0298	0.145650 ± 0.0247	0.266969 ± 0.0494
5-20-20	<b>0.064244 ± 0.0235</b>	<b>0.067385 ± 0.0264</b>	0.073628 ± 0.0282	0.074958 ± 0.0253	0.101675 ± 0.0291	0.186484 ± 0.0327
5-20-10	0.067981 ± 0.0232	0.068530 ± 0.0260	0.076746 ± 0.0273	<b>0.069477 ± 0.0280</b>	0.078583 ± 0.0302	0.097286 ± 0.0298
NSGAI1	6.539745 ± 0.2762	9.377000 ± 0.1322	9.585842 ± 0.0734	9.634870 ± 0.0831	9.666727 ± 0.0785	9.634973 ± 0.0966
DMO	6.629688 ± 0.4212	8.457203 ± 0.3208	8.720438 ± 0.3497	8.800647 ± 0.3195	8.819693 ± 0.3404	8.853029 ± 0.3145
HypE	7.188594 ± 0.2906	8.012234 ± 0.4234	8.365638 ± 0.3143	8.411360 ± 0.2328	8.484474 ± 0.2009	8.515142 ± 0.2176
MSOPS	7.910041 ± 0.2346	8.792252 ± 0.2246	8.577633 ± 0.2903	8.322629 ± 0.3015	8.140124 ± 0.3840	7.439052 ± 0.5382

TABLE IV  
MEAN AND STANDARD DEVIATION OF THE IGD METRIC. DTLZ1 PROBLEM.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
1-100-N	0.177062 ± 0.0213	0.159095 ± 0.0177	0.136568 ± 0.0090	0.122916 ± 0.0093	0.105603 ± 0.0106	0.092822 ± 0.0208
2-50-N	0.161717 ± 0.0196	0.150409 ± 0.0123	0.136469 ± 0.0127	0.123138 ± 0.0121	0.110818 ± 0.0240	0.105347 ± 0.0309
2-50-75	0.171271 ± 0.0194	0.152603 ± 0.0126	0.135265 ± 0.0090	0.121159 ± 0.0077	0.103182 ± 0.0100	0.086874 ± 0.0125
2-50-50	0.171014 ± 0.0190	0.155282 ± 0.0155	0.134580 ± 0.0096	0.120212 ± 0.0121	<b>0.101939 ± 0.0070</b>	0.090543 ± 0.0179
2-50-30	0.171814 ± 0.0232	0.154499 ± 0.0135	0.134824 ± 0.0088	0.122356 ± 0.0086	0.102709 ± 0.0077	0.090167 ± 0.0230
2-50-20	0.171616 ± 0.0232	0.155428 ± 0.0129	0.135744 ± 0.0097	0.120367 ± 0.0079	0.103809 ± 0.0109	0.086405 ± 0.0137
2-50-10	0.178185 ± 0.0228	0.153482 ± 0.0122	0.137445 ± 0.0099	0.121730 ± 0.0102	0.105750 ± 0.0145	0.092453 ± 0.0263
4-25-N	0.156838 ± 0.0180	0.158036 ± 0.0212	0.159967 ± 0.0421	0.158472 ± 0.0418	0.155411 ± 0.0571	0.177704 ± 0.0732
4-25-75	0.155603 ± 0.0208	0.145866 ± 0.0109	0.131837 ± 0.0091	0.124928 ± 0.0173	0.110485 ± 0.0169	0.098313 ± 0.0244
4-25-50	0.160452 ± 0.0185	0.148242 ± 0.0130	<b>0.131780 ± 0.0092</b>	0.121992 ± 0.0115	0.104291 ± 0.0121	0.089242 ± 0.0192
4-25-30	0.160773 ± 0.0175	0.149413 ± 0.0116	0.134253 ± 0.0101	0.120568 ± 0.0080	0.104868 ± 0.0106	0.090184 ± 0.0190
4-25-20	0.166443 ± 0.0215	0.153053 ± 0.0116	0.134024 ± 0.0132	0.121231 ± 0.0099	0.104382 ± 0.0097	<b>0.085503 ± 0.0117</b>
4-25-10	0.166155 ± 0.0213	0.150488 ± 0.0114	0.134075 ± 0.0097	0.121010 ± 0.0129	0.106047 ± 0.0140	0.085936 ± 0.0125
5-20-N	0.160733 ± 0.0254	0.169211 ± 0.0424	0.169055 ± 0.0415	0.184481 ± 0.0617	0.188111 ± 0.0810	0.216978 ± 0.1157
5-20-75	<b>0.150242 ± 0.0211</b>	0.146229 ± 0.0111	0.136746 ± 0.0169	0.124014 ± 0.0142	0.118320 ± 0.0355	0.115327 ± 0.0417
5-20-50	0.154721 ± 0.0211	0.145014 ± 0.0117	0.132247 ± 0.0099	0.122512 ± 0.0180	0.108869 ± 0.0174	0.095807 ± 0.0229
5-20-30	0.162504 ± 0.0214	<b>0.144903 ± 0.0108</b>	0.133208 ± 0.0111	0.119125 ± 0.0076	0.105505 ± 0.0148	0.093030 ± 0.0225
5-20-20	0.165147 ± 0.0210	0.148400 ± 0.0123	0.133991 ± 0.0095	<b>0.118912 ± 0.0091</b>	0.103052 ± 0.0093	0.088462 ± 0.0148
5-20-10	0.165863 ± 0.0194	0.151546 ± 0.0109	0.132680 ± 0.0078	0.120739 ± 0.0116	0.102036 ± 0.0069	0.087523 ± 0.0152
NSGAII	7.817745 ± 4.2551	10.45238 ± 7.7348	9.395123 ± 6.3933	6.513742 ± 5.6212	5.597018 ± 4.3728	3.484032 ± 2.3463
DMO	5.527289 ± 4.4799	2.985676 ± 2.5608	1.872924 ± 1.5953	1.366379 ± 0.9851	1.436062 ± 1.0061	1.087918 ± 0.6951
HypE	1.511872 ± 0.8715	2.378413 ± 1.2825	2.359147 ± 1.1410	2.024472 ± 1.0968	1.712675 ± 0.6897	1.633190 ± 0.7154
MSOPS	0.309218 ± 0.5437	0.323235 ± 0.2680	0.237655 ± 0.1358	0.216796 ± 0.1940	0.153786 ± 0.0927	0.127538 ± 0.0823

TABLE V  
MEAN AND STANDARD DEVIATION OF THE IGD METRIC. DTLZ3 PROBLEM.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
1-100-N	0.535310 ± 0.1050	0.426694 ± 0.0723	0.385149 ± 0.0947	0.350378 ± 0.0927	0.367180 ± 0.1289	0.366908 ± 0.2043
2-50-N	0.754033 ± 0.2305	0.612994 ± 0.2237	0.662429 ± 0.3328	0.651136 ± 0.3221	0.734840 ± 0.3802	0.861584 ± 0.4143
2-50-75	0.567423 ± 0.1551	0.447009 ± 0.0927	0.409886 ± 0.1159	0.374076 ± 0.0954	0.375383 ± 0.1324	0.435860 ± 0.2056
2-50-50	0.541014 ± 0.0953	0.439969 ± 0.0935	0.386285 ± 0.0885	0.367103 ± 0.1177	0.352352 ± 0.1637	0.406441 ± 0.2286
2-50-30	0.518845 ± 0.0623	0.428231 ± 0.0731	0.399763 ± 0.1118	0.360767 ± 0.1253	0.337443 ± 0.1214	0.380326 ± 0.1916
2-50-20	0.534376 ± 0.0908	0.432649 ± 0.0966	0.393532 ± 0.1092	0.363802 ± 0.1168	0.345086 ± 0.1568	0.345918 ± 0.1554
2-50-10	<b>0.512164 ± 0.0446</b>	<b>0.422133 ± 0.0654</b>	<b>0.372298 ± 0.0727</b>	0.360585 ± 0.1163	<b>0.336987 ± 0.1102</b>	0.355376 ± 0.1613
4-25-N	1.780826 ± 0.6945	1.633826 ± 0.6801	1.654740 ± 0.6627	1.749416 ± 0.5956	1.864992 ± 0.7003	1.991726 ± 0.8331
4-25-75	0.953994 ± 0.3768	0.801843 ± 0.3152	0.734799 ± 0.3508	0.729540 ± 0.3129	0.762963 ± 0.3129	0.810339 ± 0.4202
4-25-50	0.761169 ± 0.2697	0.610528 ± 0.2298	0.612161 ± 0.2058	0.574890 ± 0.2328	0.555049 ± 0.2348	0.589489 ± 0.2653
4-25-30	0.612391 ± 0.2114	0.480430 ± 0.1521	0.453242 ± 0.1384	0.406661 ± 0.1476	0.455159 ± 0.2054	0.458808 ± 0.2378
4-25-20	0.569627 ± 0.1236	0.456184 ± 0.1377	0.408151 ± 0.1136	0.386174 ± 0.1168	0.364849 ± 0.1669	0.388492 ± 0.2196
4-25-10	0.556294 ± 0.1360	0.432759 ± 0.1047	0.388524 ± 0.0959	0.377131 ± 0.1113	0.356376 ± 0.1993	<b>0.336779 ± 0.1413</b>
5-20-N	2.633198 ± 1.0425	2.203752 ± 0.8313	2.242860 ± 0.9146	2.296266 ± 0.9125	2.517761 ± 0.9335	2.522268 ± 0.8318
5-20-75	1.306867 ± 0.4996	1.133198 ± 0.4775	1.071321 ± 0.4282	1.079298 ± 0.4244	1.126382 ± 0.5158	1.101353 ± 0.4628
5-20-50	0.904009 ± 0.3379	0.820545 ± 0.3736	0.832386 ± 0.4379	0.759209 ± 0.3547	0.790013 ± 0.4045	0.769001 ± 0.4025
5-20-30	0.718198 ± 0.2555	0.573812 ± 0.1855	0.517420 ± 0.1846	0.479865 ± 0.1681	0.526434 ± 0.2956	0.520433 ± 0.2292
5-20-20	0.571273 ± 0.1632	0.494359 ± 0.1586	0.428431 ± 0.1251	0.397700 ± 0.1399	0.438716 ± 0.2094	0.434748 ± 0.2318
5-20-10	0.571717 ± 0.1353	0.428256 ± 0.0810	0.405304 ± 0.1129	<b>0.347951 ± 0.0947</b>	0.351156 ± 0.1414	0.369323 ± 0.1959
NSGAII	46.94408 ± 14.740	112.7081 ± 40.539	105.8747 ± 44.895	90.76743 ± 36.006	66.19926 ± 25.745	52.78022 ± 21.293
DMO	25.57924 ± 8.1735	70.16373 ± 22.775	56.50639 ± 20.880	45.01976 ± 21.032	34.75309 ± 15.018	23.43633 ± 12.002
HypE	42.75160 ± 11.813	54.17240 ± 16.208	46.47087 ± 11.327	40.90931 ± 9.9673	36.51184 ± 8.0518	29.05640 ± 7.1596
MSOPS	9.207426 ± 5.8053	9.082625 ± 5.2078	6.882234 ± 5.5470	3.789015 ± 3.8713	3.278891 ± 2.6239	3.303975 ± 2.0285

TABLE VI  
MEAN AND STANDARD DEVIATION OF THE IGD METRIC. DTLZ6 PROBLEM.

	5 Obj.	10 Obj.	15 Obj.	20 Obj.	30 Obj.	50 Obj.
1-100-N	0.523611 ± 0.0075	0.416510 ± 0.0067	0.354409 ± 0.0056	0.313212 ± 0.0049	0.260330 ± 0.0043	0.205367 ± 0.0034
2-50-N	0.522026 ± 0.0069	0.415783 ± 0.0058	0.352877 ± 0.0048	0.311010 ± 0.0041	0.259065 ± 0.0038	0.205598 ± 0.0029
2-50-75	0.521994 ± 0.0070	0.415231 ± 0.0055	0.353149 ± 0.0046	0.312062 ± 0.0044	0.259791 ± 0.0043	0.204960 ± 0.0032
2-50-50	0.522556 ± 0.0068	0.415724 ± 0.0060	0.353209 ± 0.0051	0.312021 ± 0.0041	0.259888 ± 0.0034	<b>0.204289 ± 0.0034</b>
2-50-30	0.523041 ± 0.0072	0.415338 ± 0.0062	0.353142 ± 0.0043	0.311959 ± 0.0051	0.259559 ± 0.0041	0.205083 ± 0.0030
2-50-20	0.522394 ± 0.0077	0.416448 ± 0.0061	0.352933 ± 0.0047	0.311740 ± 0.0046	0.260473 ± 0.0039	0.205030 ± 0.0032
2-50-10	0.522635 ± 0.0075	0.415074 ± 0.0059	0.354284 ± 0.0054	0.312296 ± 0.0048	0.260526 ± 0.0040	0.205170 ± 0.0029
4-25-N	0.521550 ± 0.0084	0.417464 ± 0.0108	0.358779 ± 0.0063	0.323804 ± 0.0097	0.281540 ± 0.0135	0.247818 ± 0.0177
4-25-75	0.520948 ± 0.0055	0.414376 ± 0.0043	0.352843 ± 0.0042	0.313390 ± 0.0045	0.267103 ± 0.0046	0.226218 ± 0.0058
4-25-50	0.519284 ± 0.0060	0.413177 ± 0.0048	0.351745 ± 0.0039	0.311265 ± 0.0044	0.263995 ± 0.0040	0.219269 ± 0.0049
4-25-30	0.519385 ± 0.0073	0.413640 ± 0.0054	0.351458 ± 0.0049	0.310431 ± 0.0040	0.259876 ± 0.0031	0.210968 ± 0.0038
4-25-20	0.519912 ± 0.0068	0.414010 ± 0.0053	0.351269 ± 0.0052	0.310634 ± 0.0043	<b>0.258310 ± 0.0034</b>	0.207083 ± 0.0030
4-25-10	0.521872 ± 0.0071	0.415865 ± 0.0058	0.351756 ± 0.0055	0.311203 ± 0.0042	0.258649 ± 0.0033	0.204970 ± 0.0037
5-20-N	<b>0.517652 ± 0.0350</b>	0.428996 ± 0.0151	0.374568 ± 0.0139	0.340917 ± 0.0164	0.308309 ± 0.0200	0.286125 ± 0.0292
5-20-75	0.520241 ± 0.0061	0.417140 ± 0.0058	0.358626 ± 0.0055	0.323931 ± 0.0068	0.281619 ± 0.0089	0.244184 ± 0.0100
5-20-50	0.519605 ± 0.0068	0.414235 ± 0.0049	0.354091 ± 0.0049	0.317292 ± 0.0043	0.274444 ± 0.0060	0.233085 ± 0.0062
5-20-30	0.518568 ± 0.0062	0.414074 ± 0.0055	0.351289 ± 0.0045	0.311522 ± 0.0043	0.265340 ± 0.0045	0.220696 ± 0.0051
5-20-20	0.518932 ± 0.0064	<b>0.412683 ± 0.0051</b>	<b>0.350793 ± 0.0046</b>	<b>0.309964 ± 0.0040</b>	0.260086 ± 0.0036	0.212830 ± 0.0033
5-20-10	0.520528 ± 0.0061	0.414114 ± 0.0054	0.352705 ± 0.0048	0.310082 ± 0.0044	0.259064 ± 0.0039	0.204938 ± 0.0030
NSGAII	2.400076 ± 0.1270	2.717792 ± 0.1028	2.368454 ± 0.0655	2.101530 ± 0.0672	1.745590 ± 0.0563	1.365944 ± 0.0576
DMO	2.591950 ± 0.2159	2.619031 ± 0.1122	2.271961 ± 0.1037	2.024427 ± 0.0827	1.685581 ± 0.0736	1.326487 ± 0.0527
HypE	2.446891 ± 0.1441	2.177596 ± 0.1688	1.945053 ± 0.1005	1.725434 ± 0.0729	1.463114 ± 0.0568	1.174448 ± 0.0492
MSOPS	2.657995 ± 0.1722	2.249516 ± 0.1854	1.545002 ± 0.2235	1.154024 ± 0.1613	0.955169 ± 0.1296	0.799487 ± 0.0756

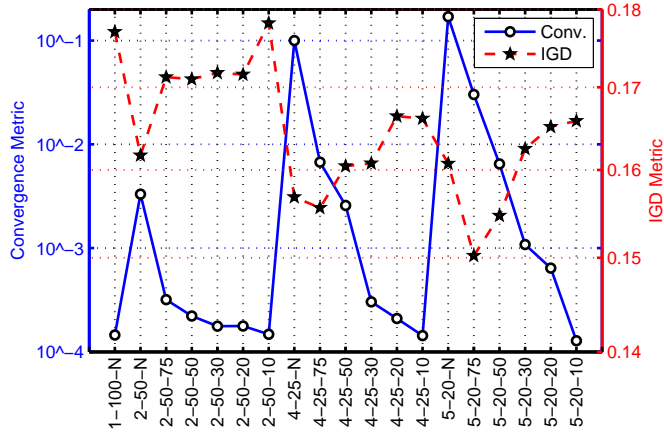


Fig. 4. Average performance of PGA configurations for the 5-objectives DTLZ1 problem. The solid blue line with circle marks corresponds to results for the convergence metric, while the dotted red line with star marks refers to IGD.

### E. IGD metric results

Tables IV, V and VI present the mean and standard deviation for the IGD metric regarding problems DTLZ1, DTLZ3 and DTLZ6, respectively. The best results for each problem size have been highlighted in these tables.

From Table IV, it is possible to note that most PGA configurations performed better than the conventional model in all instances of problem DTLZ1. For each instance of this problem, a different PGA configuration achieved the best results (5-20-75, 5-20-30, 5-20-20, 4-25-50, 4-25-20 and 2-50-50). Regarding problem DTLZ3, the best values for the IGD metric were obtained by configurations 2-50-10, 4-25-10 and 5-20-10. As stated in Section IV-D, problem DTLZ3 involves higher convergence difficulties. Thus, note that a frequent migration is common in the best performing configurations. Finally, results for problem DTLZ6 show an interesting behavior (Table VI). The 5-objectives instance was better solved by using the smallest subpopulations with no migration (5-20-N). However, migration and larger subpopulations became required as the problem size was raised.

The results obtained for the IGD metric suggest that isolation, as well as the increase in the number of subpopulations, lead to improve diversity. This can be observed for the smallest instances of problems DTLZ1 and DTLZ6. However, such a behavior does not hold as the problem becomes harder, where larger subpopulations and frequent migration are needed. With the aim of clarifying this point, let us analyze Figure 4 again, now focusing on the IGD measure. Figure 4 shows the average performance of each PGA configuration regarding the 5-objectives instance of problem DTLZ1. As we can see from this figure, the best results (lowest IGD values) were obtained when using little or no migration. Contrasting with the behavior for the convergence metric, the performance of the approach regarding the IGD measure deteriorates as the frequency of migrations is increased.

Among the adopted state-of-the-art MOEAs, NSGA-II per-

formed worst for most instances of this experiment. MSOPS performed best in most cases, while the ranking between DMO and HypE is not clear.

## V. CONCLUSIONS AND FUTURE WORK

In this study, the suitability of parallel genetic algorithms (PGAs) for many-objective optimization was explored. In PGAs, individuals are organized into multiple subpopulations which evolve in isolation most of the time, but individuals are occasionally exchanged (migration). Isolation favors speciation, which can be exploited as a potential source of diversity. An elitist genetic algorithm was adopted as the search engine to evolve each subpopulation. The adopted genetic algorithm implements a fine-grained ranking strategy, which has been identified in our previous work as an essential requirement to perform an effective search in many-objective optimization.

For our experimental study, 19 different configurations of the implemented PGA were explored, varying the number and size of the subpopulations as well as the frequency of migrations. However, the total number of individuals was kept constant for all cases, so that the number of objective functions evaluations does not increase as a consequence of using multiple subpopulations. Also, four state-of-the-art approaches were considered as a reference. Problems DTLZ1, DTLZ3 and DTLZ6 were adopted, ranging in objectives from 5 to 50.

Our results indicate that the implemented PGA is a convenient approach for many-objective optimization. For most instances of the adopted test problems, the use of multiple subpopulations outperformed the conventional model in terms of convergence. In fact, a small population is not expected to provide, by itself, an acceptable convergence. However, the collaborative behavior among subpopulations that emerges as a result of migration seems to improve the search capabilities of the approach. Thus, migration becomes more important as the size of the subpopulations decreases, which was clearly demonstrated through Figure 4.

Regarding the IGD metric, the best results for all the instances of the adopted test problems were obtained by using multiple subpopulations. As expected, the obtained results suggest that isolation favors diversity. As illustrated in Figure 4, the best IGD values were achieved when using little or no migration, but the performance gradually declined with the increase in the frequency of migrations.

Therefore, a frequent migration tends to improve convergence, while isolation favors the diversity in the final approximation set. The fact that the convergence and IGD measures contradict each other in Figure 4 supports these claims (both measures were computed on the same data).

With the aim of exploring the intrinsic ability of PGAs to favor diversification, no additional diversity promotion mechanisms were adopted. Nevertheless, by using such additional mechanisms the performance of the approach could be better improved in this regard.

Although the focus of this study was not on computational efficiency, an additional advantage of PGAs is that they can

naturally exploit parallel architectures.

Due to space limitations, we have reported in this paper results for only three test problems. However, it is important to extend these experiments to a larger set of test cases as well as to adopt real-world many-objective problems in order to generalize our results. Also, the consideration of a wider set of performance measures is required in order to derive more general conclusions about the behavior of the studied approach.

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