CONVEX HULLS

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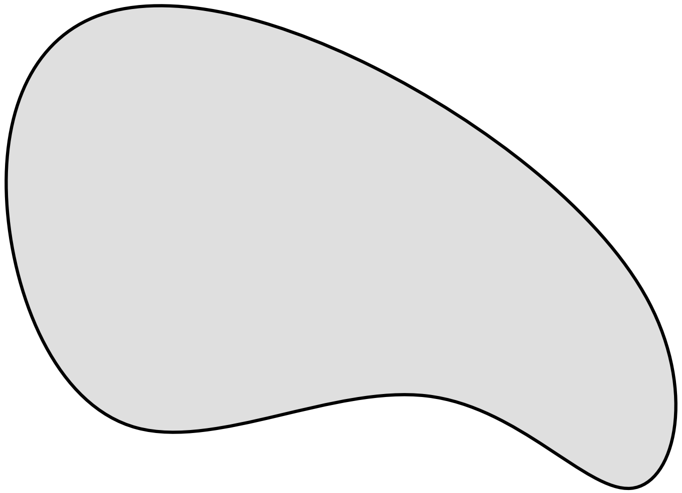

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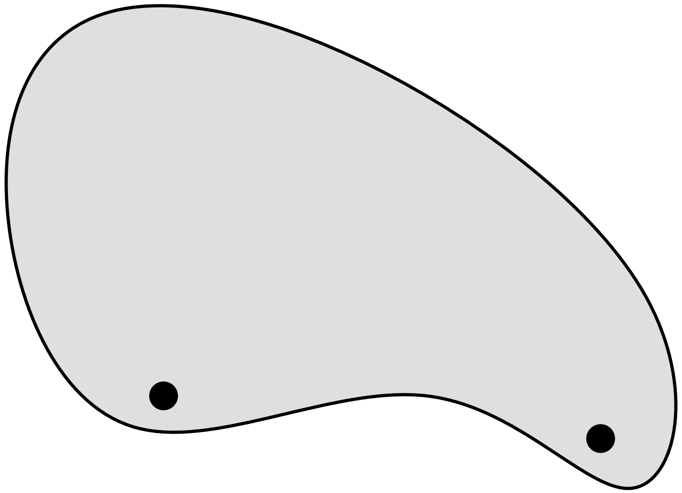

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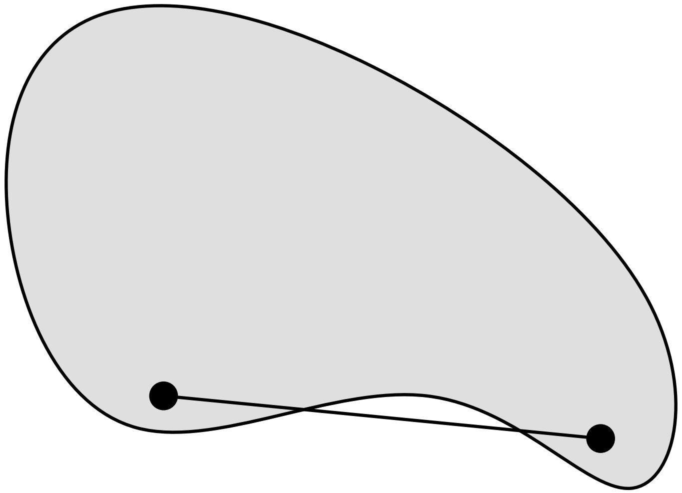

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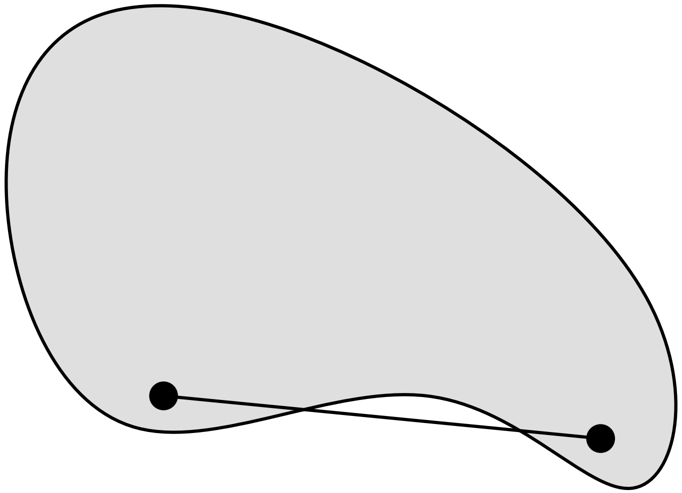

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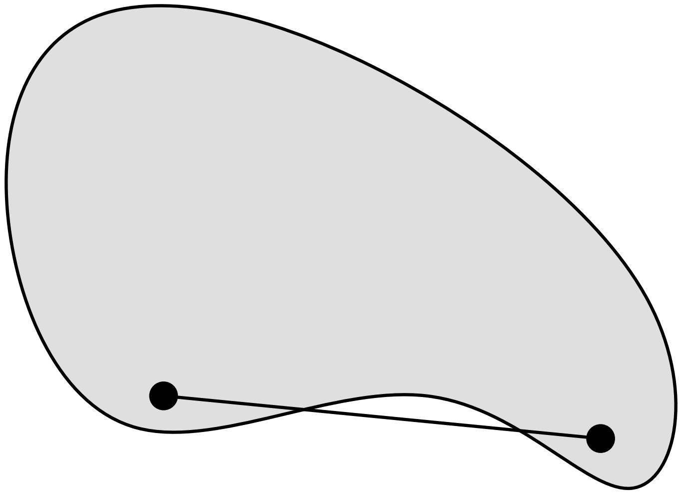

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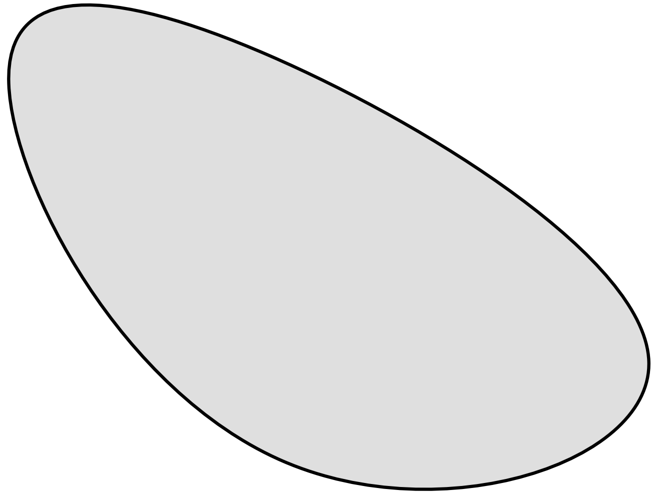

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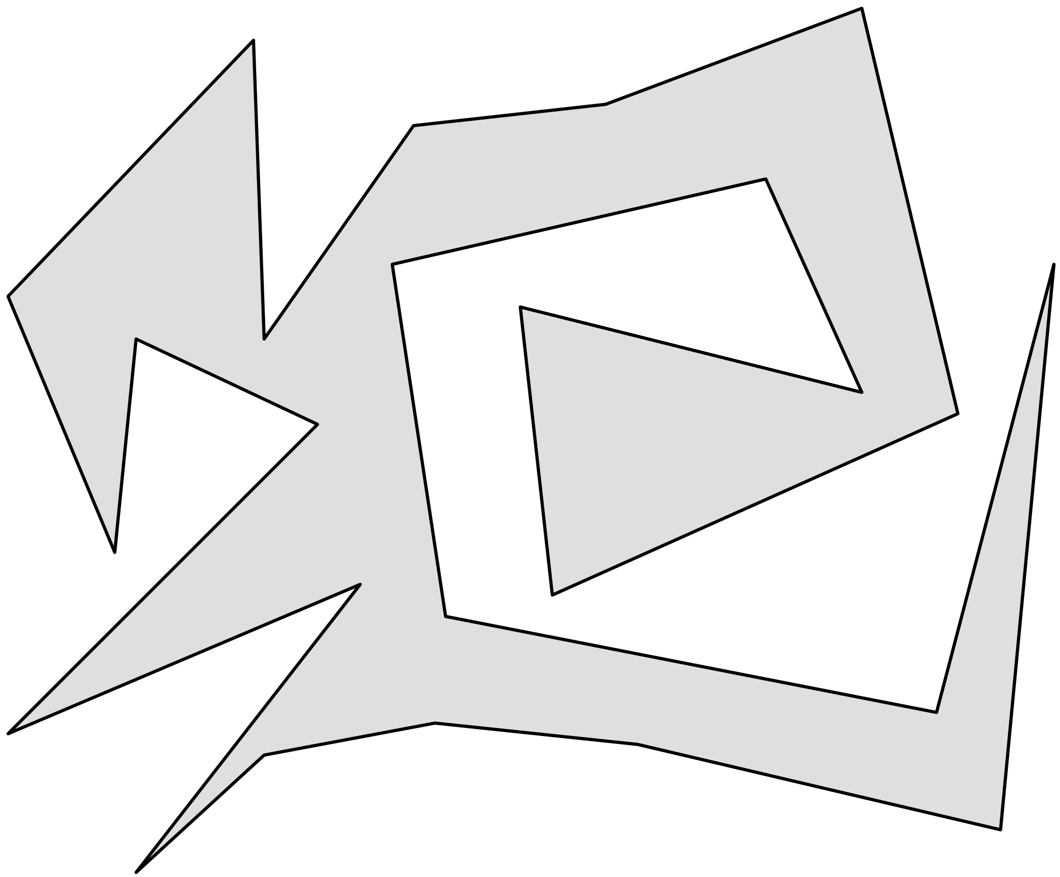

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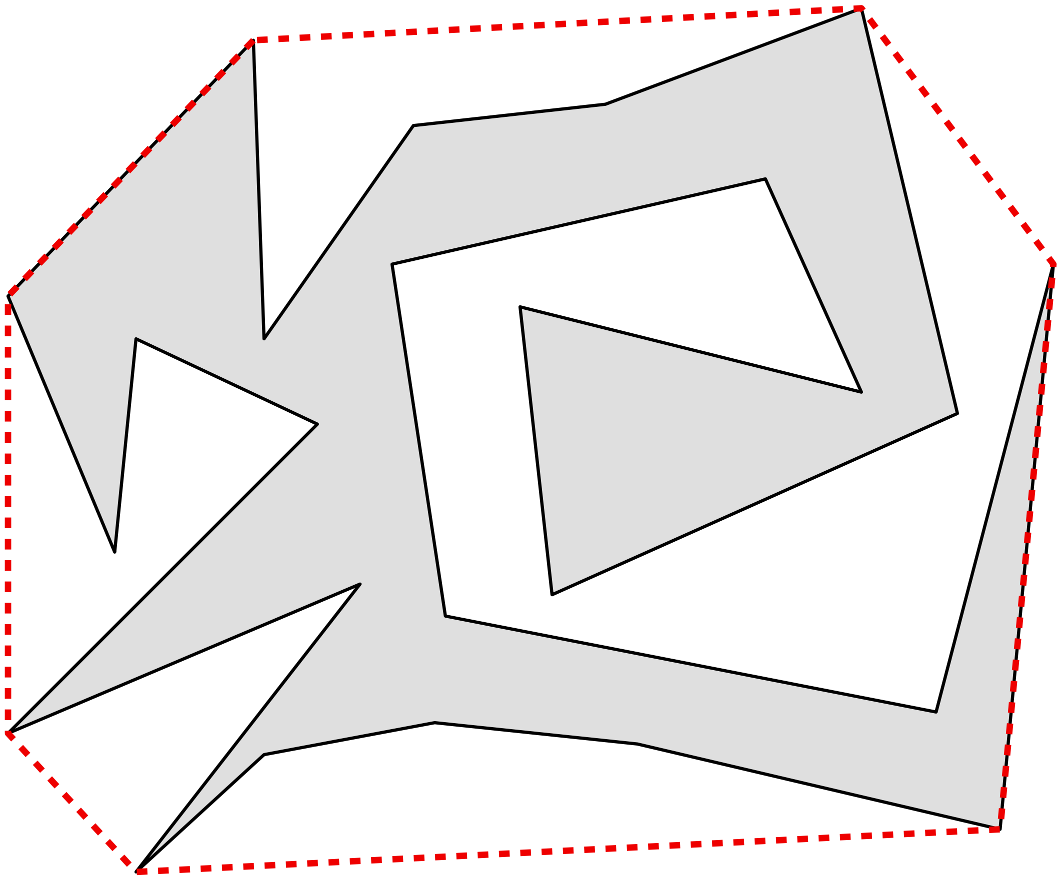

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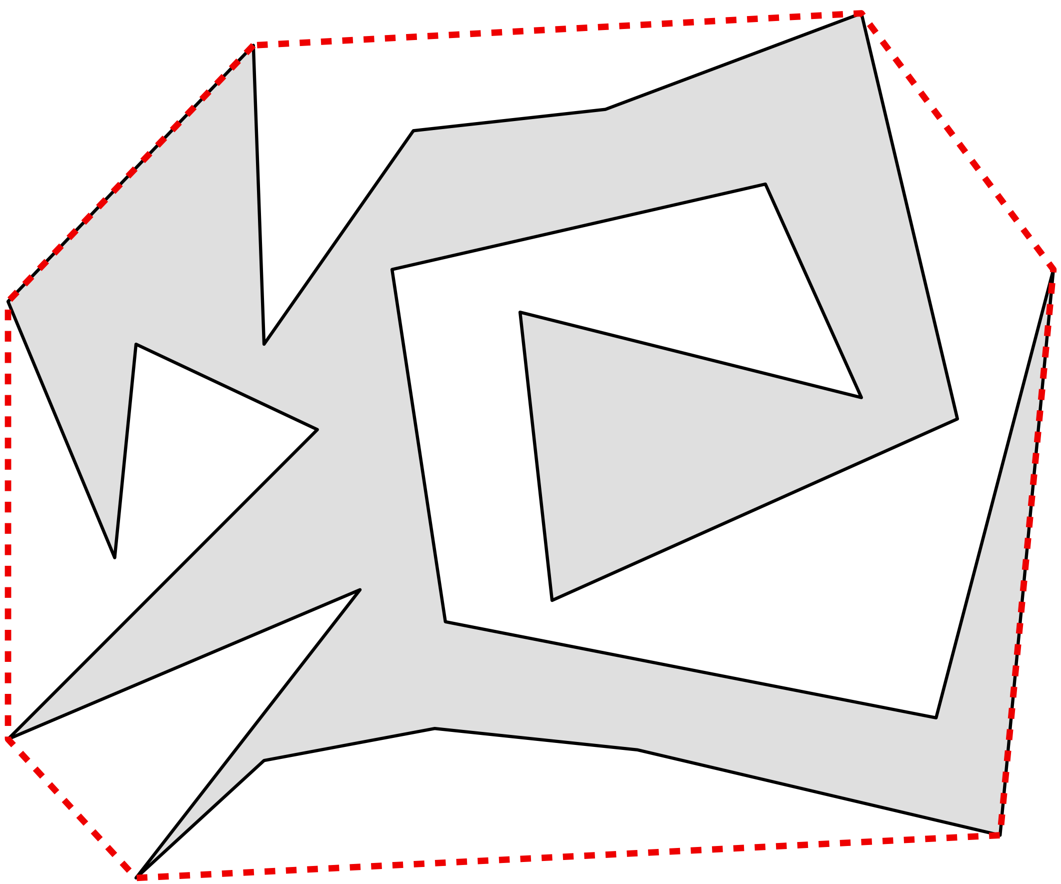

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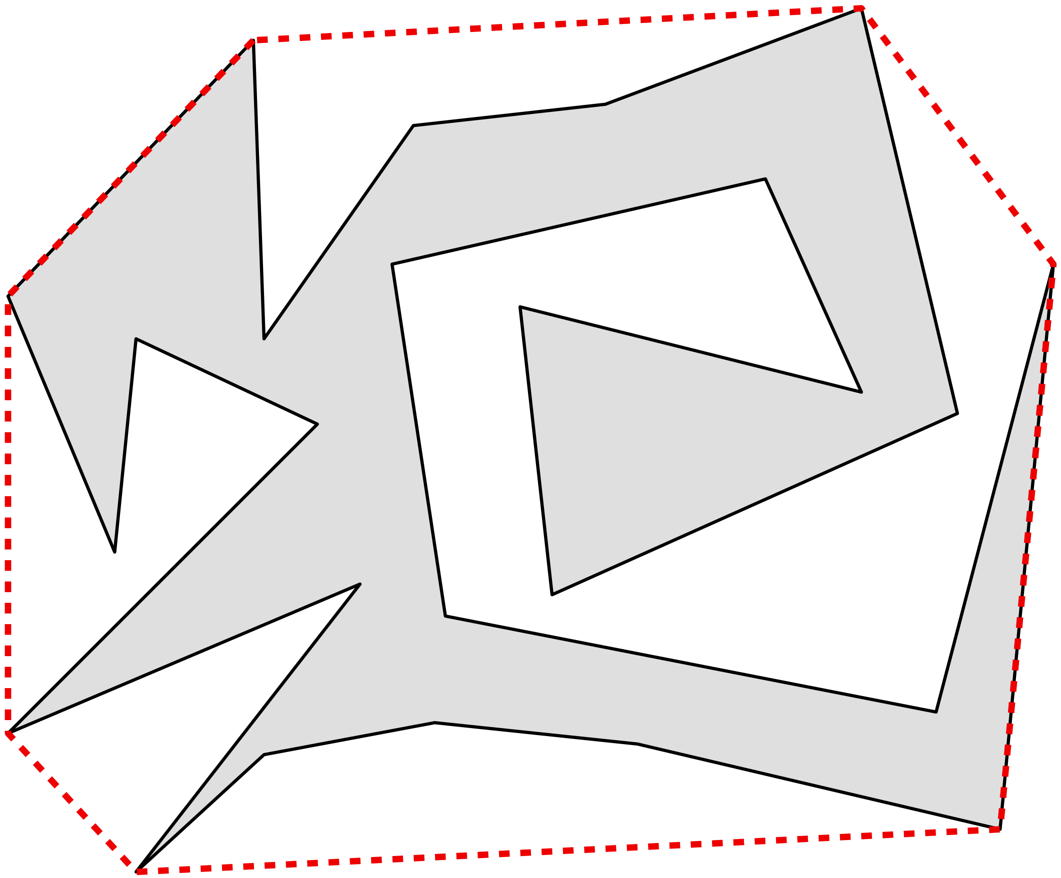

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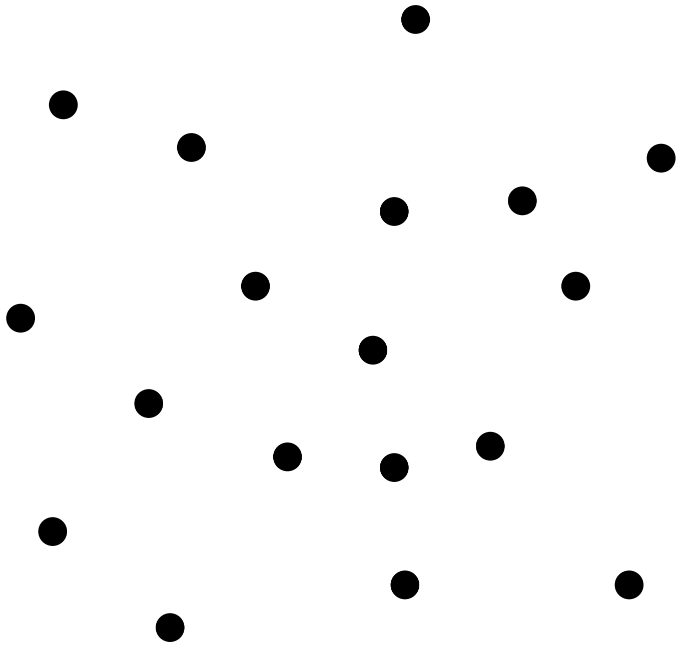

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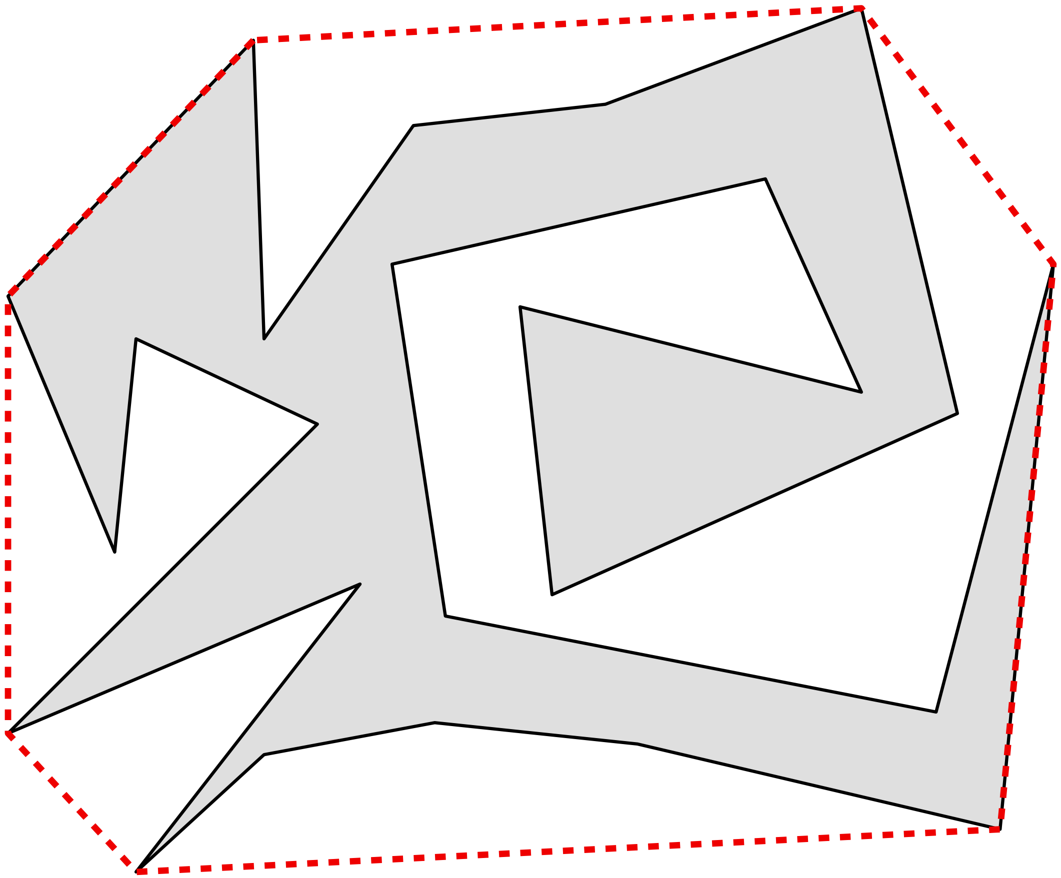

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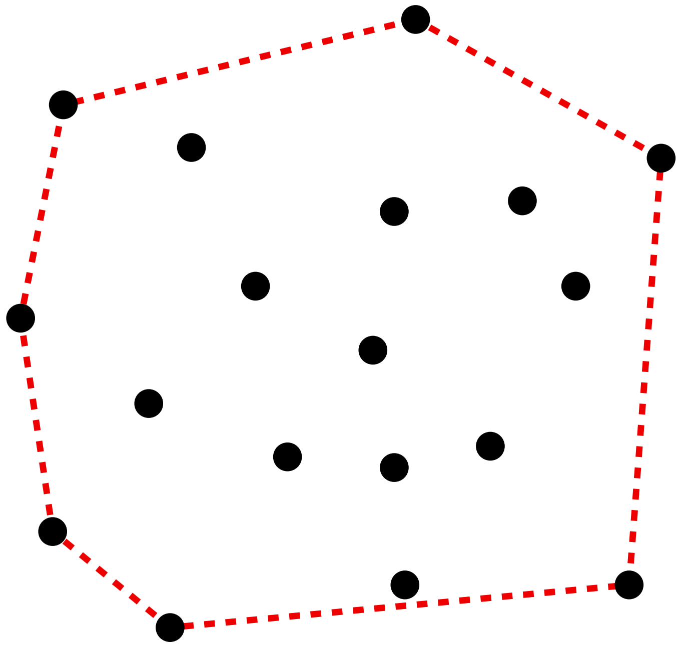

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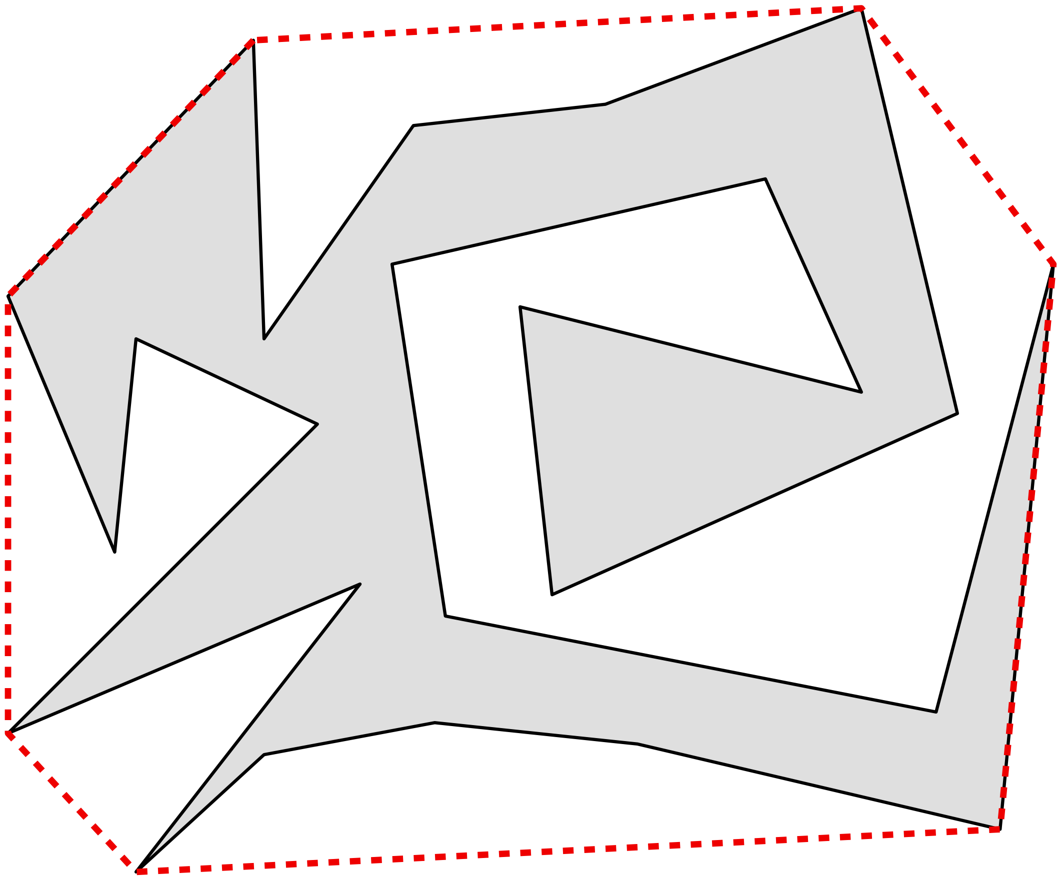

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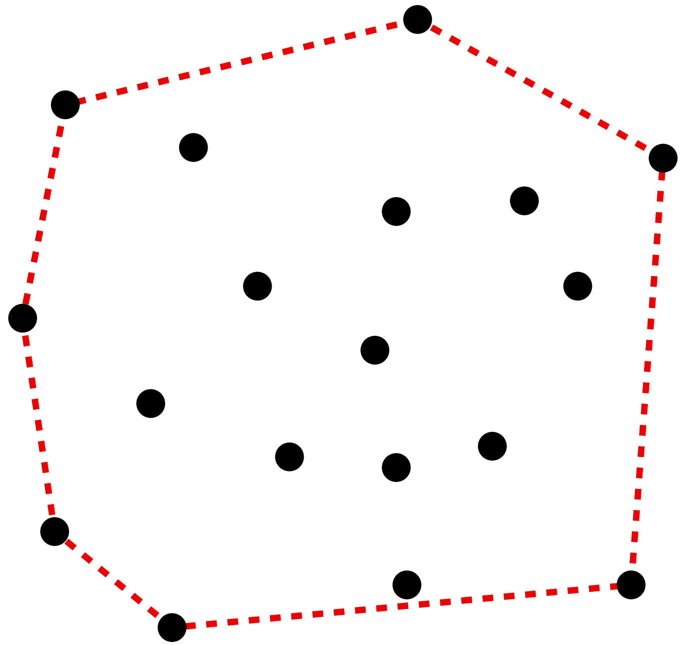

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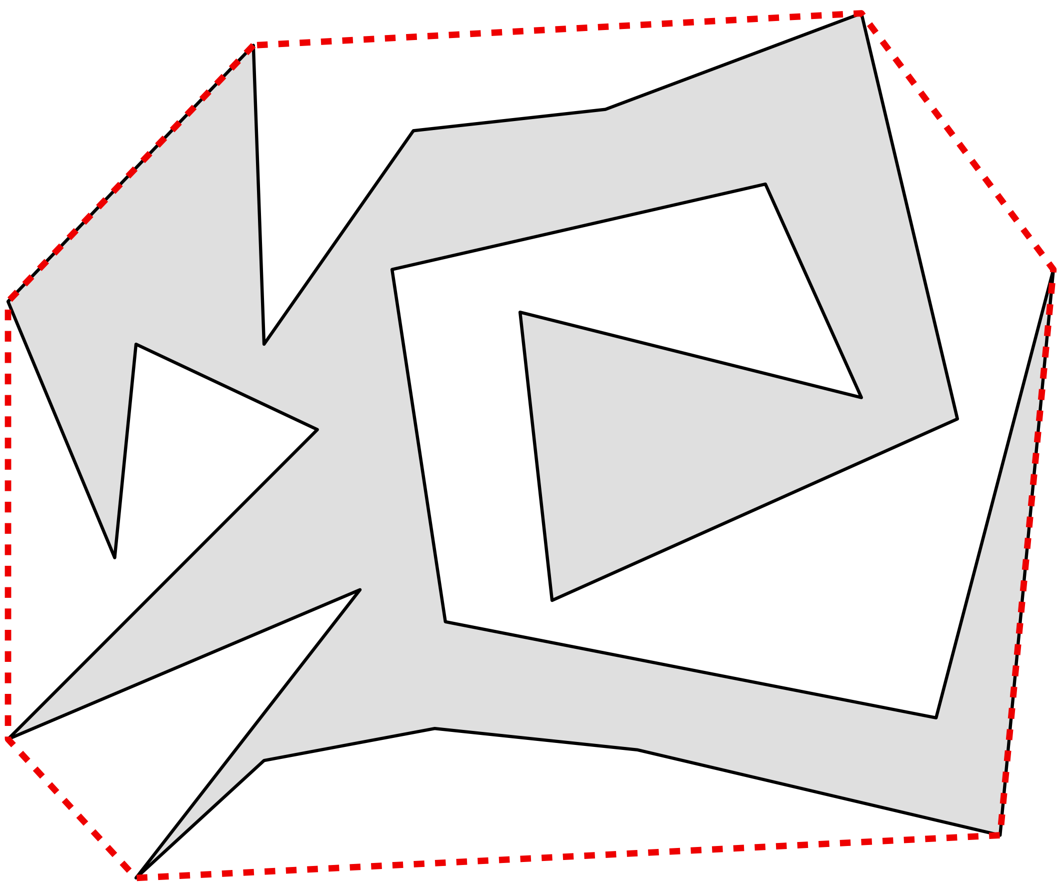

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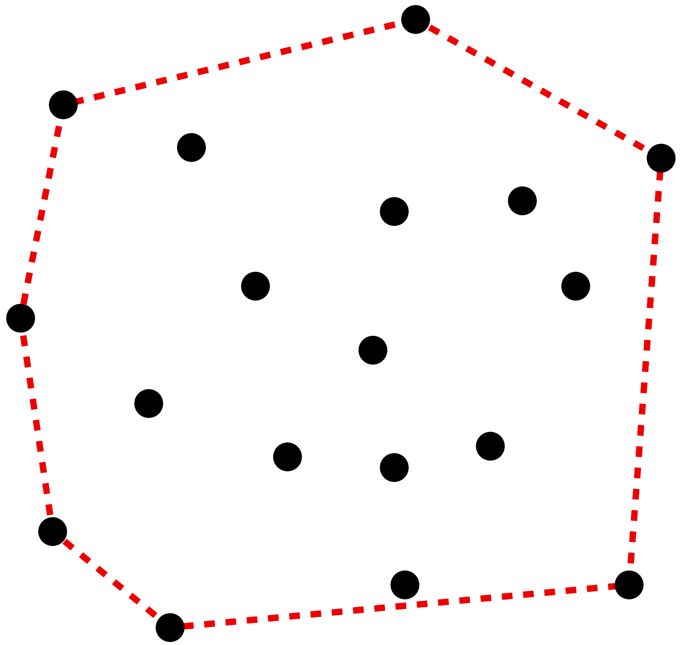


The convex hull of a finite set of points in the plane is a convex polygon.

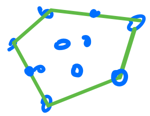
In both cases, the vertices of $ch(X)$ are points of X.

Computing the extreme points

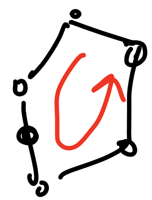
Deseamos dar un algoritmo para calcular el cierre convexo de un conjunto S de n puntos en el plano.

Podríamos dar como salida:

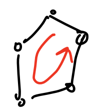
1. Todos los puntos de la frontera ch(S), en orden arbitrario.



2. Los puntos extremos, es decir los vértices del cierre convexo, en orden arbitrario

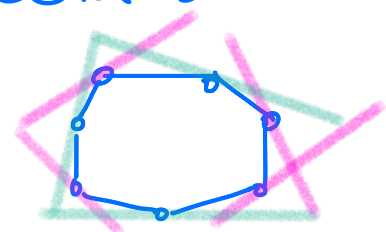


3. Todos los puntos de ∂ch(S) en el orden de recorrido
4. Los puntos extremos, es decir los vértices del cierre convexo, en orden el orden de recorrido.



Si no están en pos gral podrían ser distintos,

Puntos extremos: más alto, más bajo, más a la izq, más a la derecha...
i.e. si existe una recta que pasa por este y únicamente interseca al ch(S) en ese punto.



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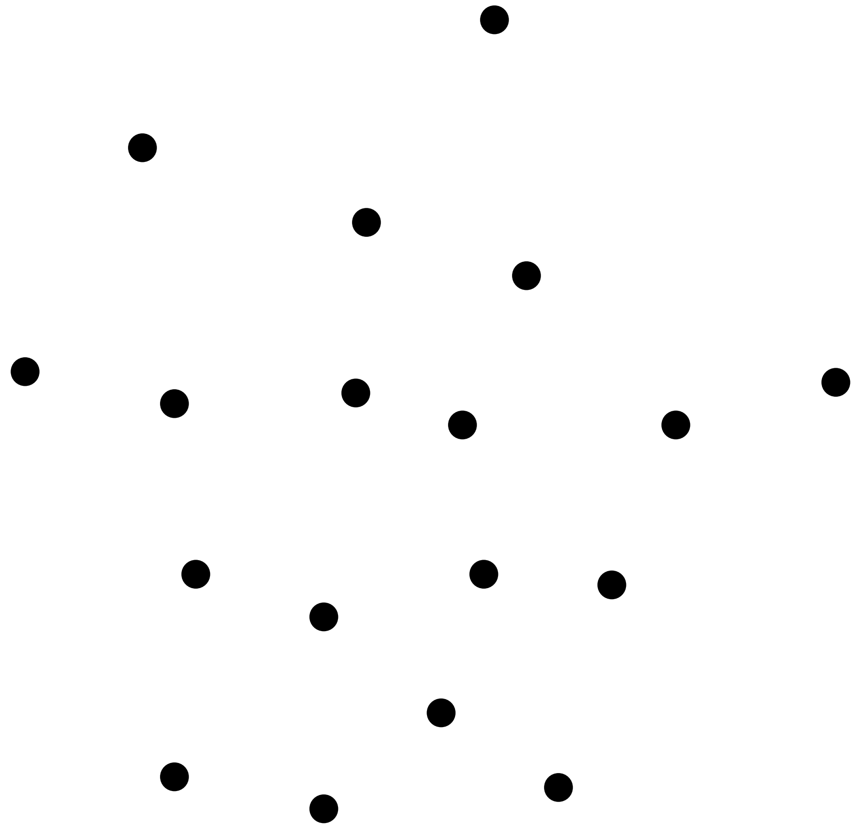

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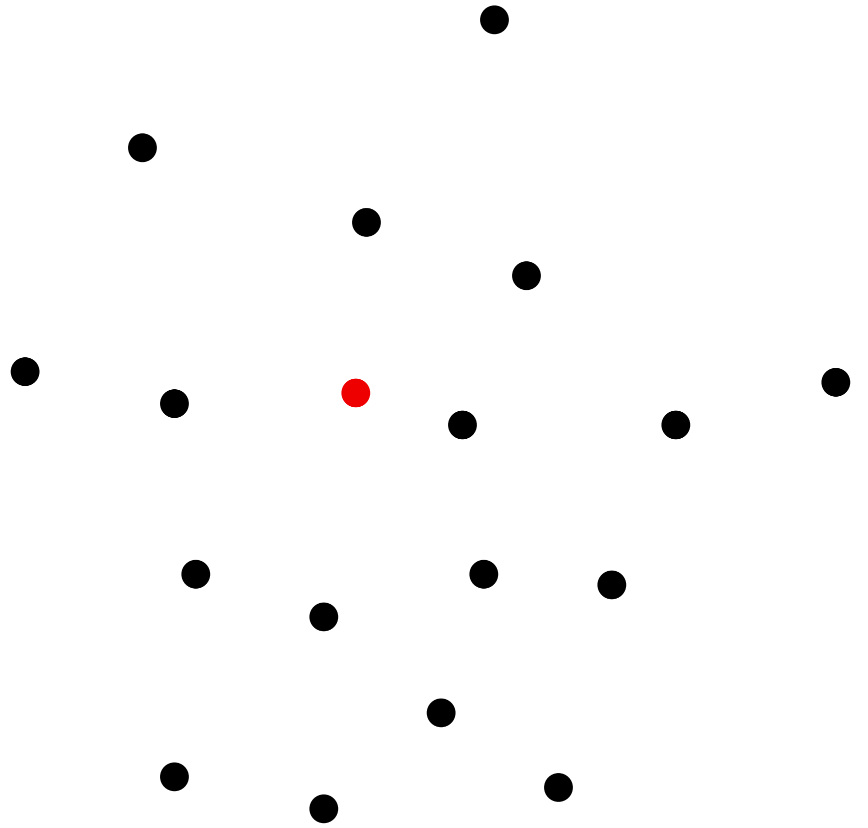

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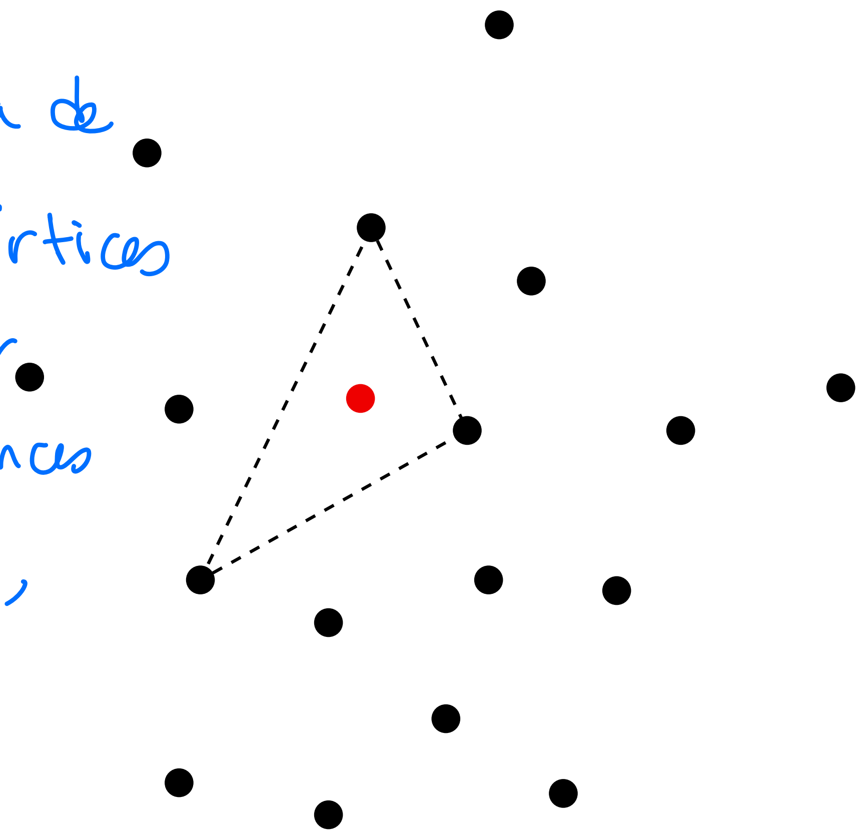
Demostración.

Sabemos que ch(S) es la combinación convexa de todos los subconjuntos de S de tamaño 3, i.e. Es la unión de todos los Δ's con vértices en S.

Sea p ∈ S, tq p está en ch(S), si p está en el interior de algún triángulo, entonces no es extremo.

Si no existe ningún Δ que contenga a p, entonces p es extremo.

Es fácil ver.



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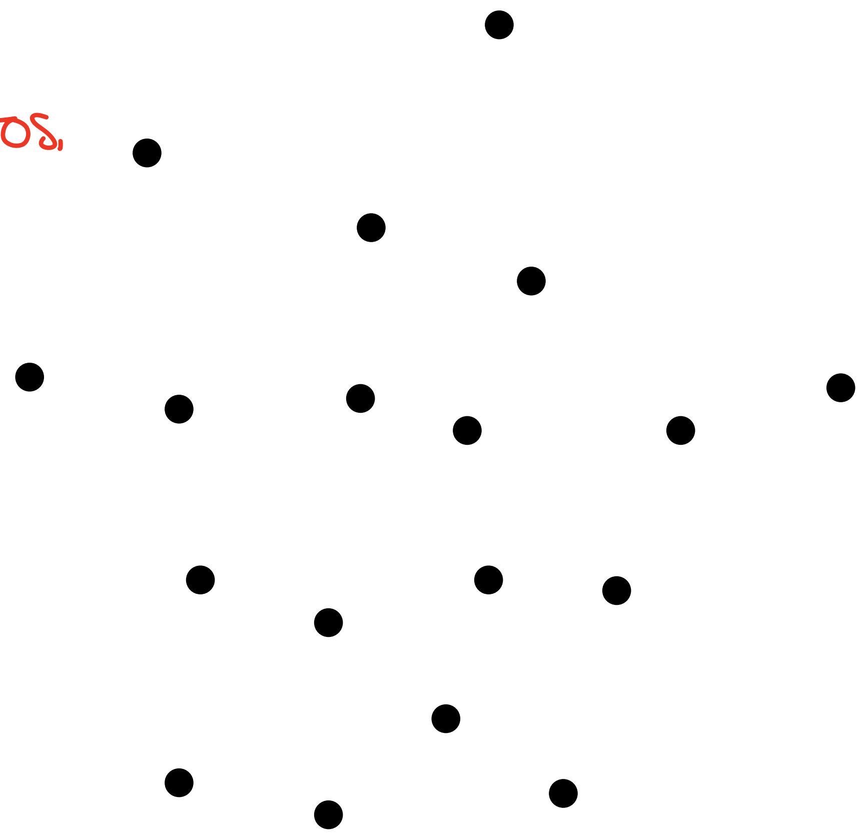
Algorithm

Input: $p_1, \ldots, p_n$ ← todos los pontos distintos.

Output: set of the extreme points

Procedure:

- For each i,
  - For each $j, k, l \neq i$,
  - If p_i lies in the triangle p_j, p_k, p_l, eliminate p_i.
- Return the set of surviving p_i's.



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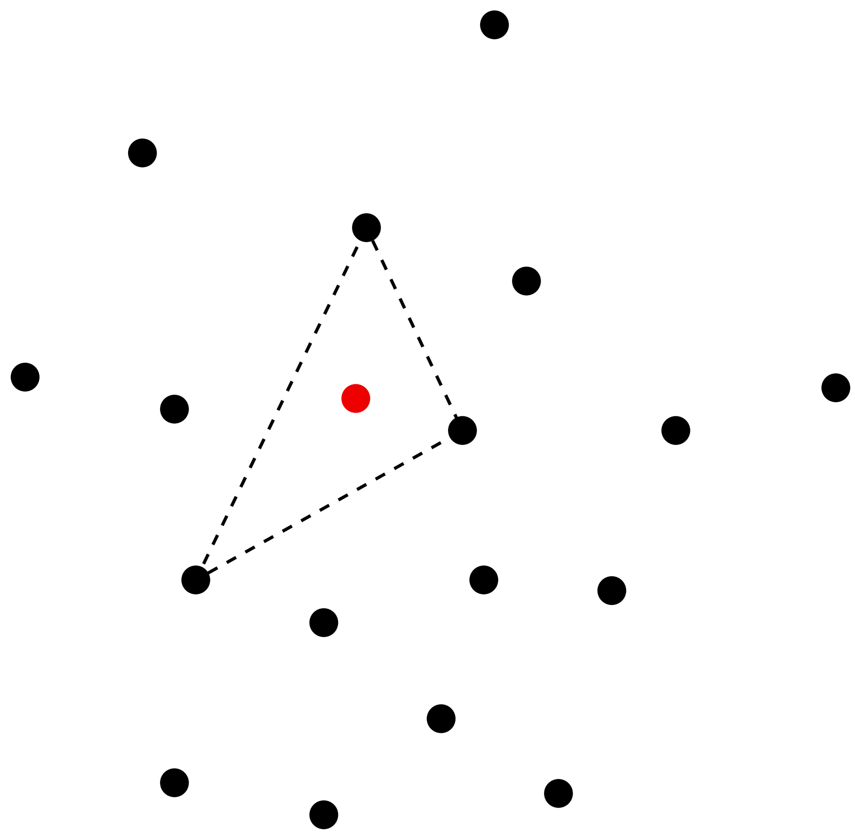
Algorithm: INTERIOR POINTS
for each i do
for each $j \neq i$ do
for each $k \neq i \neq j$ do
  for each $l \neq i \neq j \neq k$ do
    if $p_l \in \Delta(p_i, p_j, p_k)$
      then p_l is nonextreme

O'Rourke



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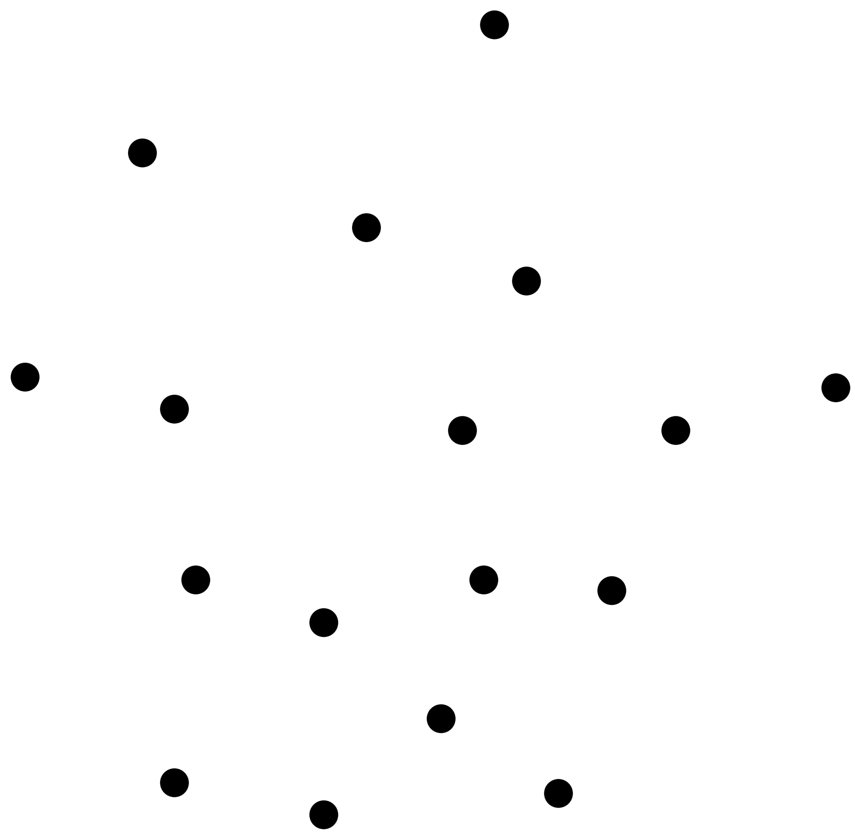

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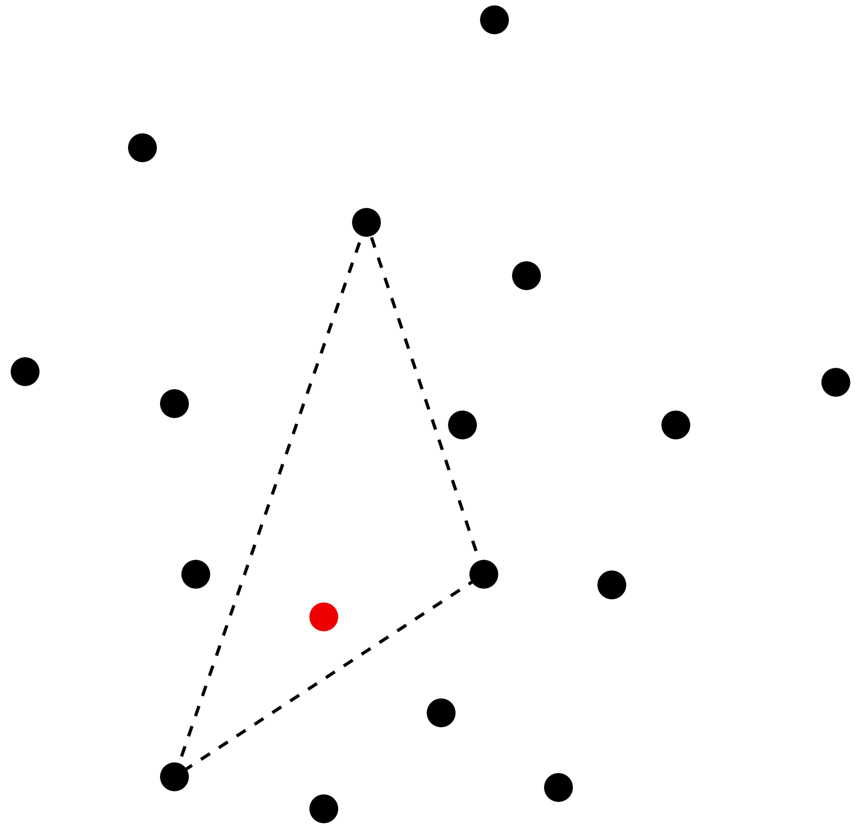

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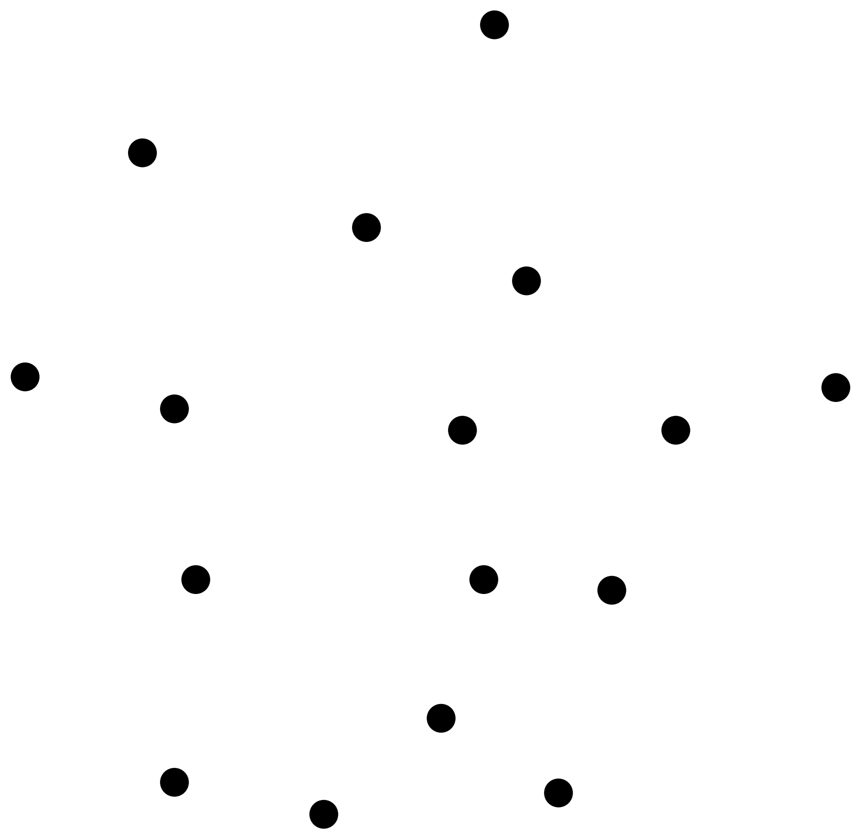

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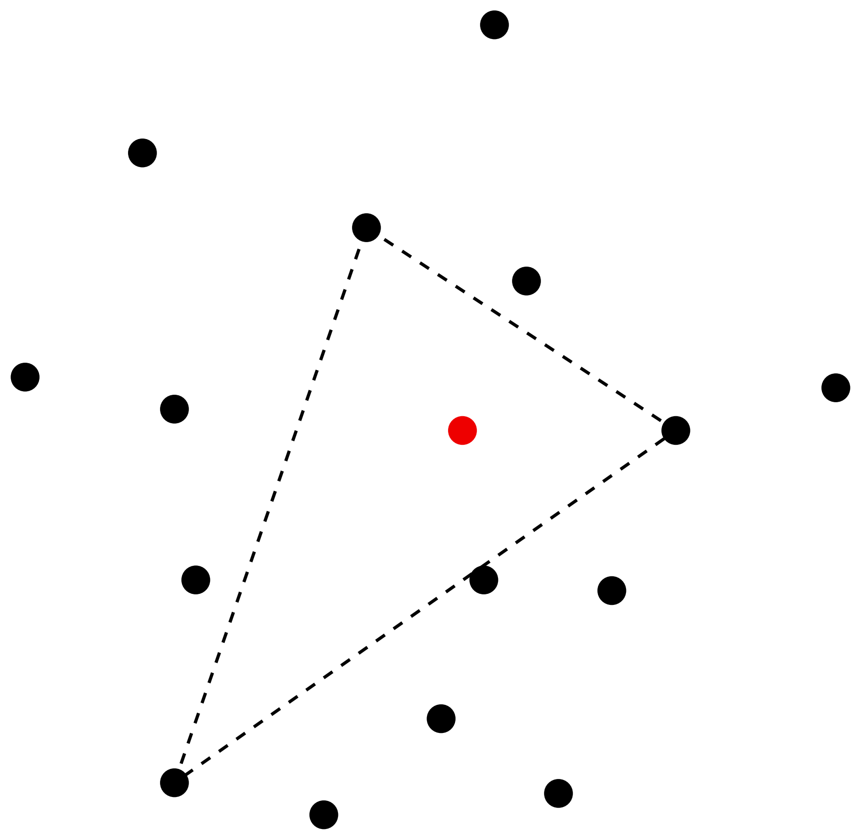

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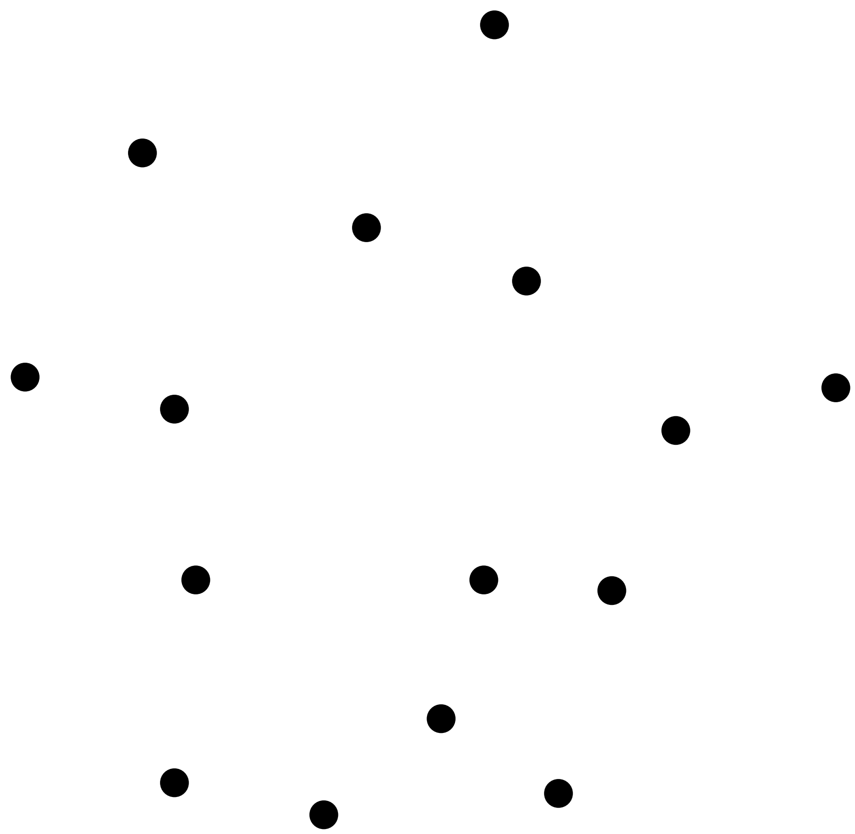

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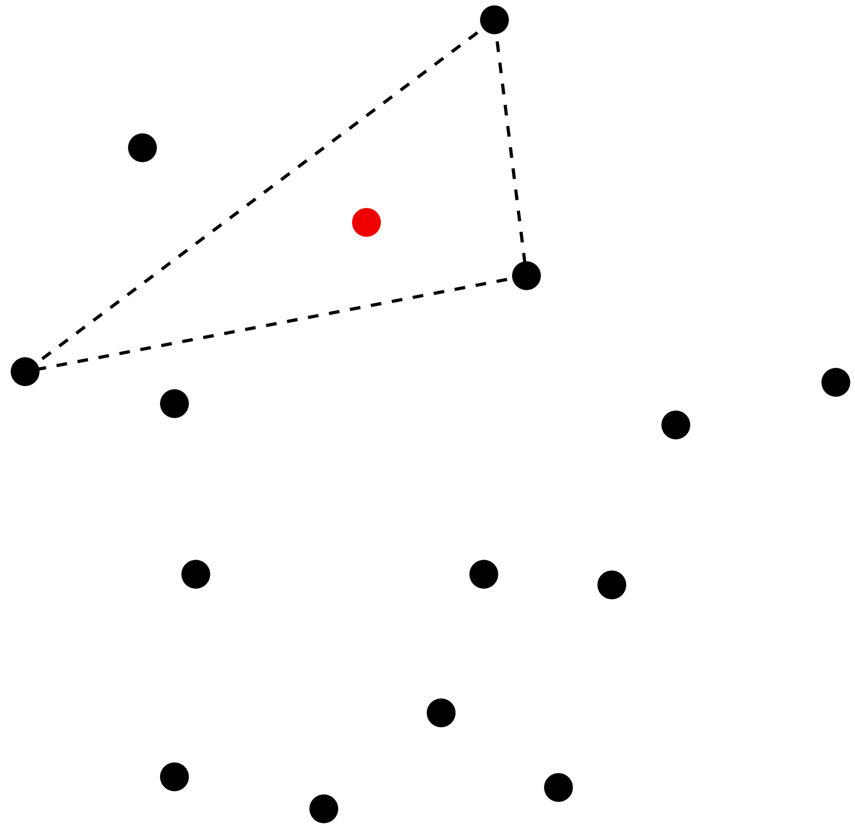

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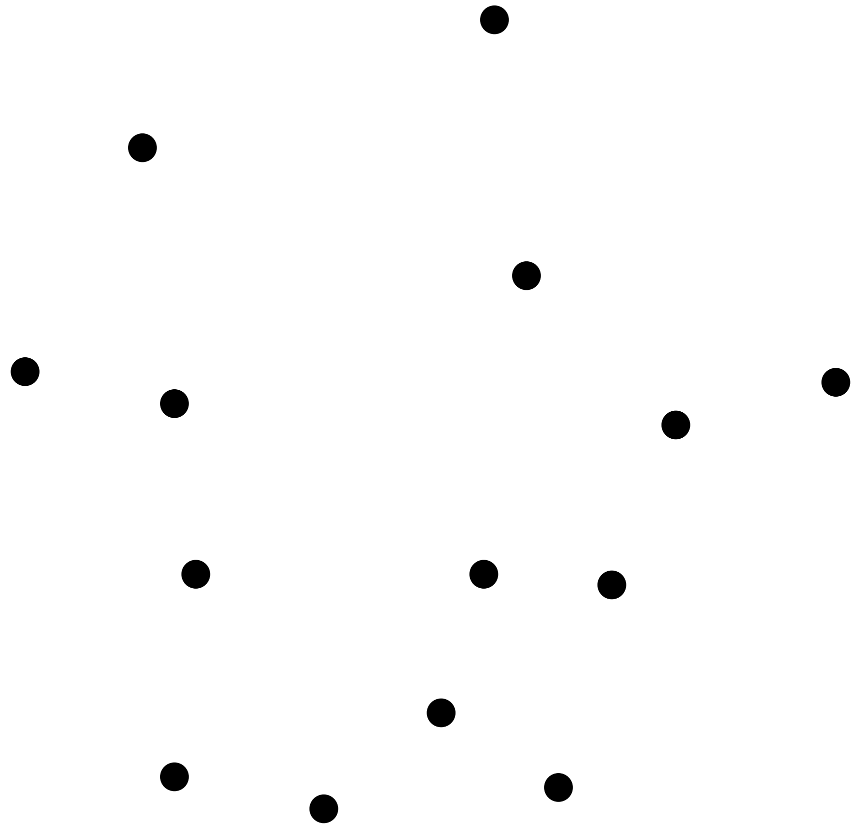

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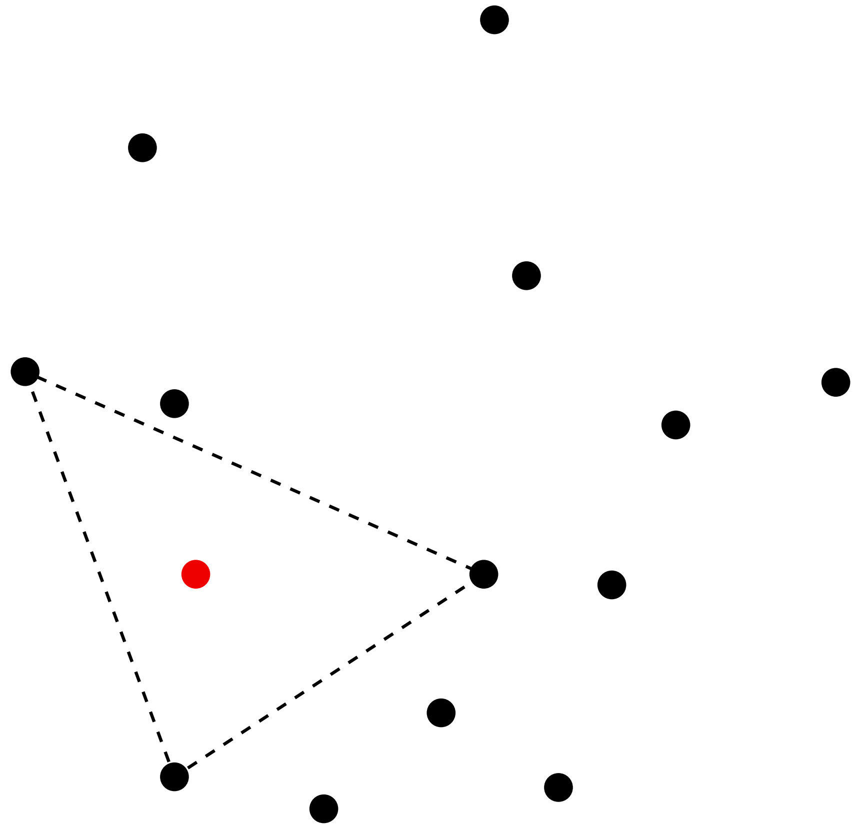

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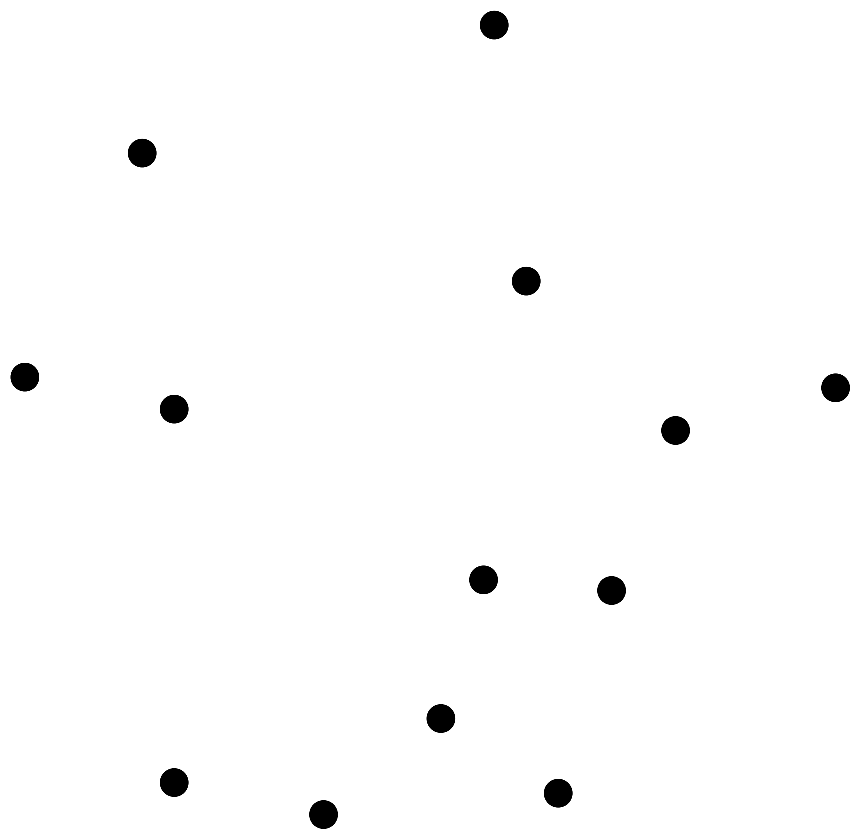

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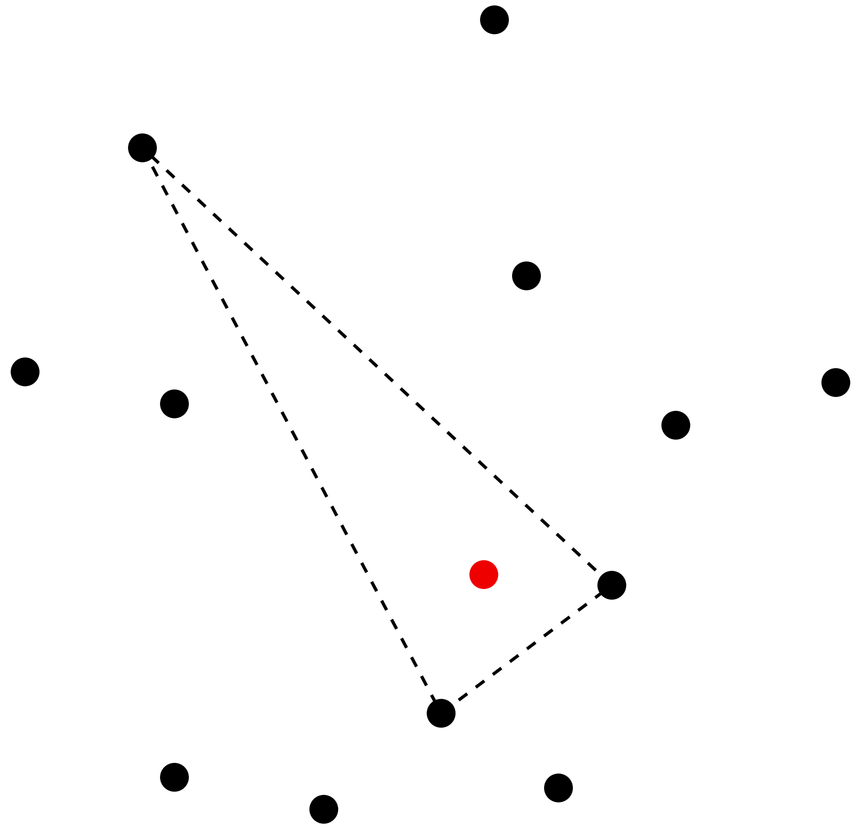

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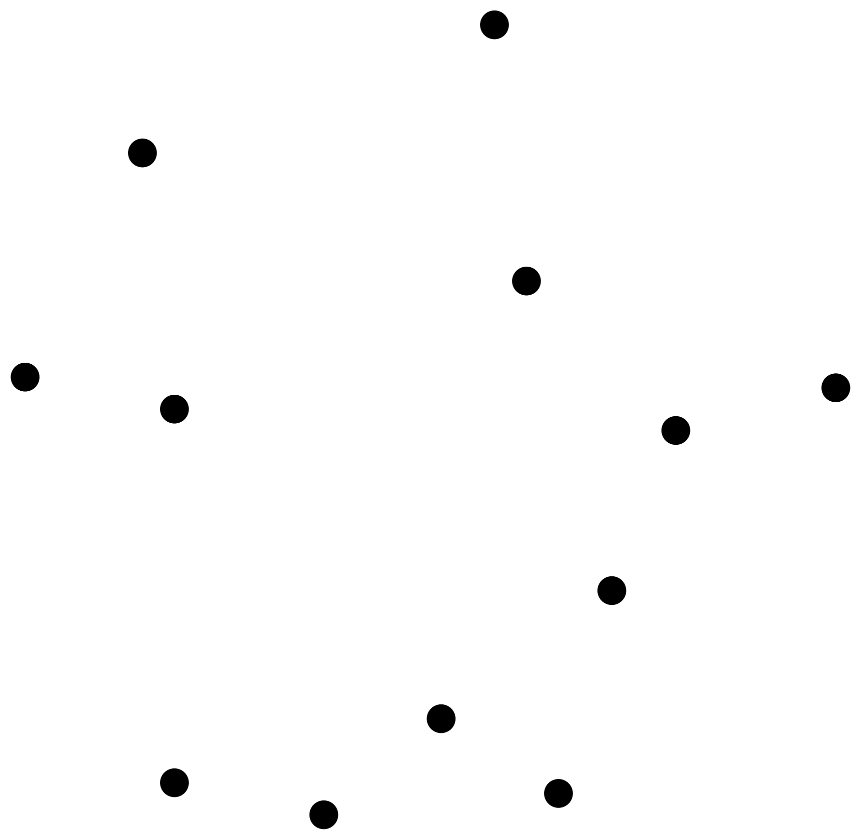

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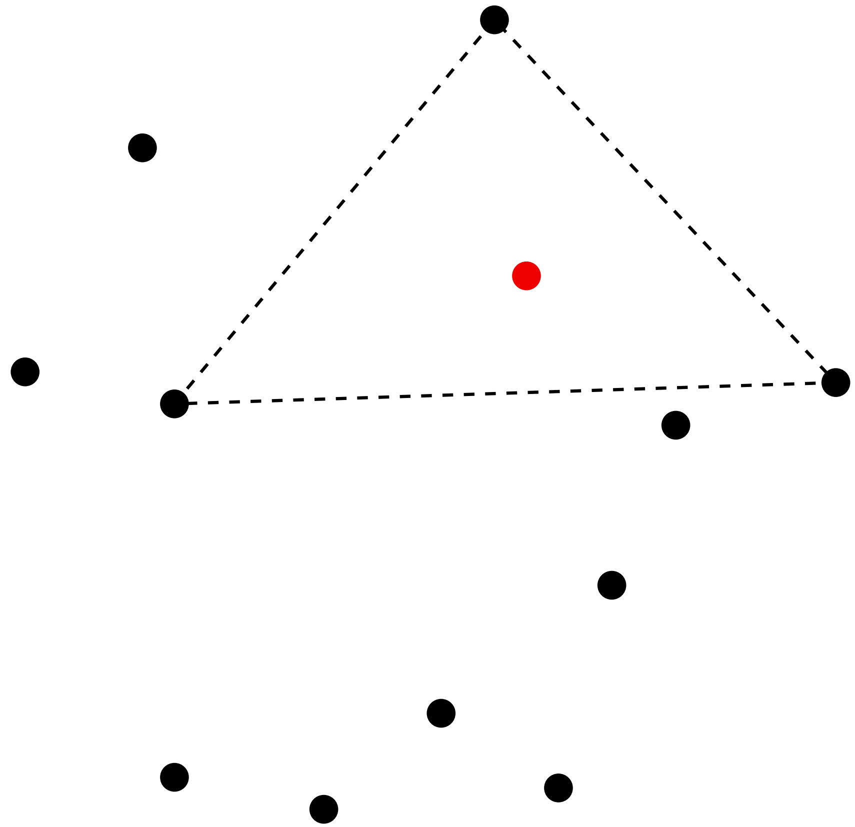

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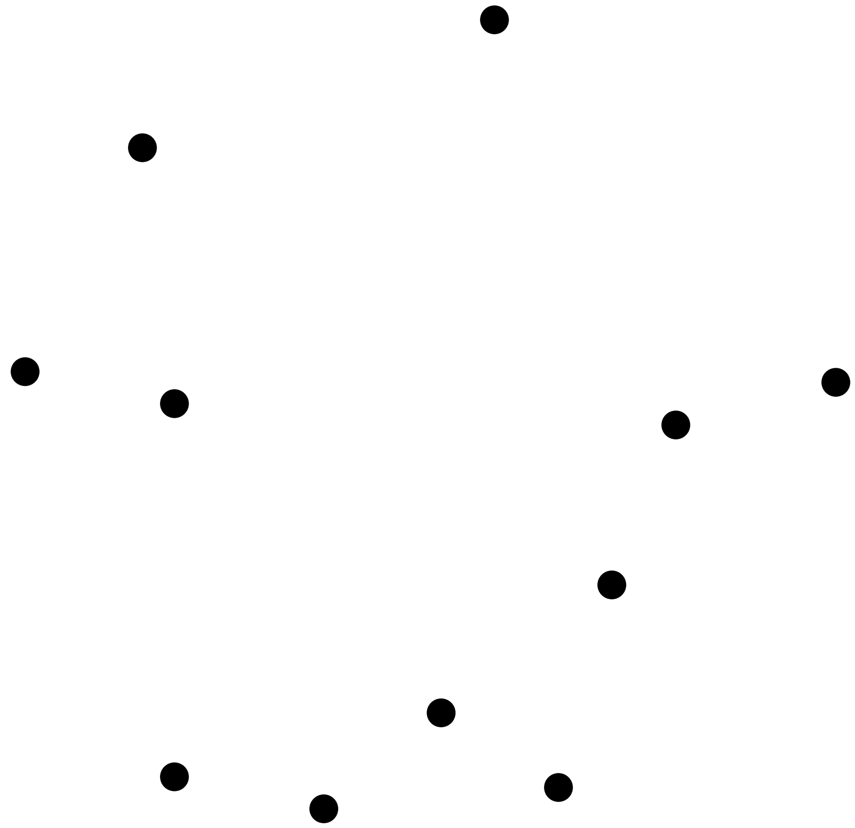

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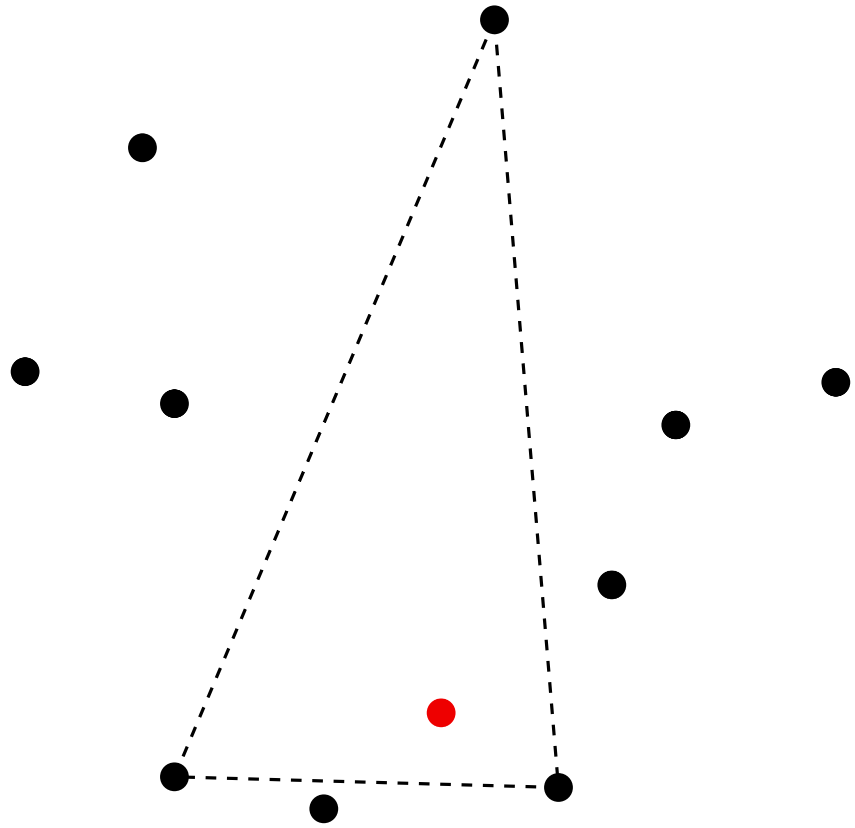

CONVEX HULL

Computing the extreme points

Characterization

Given $X = \{p_1, \ldots, p_n\}$, the point p_i belongs to the boundary of the convex hull of X if and only if p_i does not lie in any of the triangles $p_j p_k p_l$.

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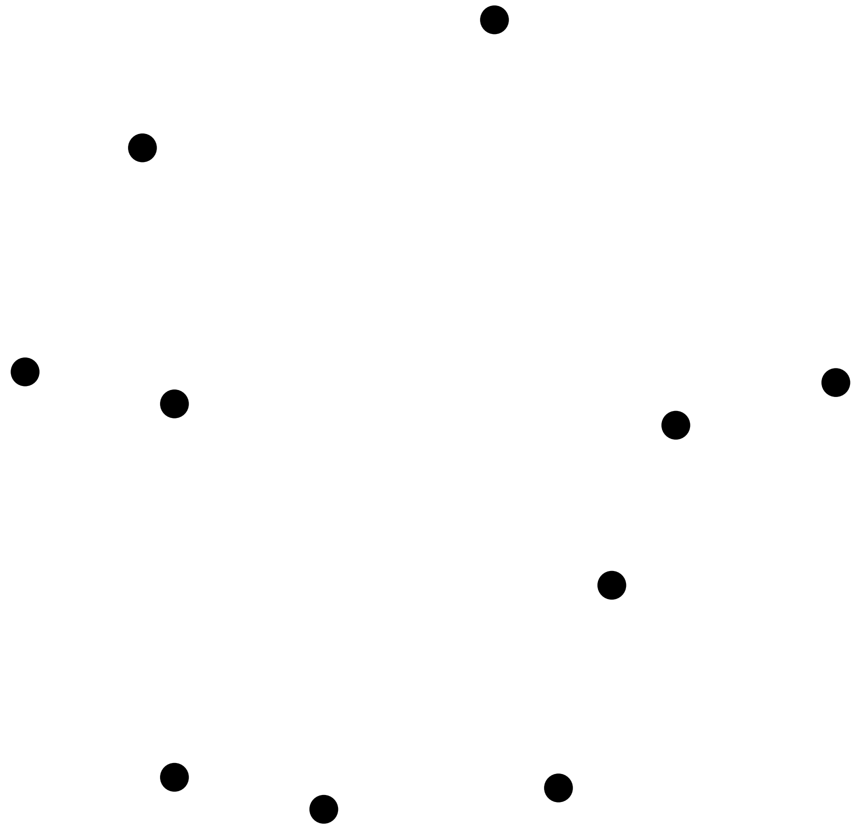

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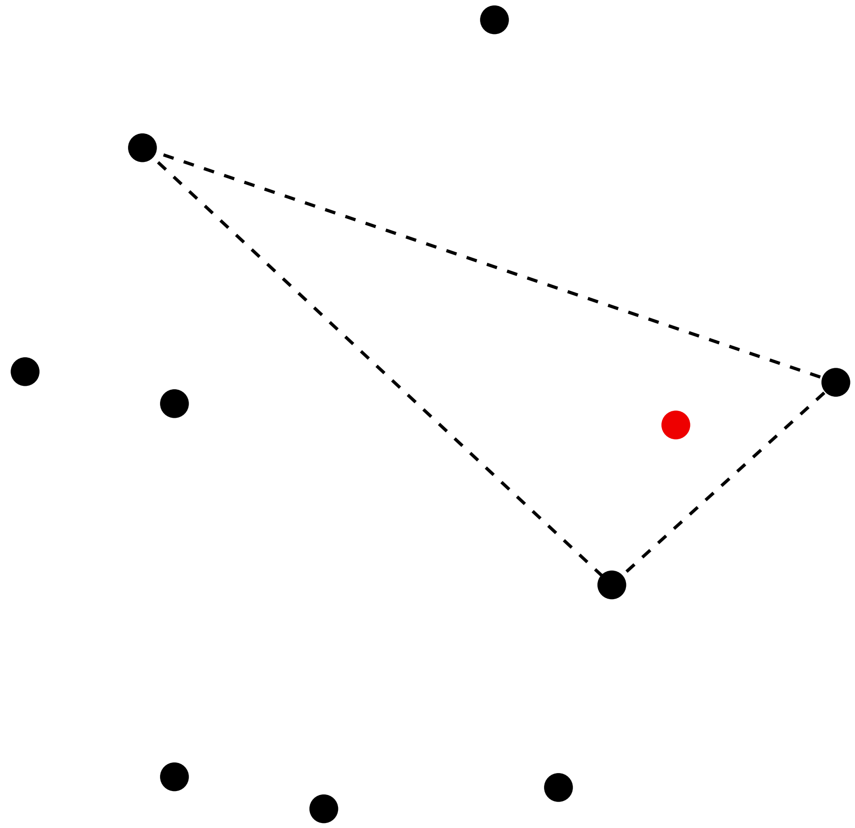

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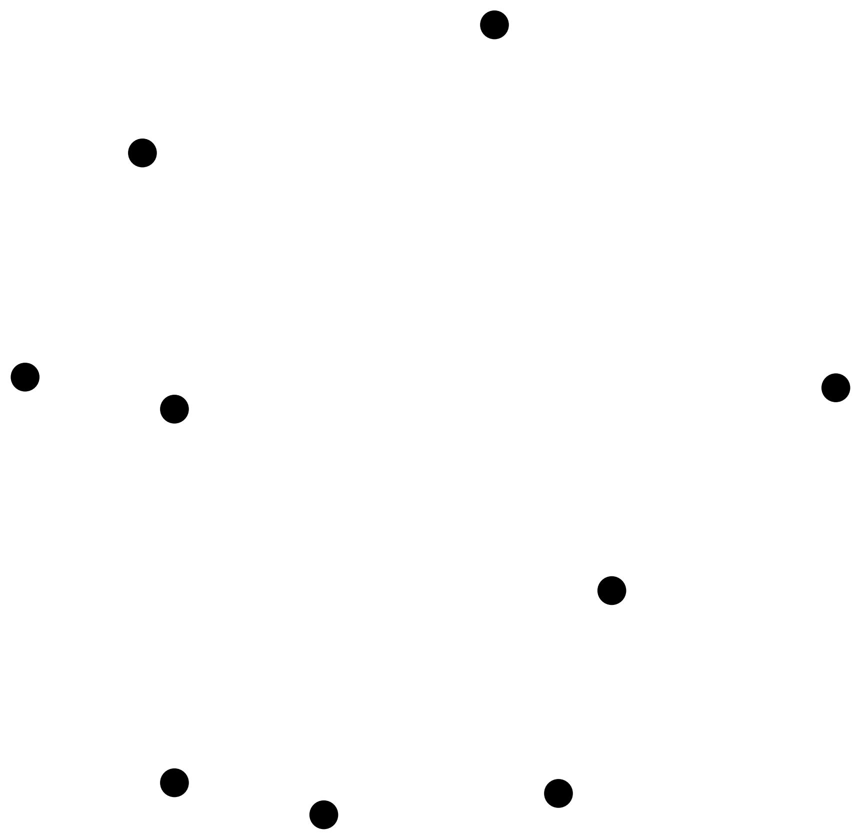

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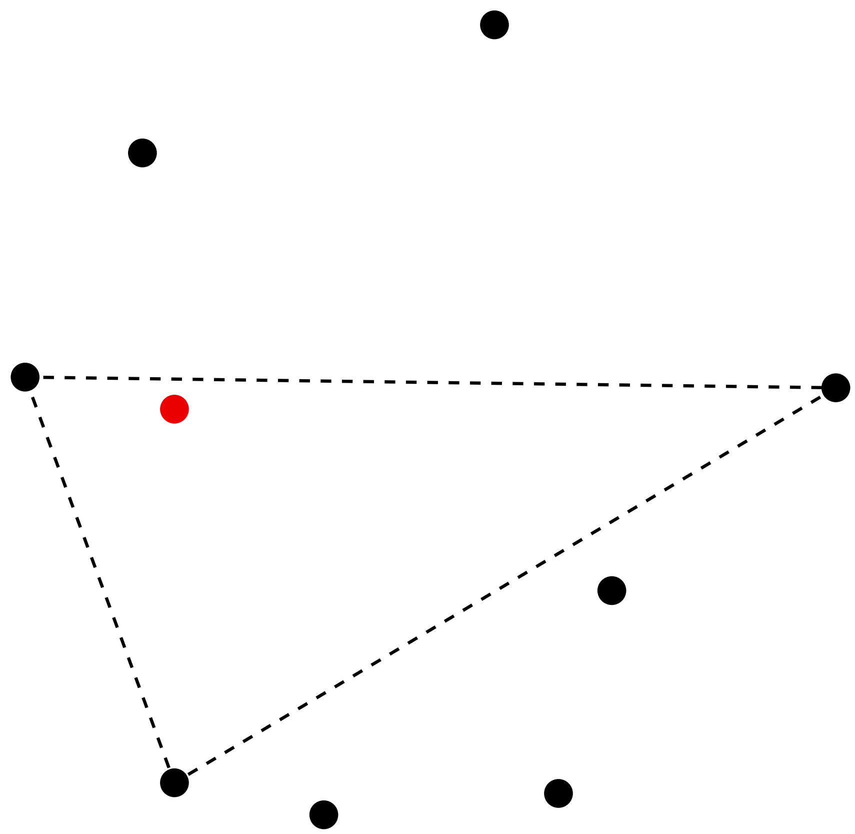

CONVEX HULL

Computing the extreme points

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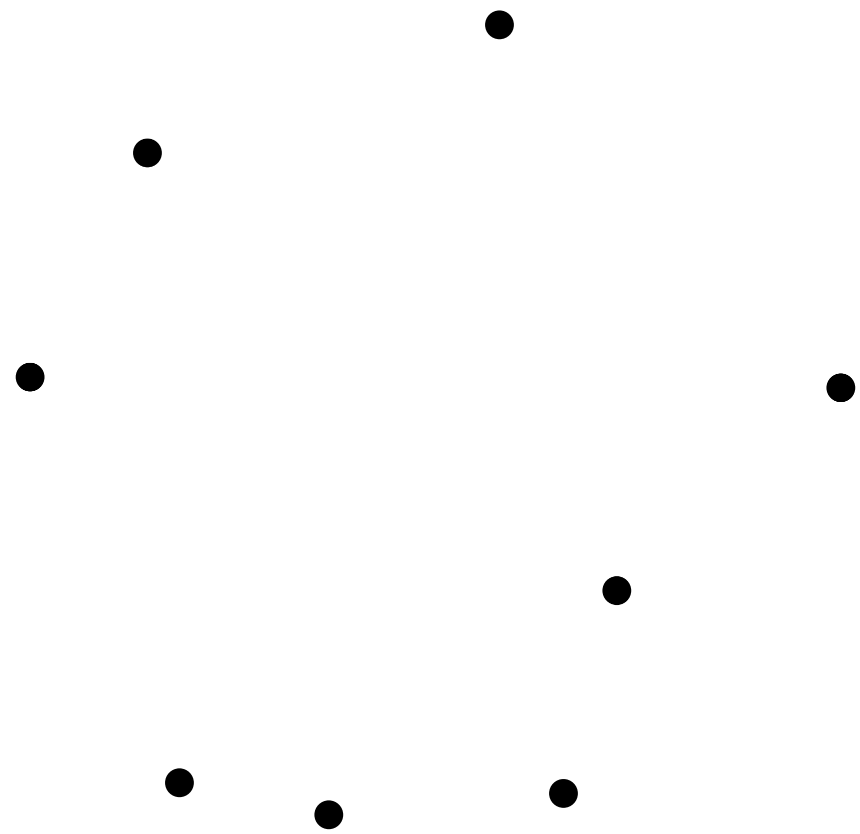

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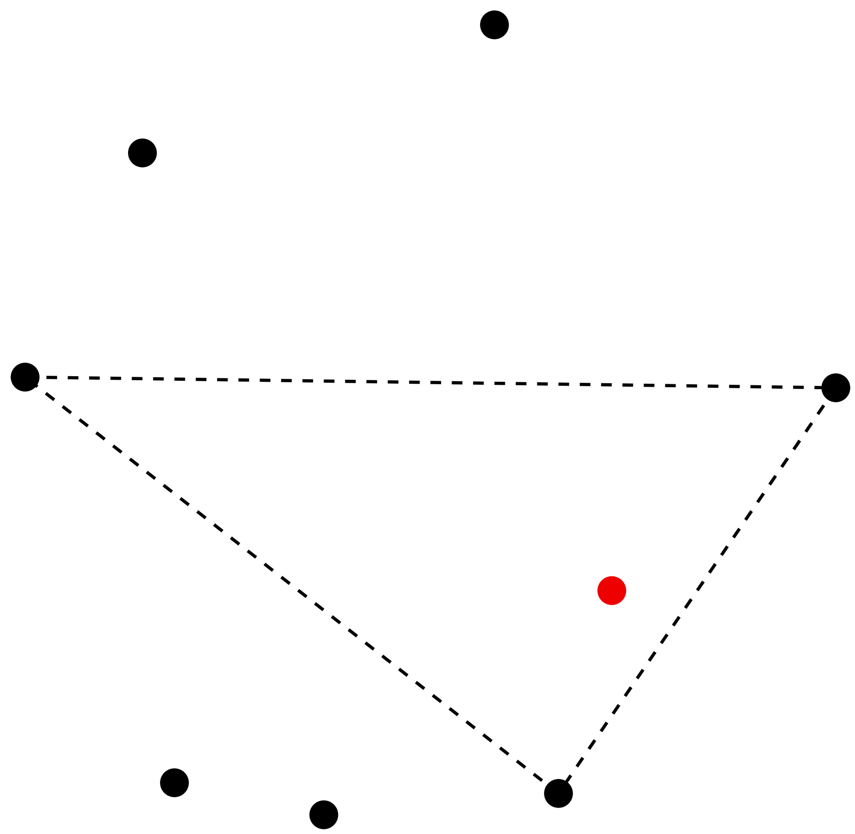

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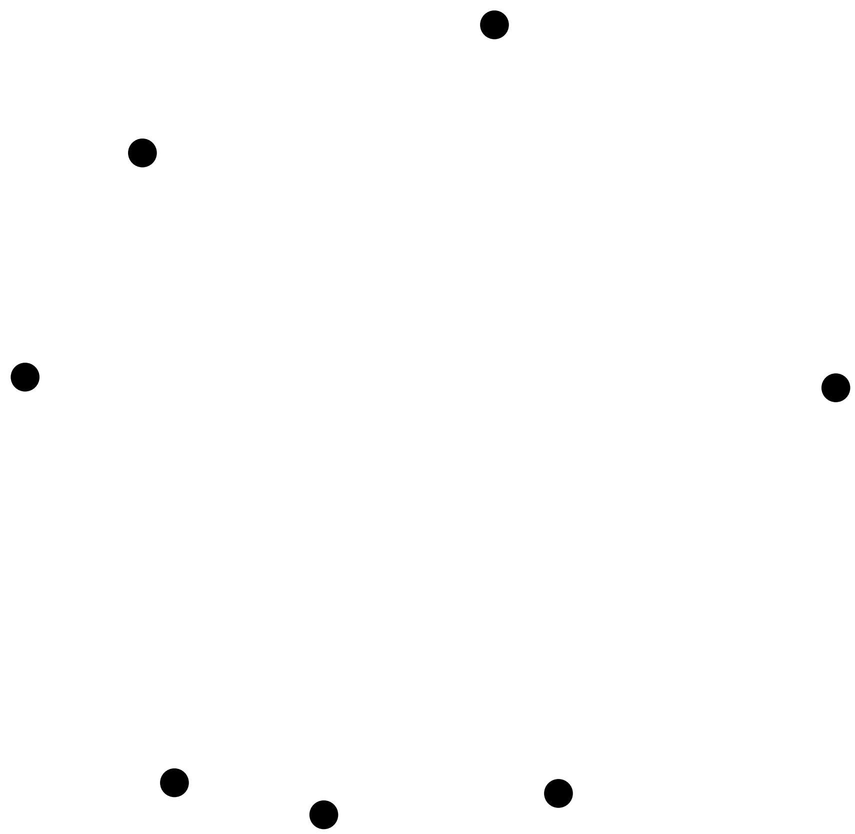

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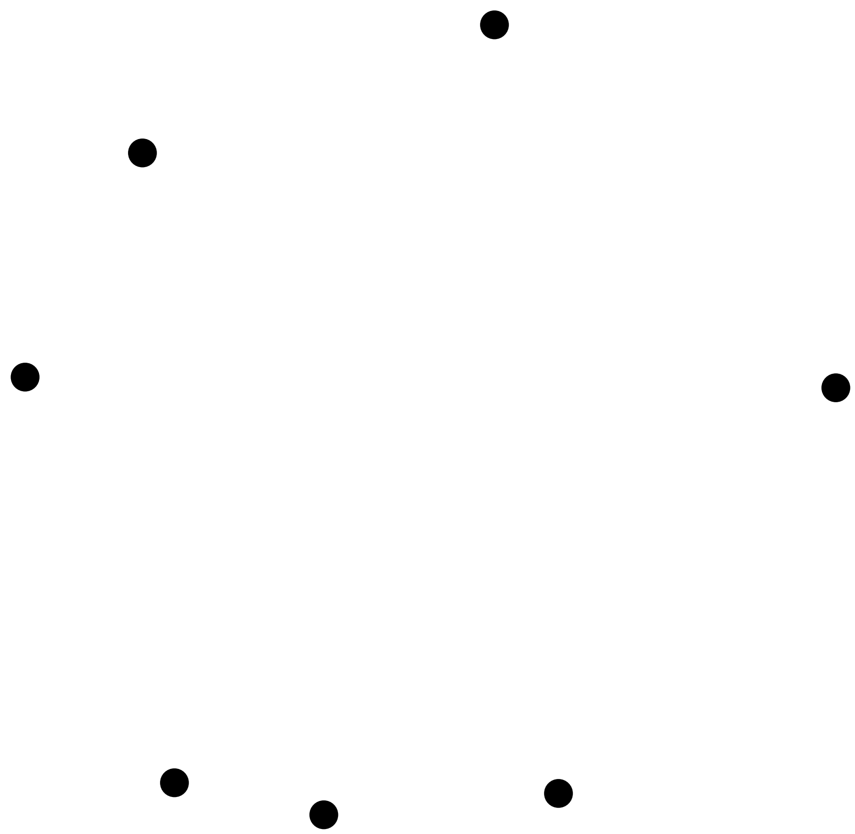


Running time: $O(n^4)$

Computing the extreme segments

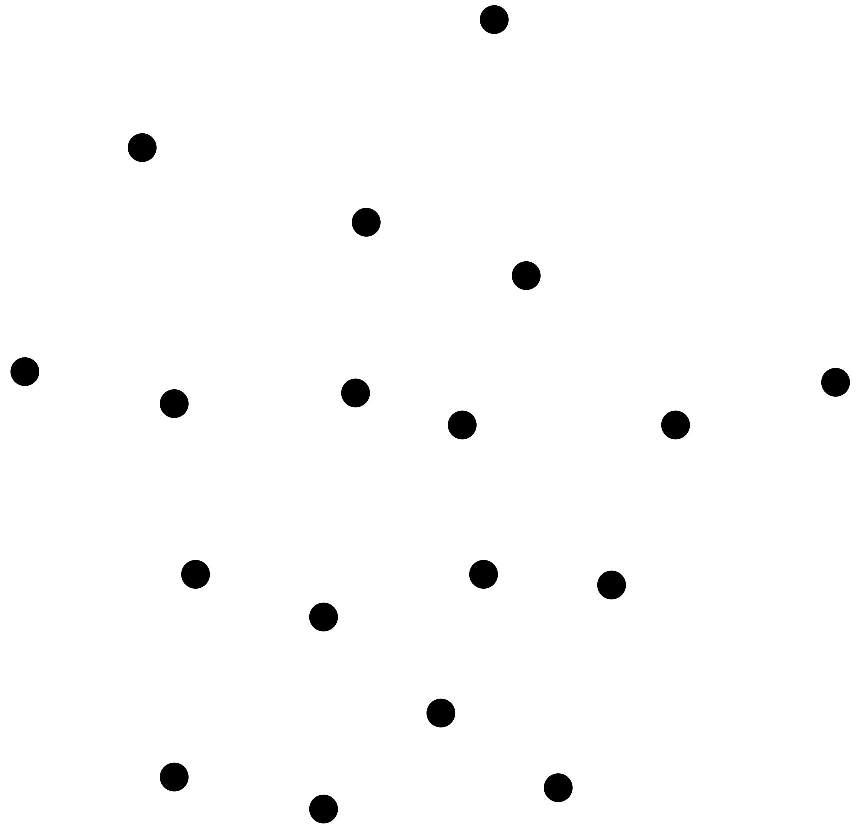
Computing the extreme segments

Characterization

Given $X = \{p_1, \ldots, p_n\}$, the segment $p_i p_j$ is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line $p_i p_j$.

Computing the extreme segments

Characterization
Given $X = \{p_1, \ldots, p_n\}$, the segment p_ip_j is an extreme segment if and only if all the points p_k with $k \neq i, j$ lie in the same halfplane defined by the line p_ip_j.



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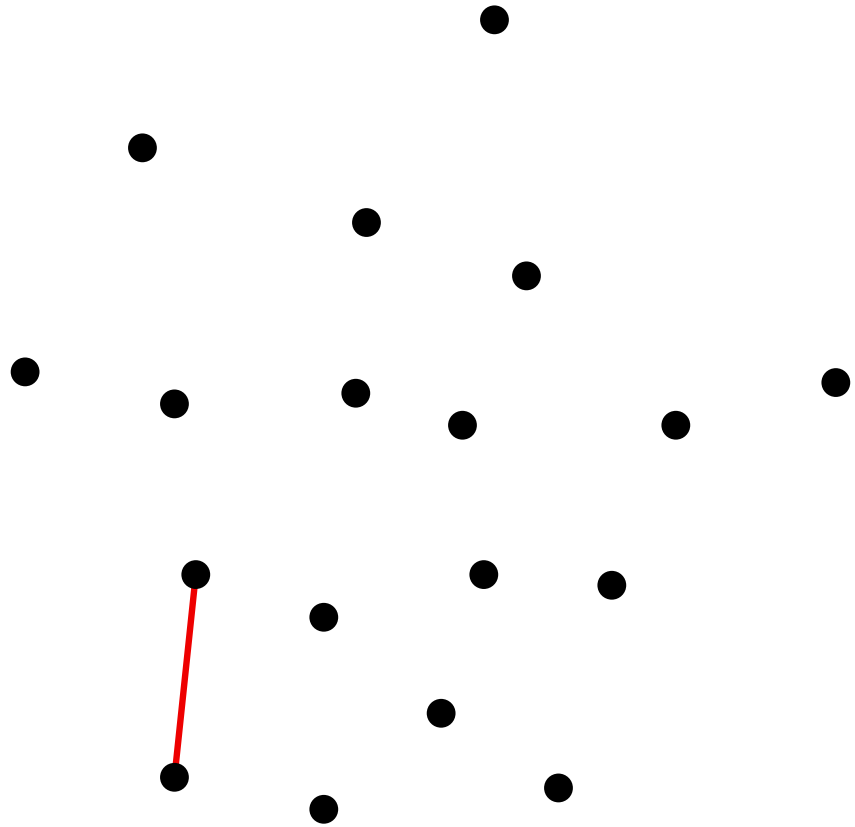

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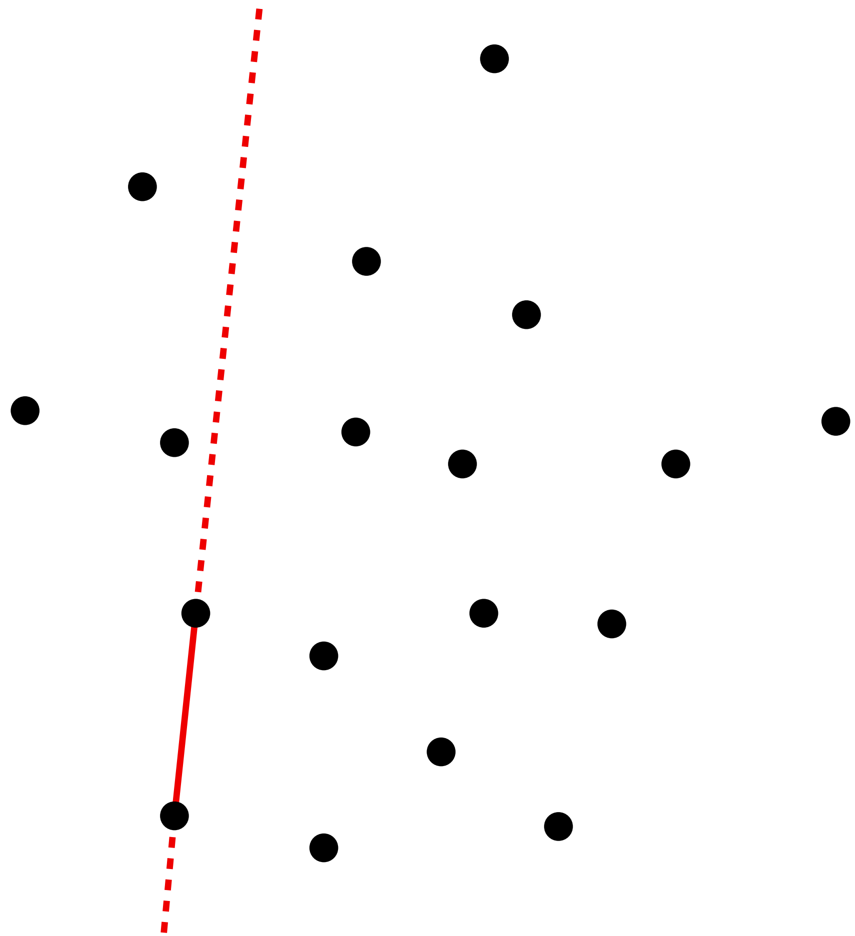

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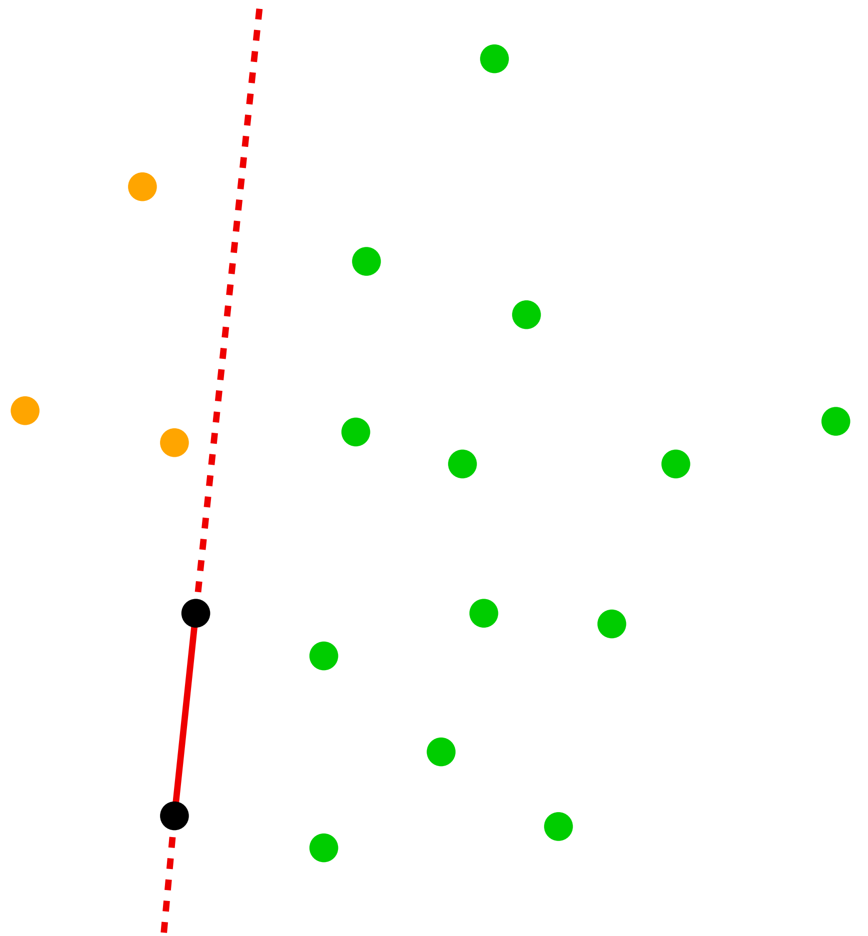

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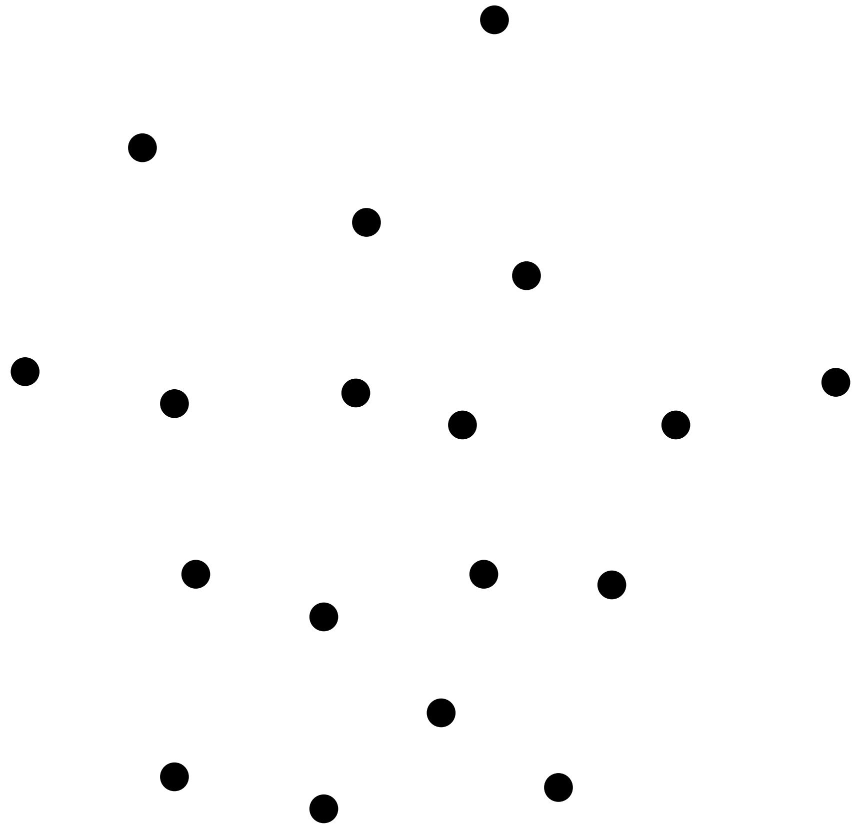

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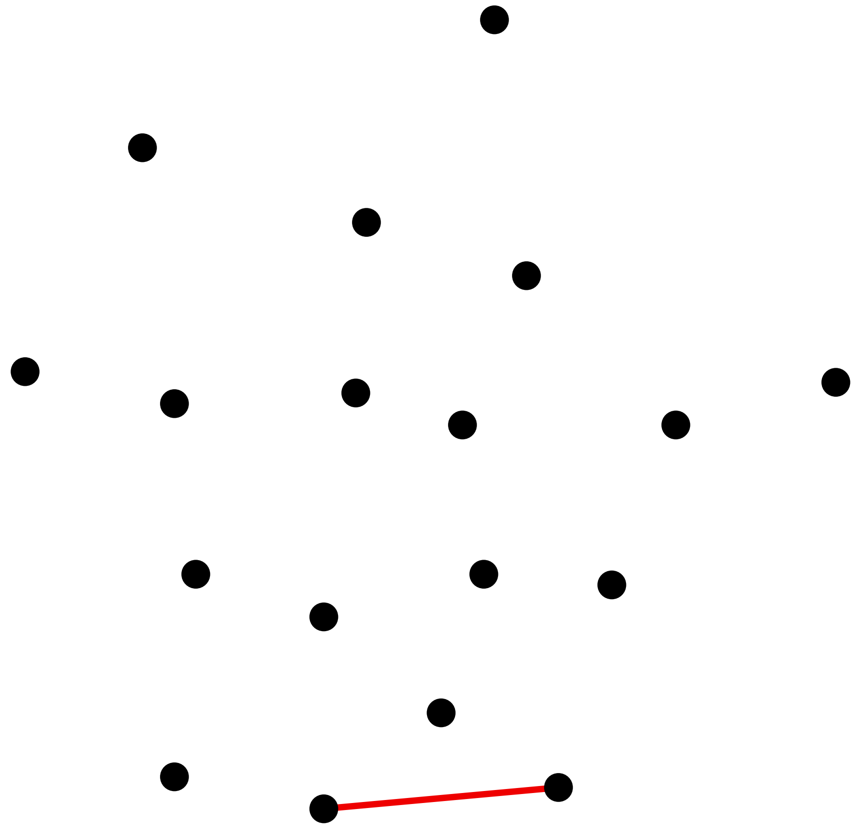

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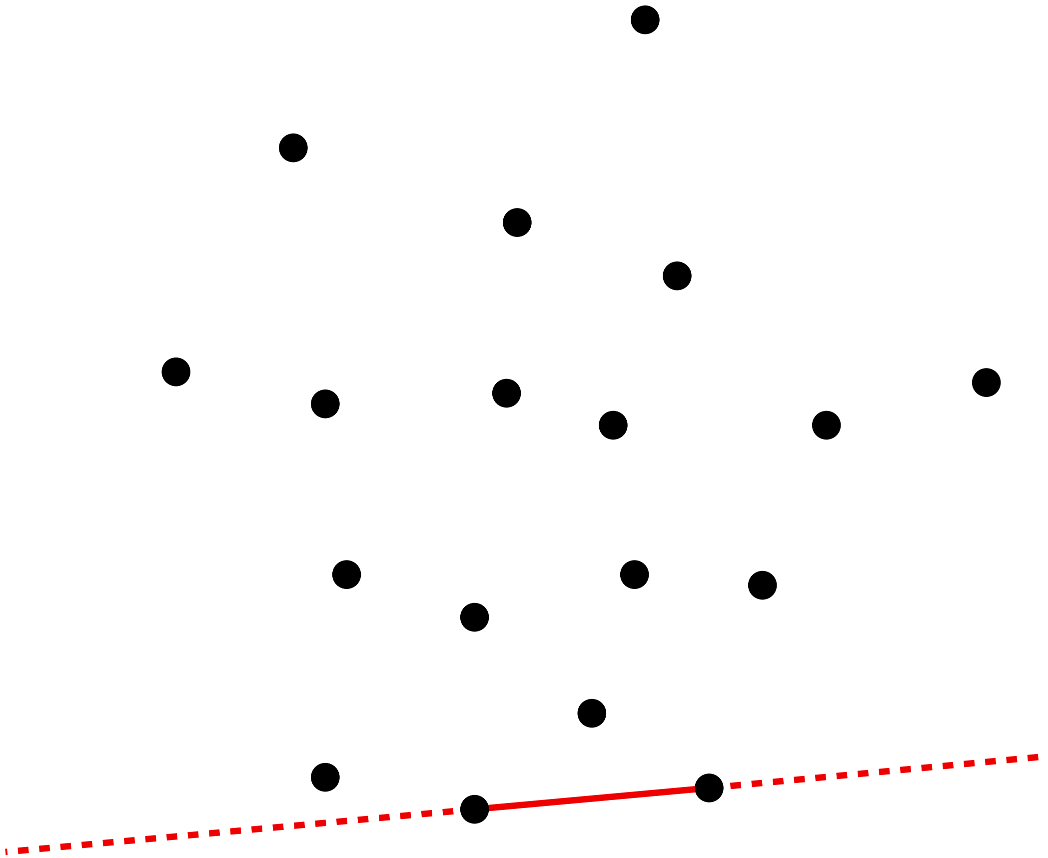

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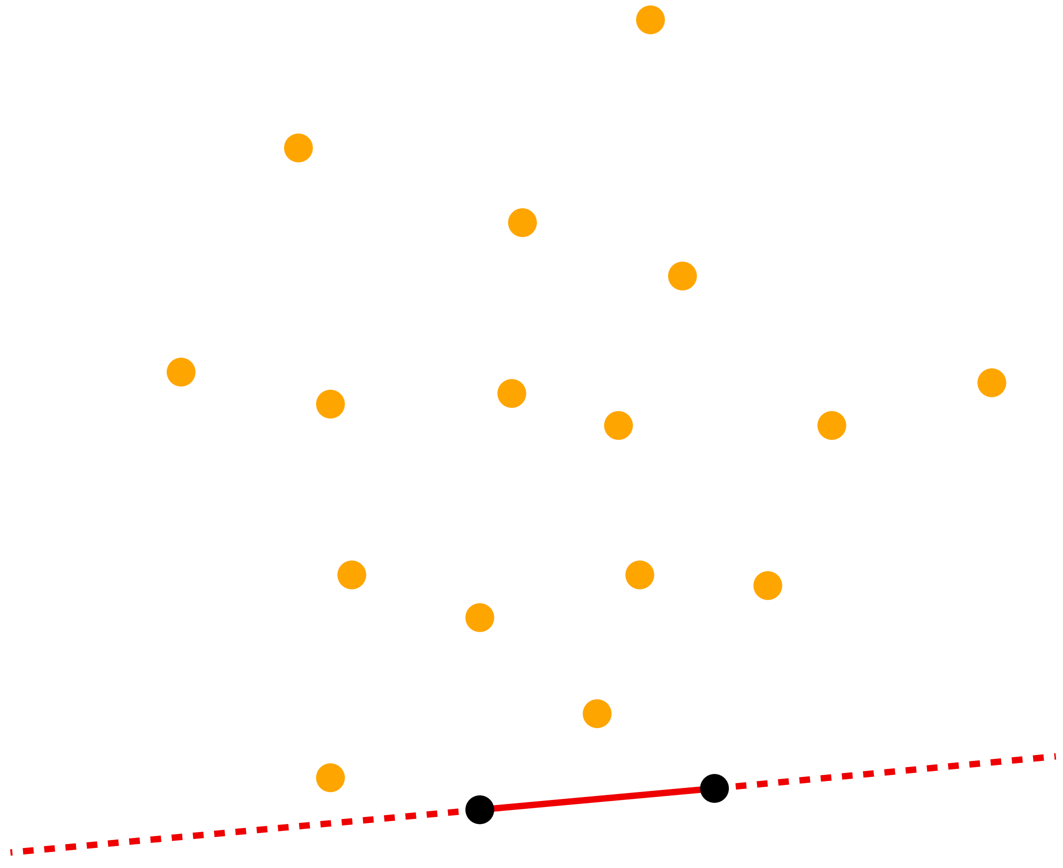

CONVEX HULL

Computing the extreme segments

Characterization

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Algorithm

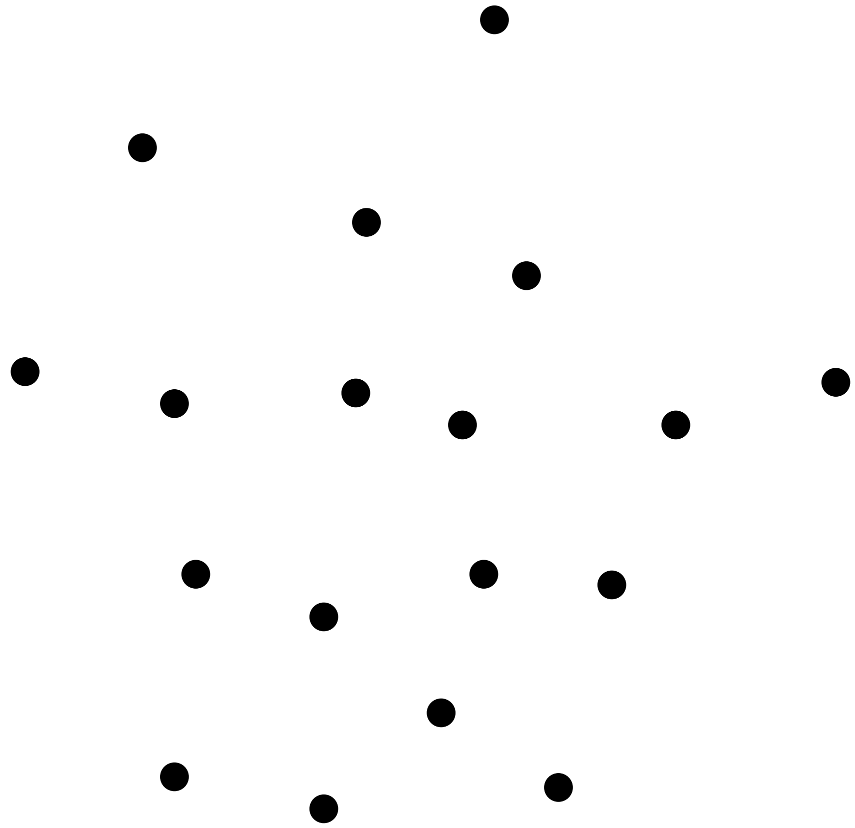
Input: $p_1, \ldots, p_n$
Output: set of the extreme segments

Procedure:

For each i, j,

Check whether all p_k with $k \neq i, j$ lie in the same halfplane defined by $p_i p_j$.

In the affirmative, return the segment $p_i p_j$.



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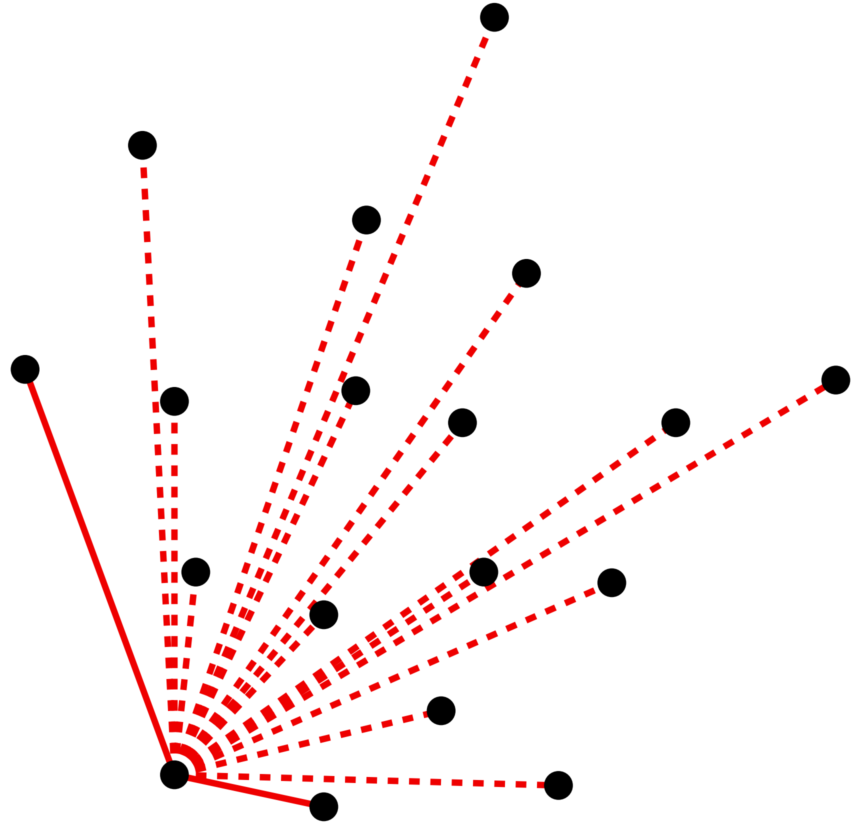

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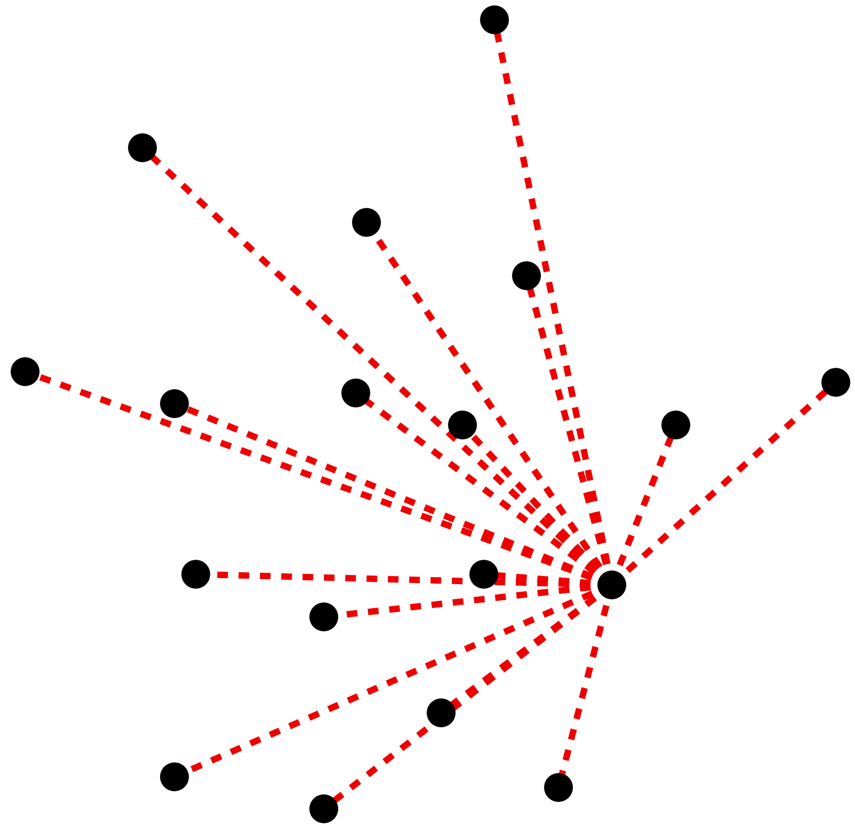

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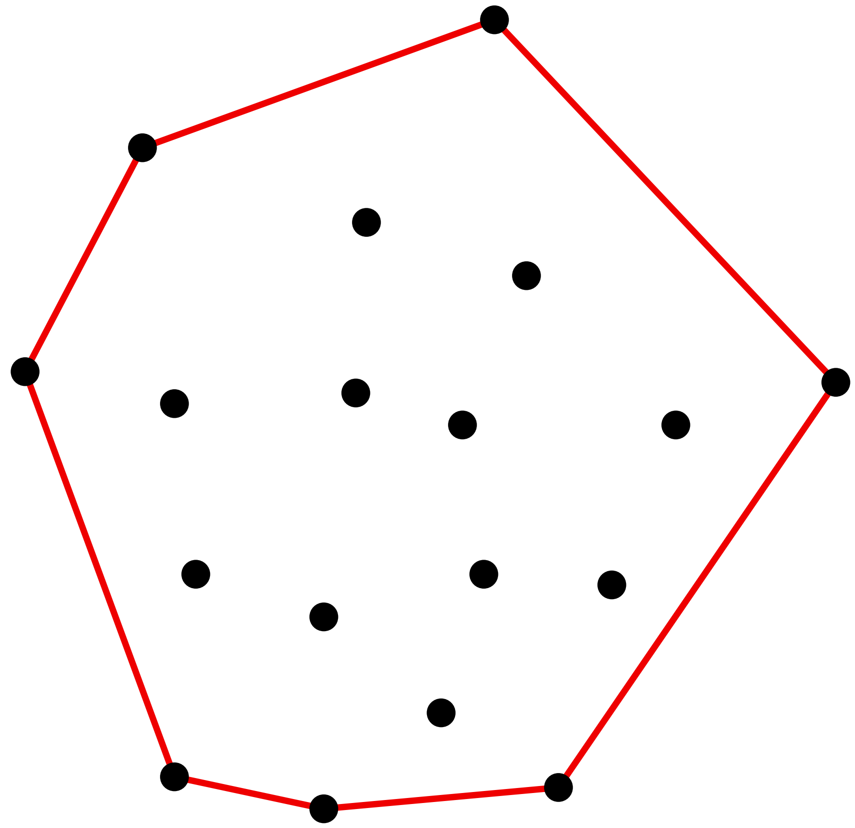

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O'Rourke.

Algorithm

Input: $p_1, \ldots, p_n$
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  For each i, j,
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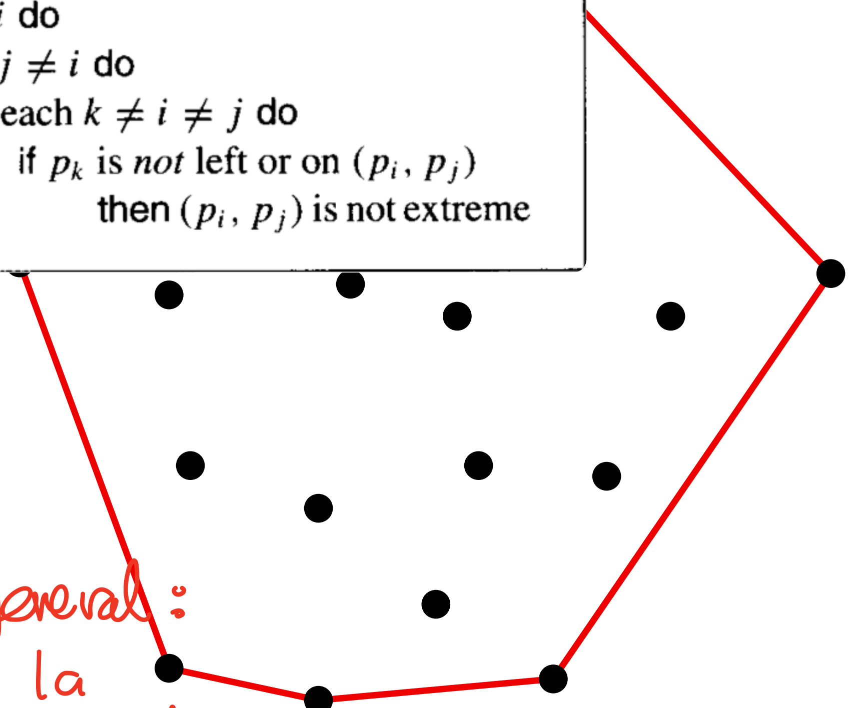
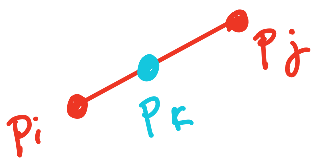
     lie in the same halfplane defined by $p_i p_j$.
    In the affirmative, return the segment $p_i p_j$.

Algorithm: EXTREME EDGES
for each i do
for each $j \neq i$ do
  for each $k \neq i \neq j$ do
    if p_k is *not* left or on (p_i, p_j)
      then (p_i, p_j) is not extreme



Running time: $\Theta(n^3)$

Problemas si los pontos no están en posición general:



el algoritmo produce la arista pi pj, pi pk y pk pj ∀ → Produce salida (2)

Computing the convex hull

Computing the convex hull **(sorted list of its vertices)**

Computing the convex hull

Input:
$P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:
l, the list of the vertices of $ch(P)$ sorted in counterclockwise order

↑ en el algoritmo original se genera como salida la lista de aristas, esta es una pequeña variante: generar los vértices como salida.

* consultar original en O'Rourke

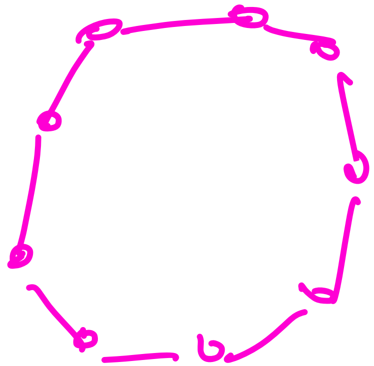
Computing the convex hull

Input:
$P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$ a set of n points in the plane

Output:
l, the list of the vertices of $ch(P)$ sorted in counterclockwise order

Characterization
Given $X = \{p_1, \ldots, p_n\}$, the segment $p_i p_j$ is an edge of the convex hull of X if and only if all the points p_k with $k \neq i, j$ lie to the left of the oriented line $p_i p_j$.

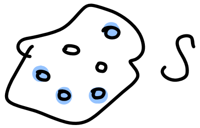
Podemos mejorar el algoritmo anterior
Hay exactamente n aristas.



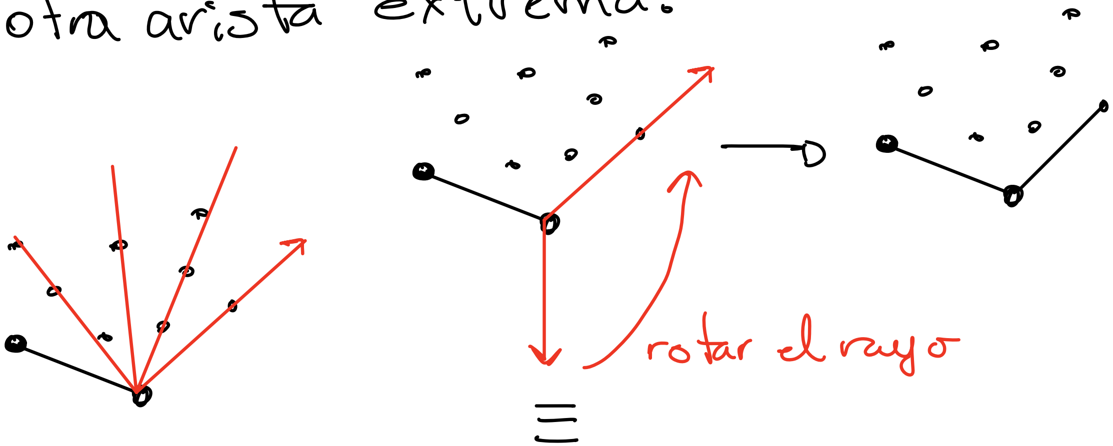
Jarvis march, 1973. Raymond Austin Jarvis, Ing. Electrico Australiano murió en 2013,
= Gift wrapping.

Suponemos posición general.

Idea: Usar la información ya encontrada para encontrar nueva información. Esta es una técnica muy común.



Podemos replantear el problema:
Dada una arista extrema de ch(S), encontrar otra arista extrema.



Jarvis march

El de menor coord y, por ejemplo.

O(n)

1. Find a vertex of $ch(P)$ (for example, the lexicographically smaller point $p_i \in P$) and add it to l
2. While $v = \mathrm{Last}(l) \neq \mathrm{First}(l)$, do:
   (a) Detect the angularly rightmost point $p_j \in P$ with respect to v.
   (b) Add p_j to l
3. Return l

Jarvis march

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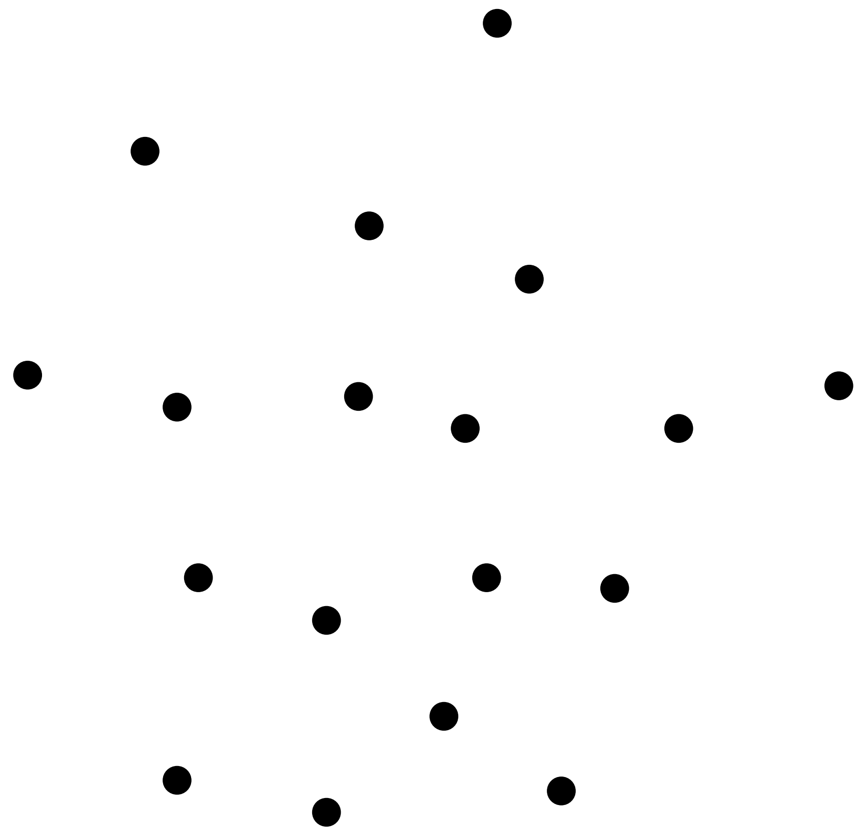

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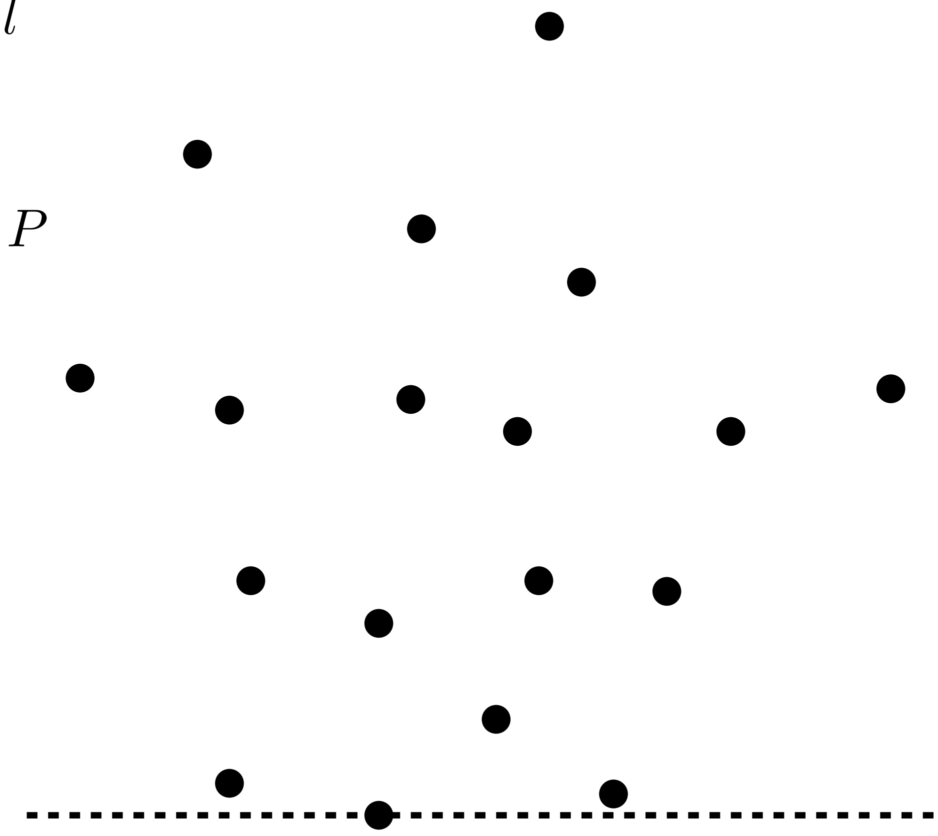
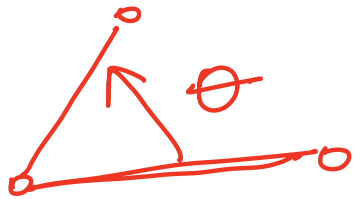


Jarvis march



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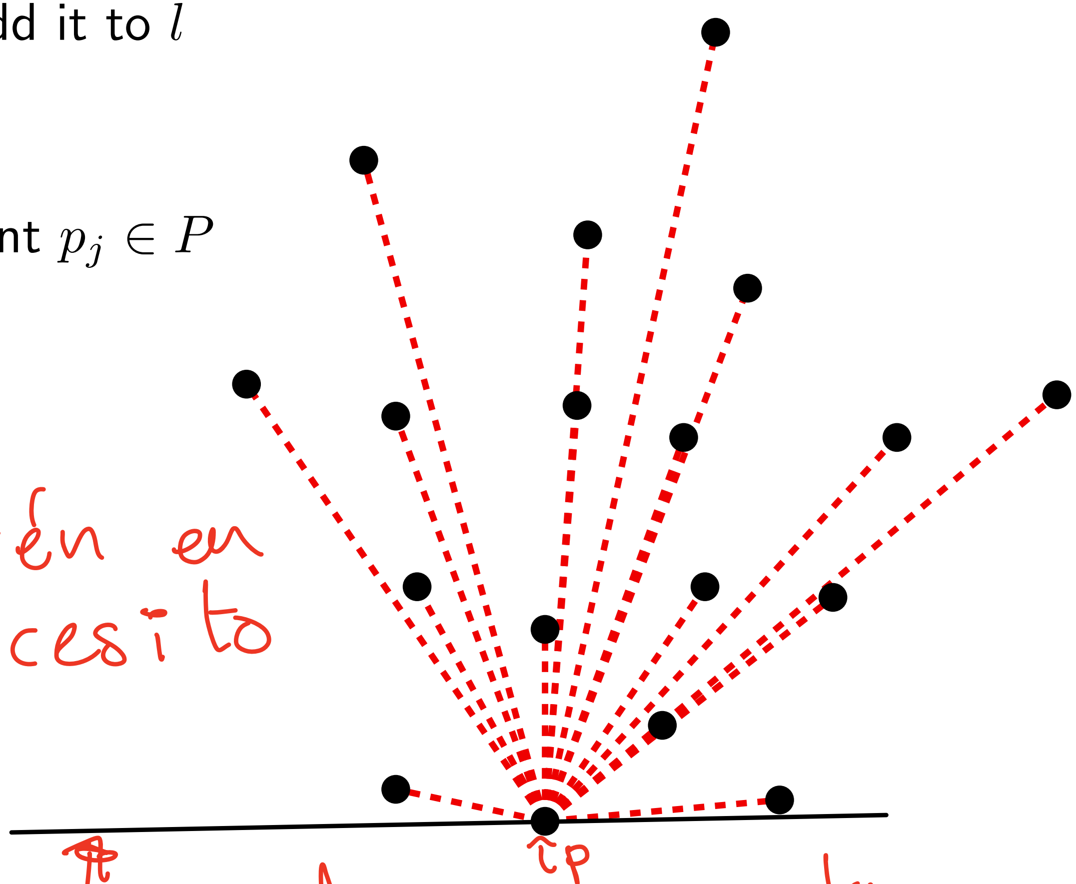
no necesito que estén en orden, únicamente necesito el mínimo



para el arrangue uso la recta horizontal que pasa por p.

Jarvis march

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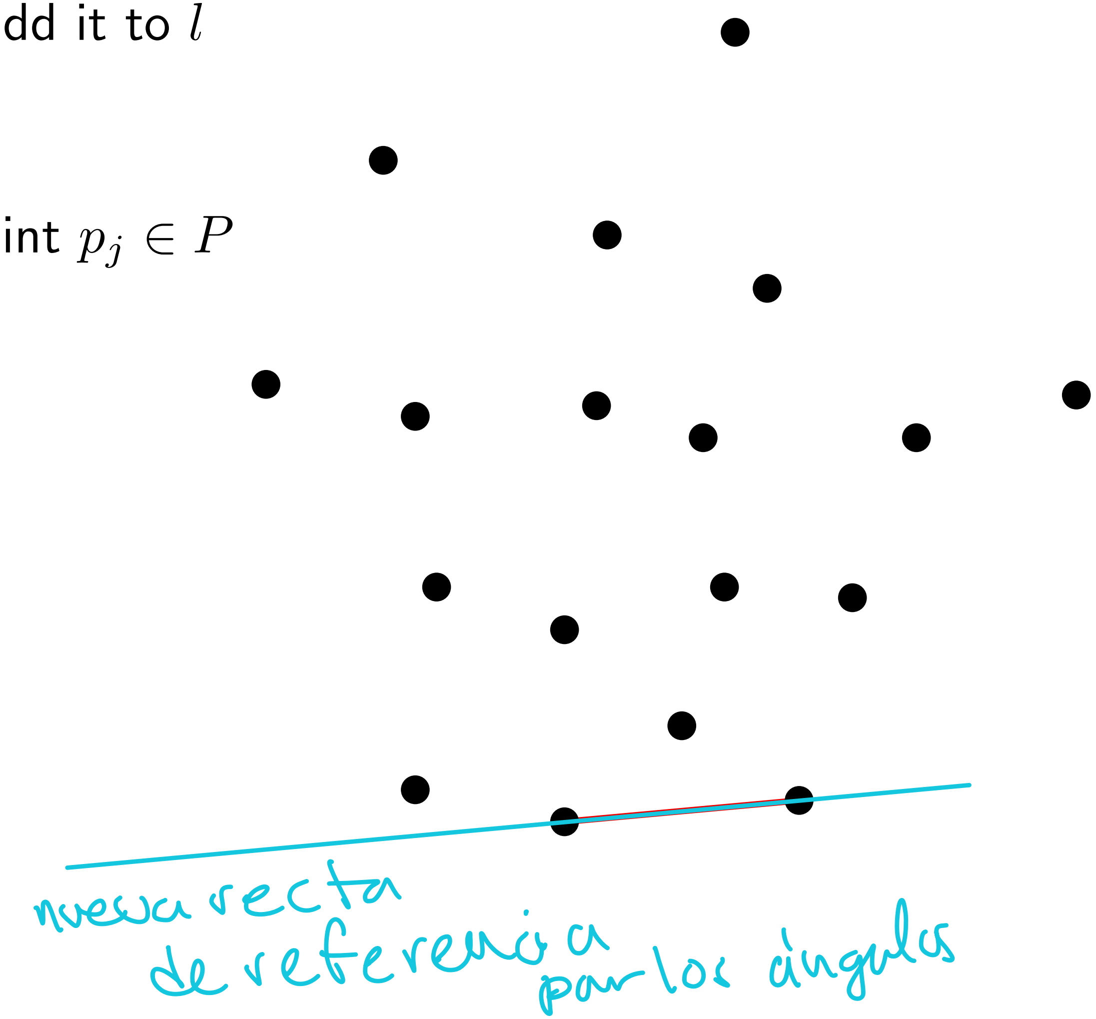

CONVEX HULL

Jarvis march

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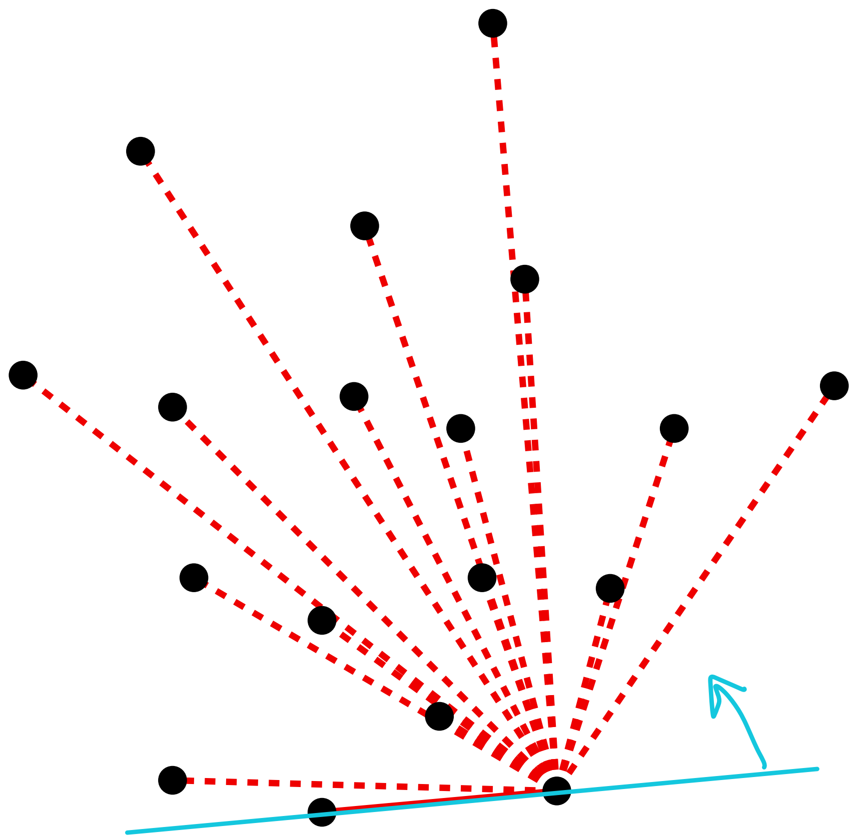

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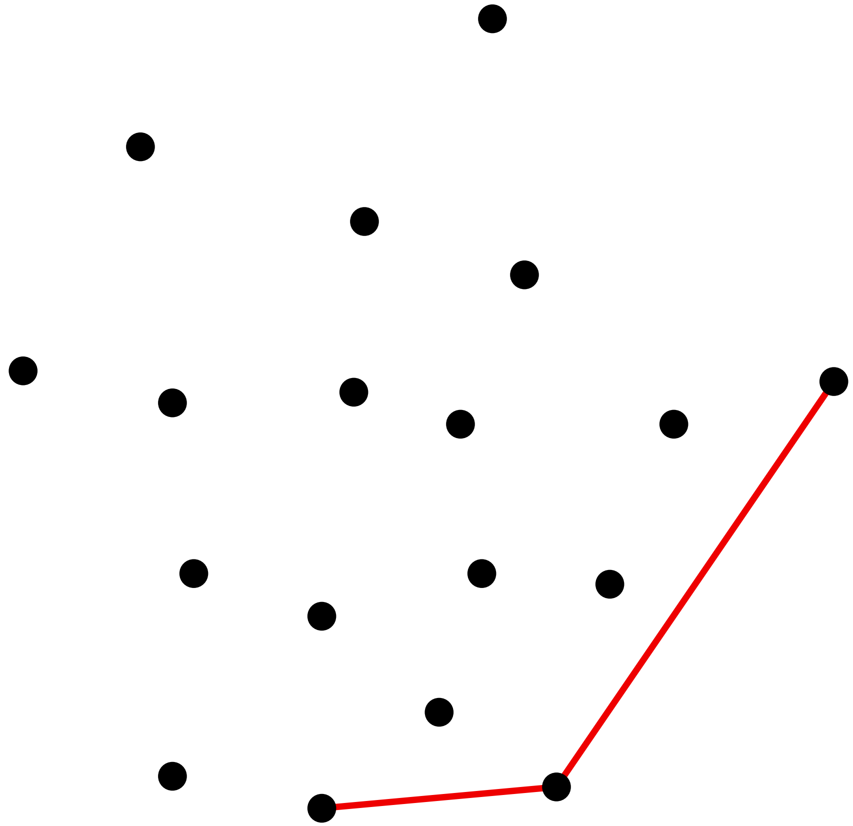

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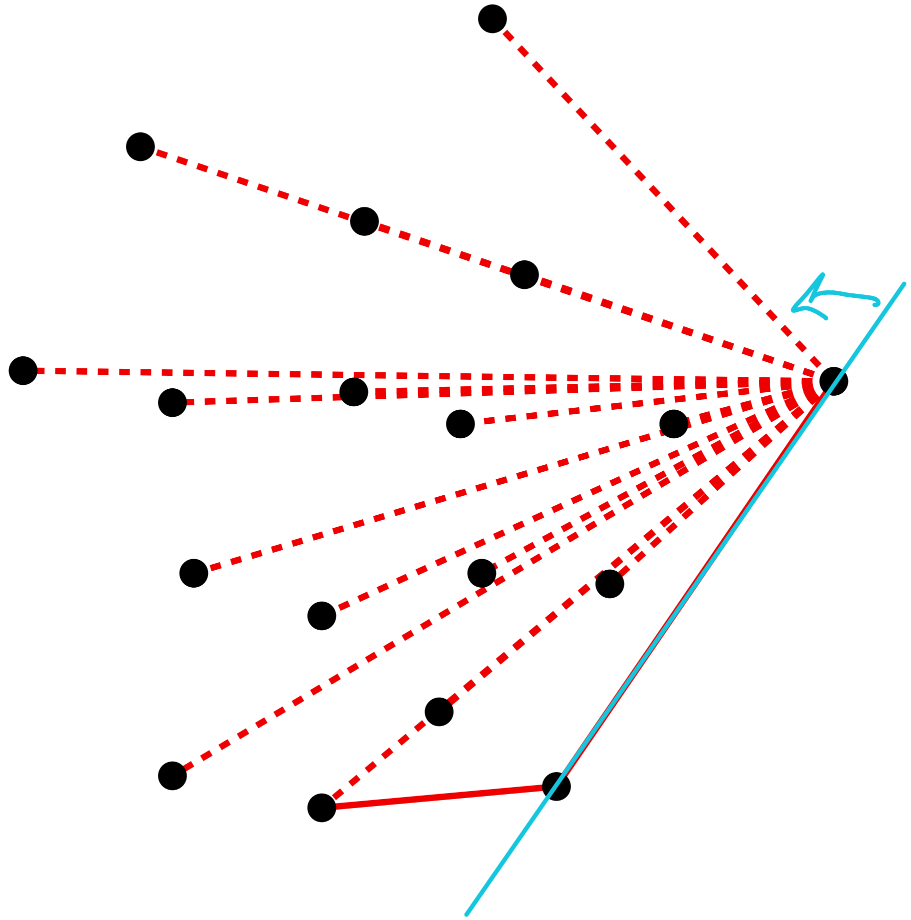

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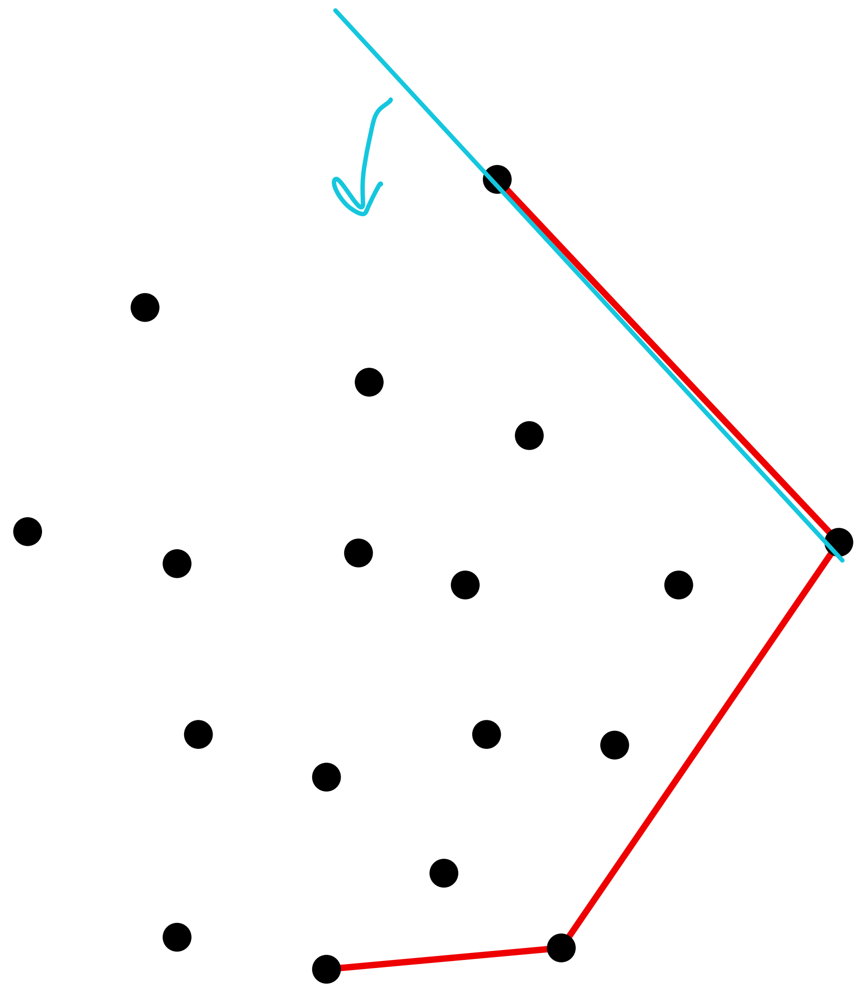

Jarvis march

1. Find a vertex of $ch(P)$ (for example, the lexicographically smaller point $p_i \in P$) and add it to l
2. While $v = \mathrm{Last}(l) \neq \mathrm{First}(l)$, do:
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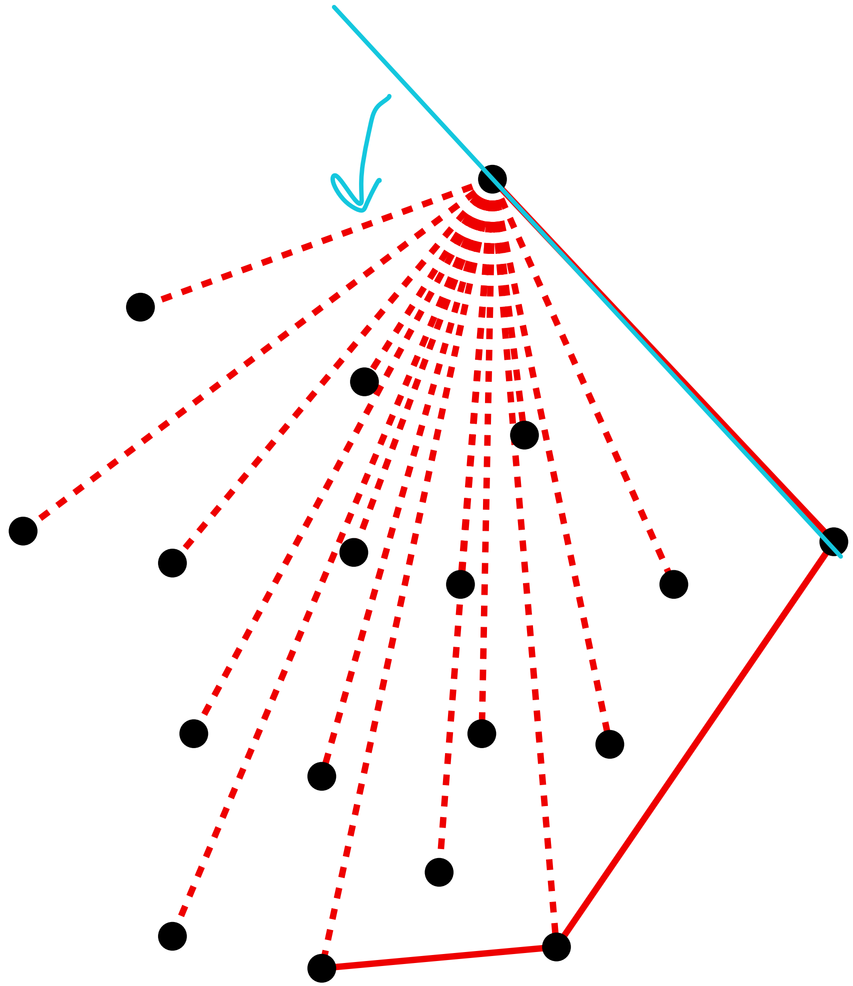

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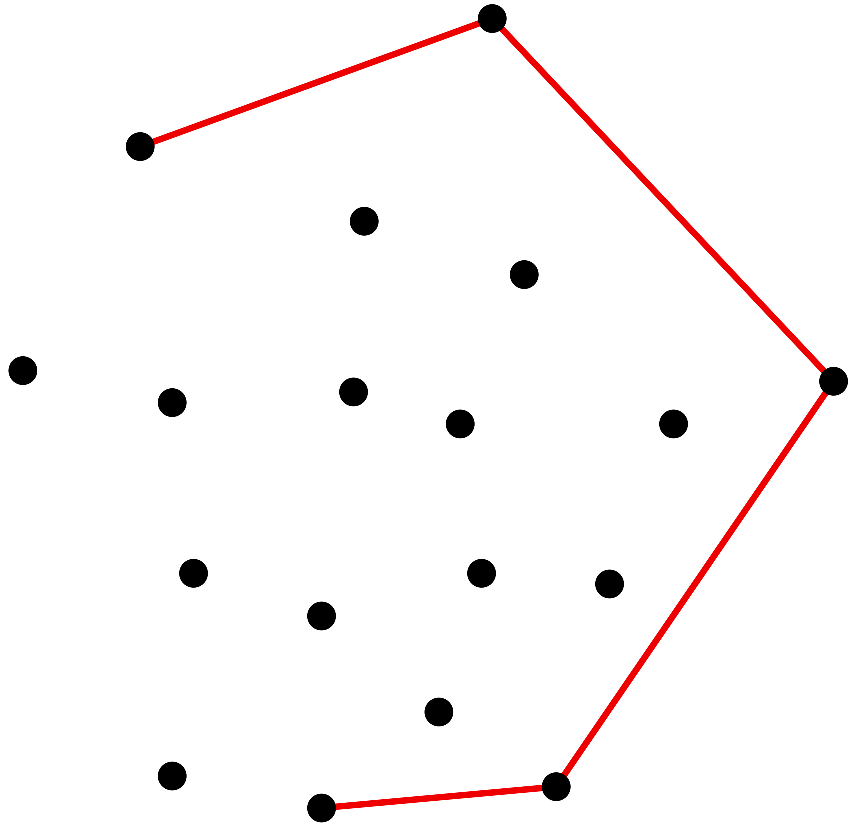

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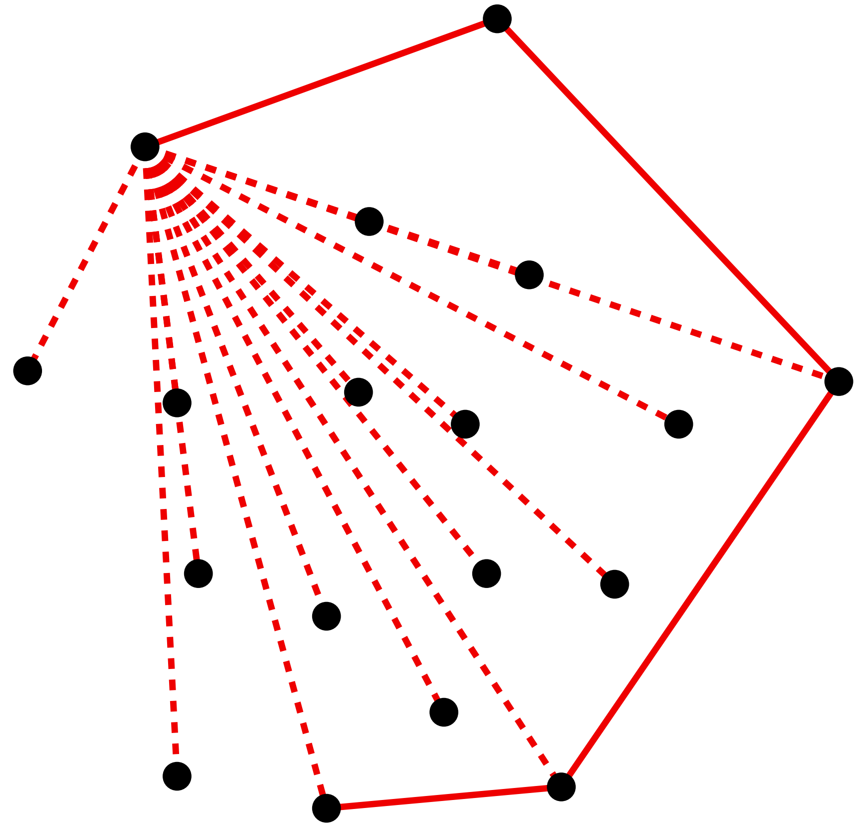

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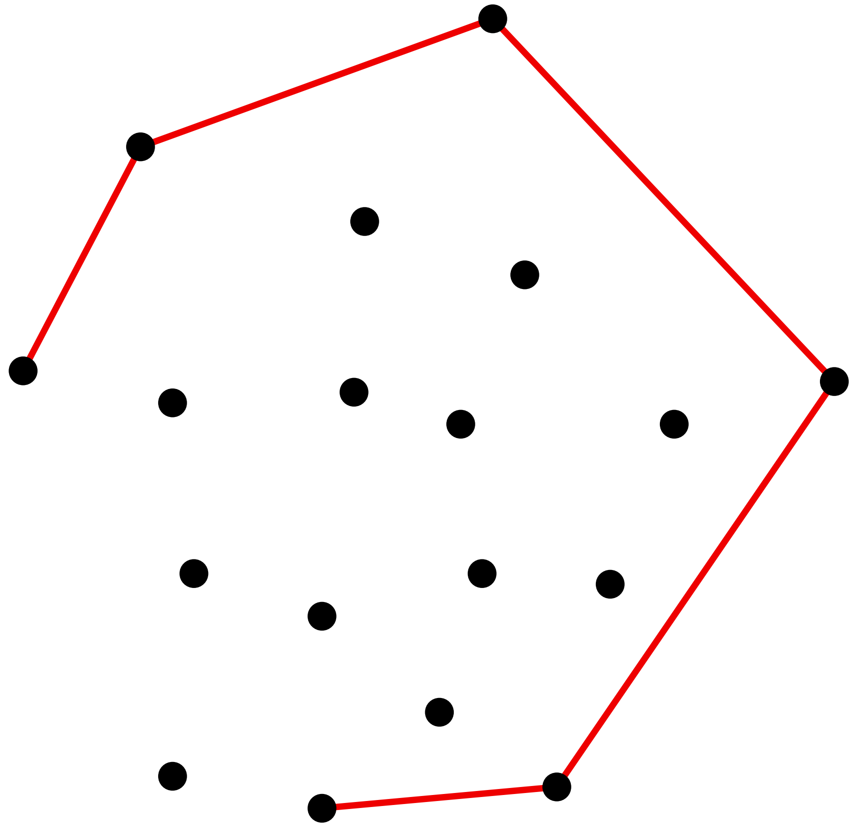

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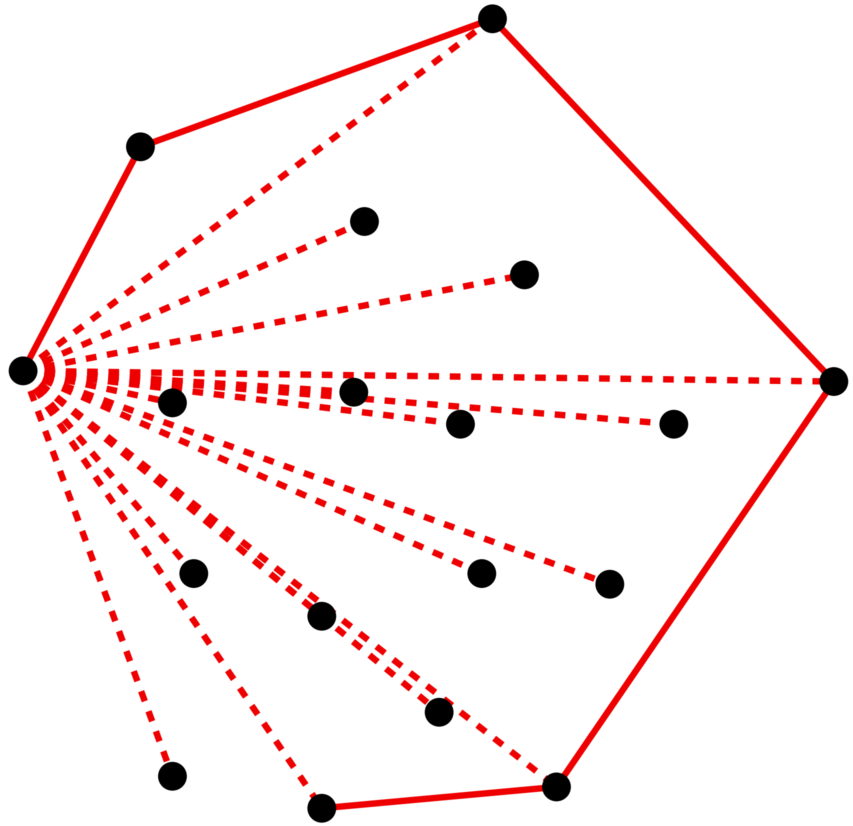

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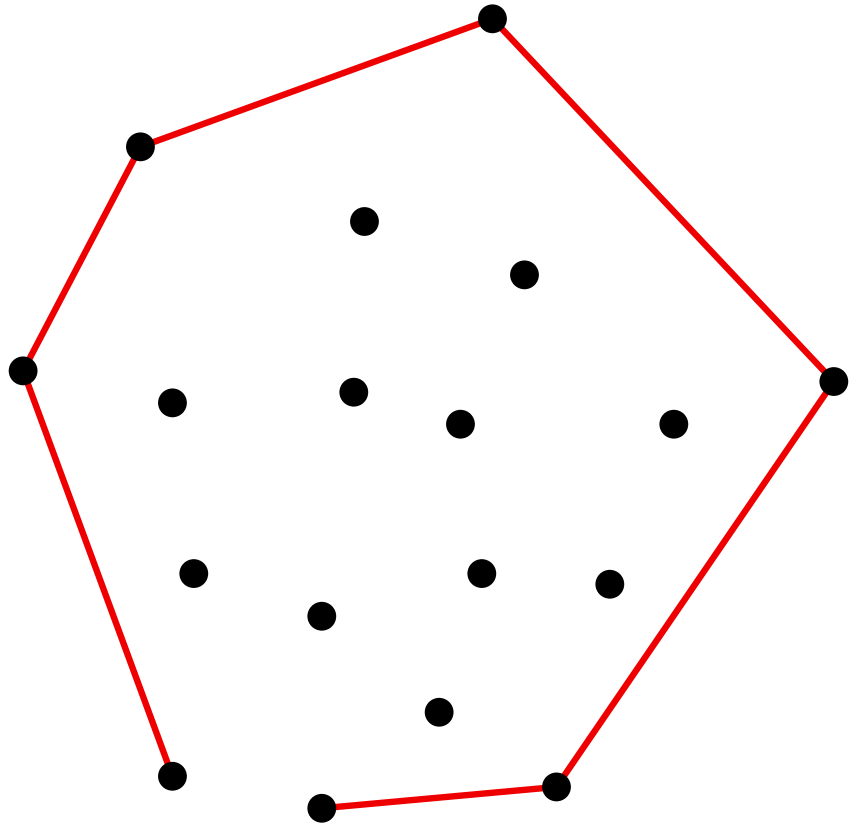

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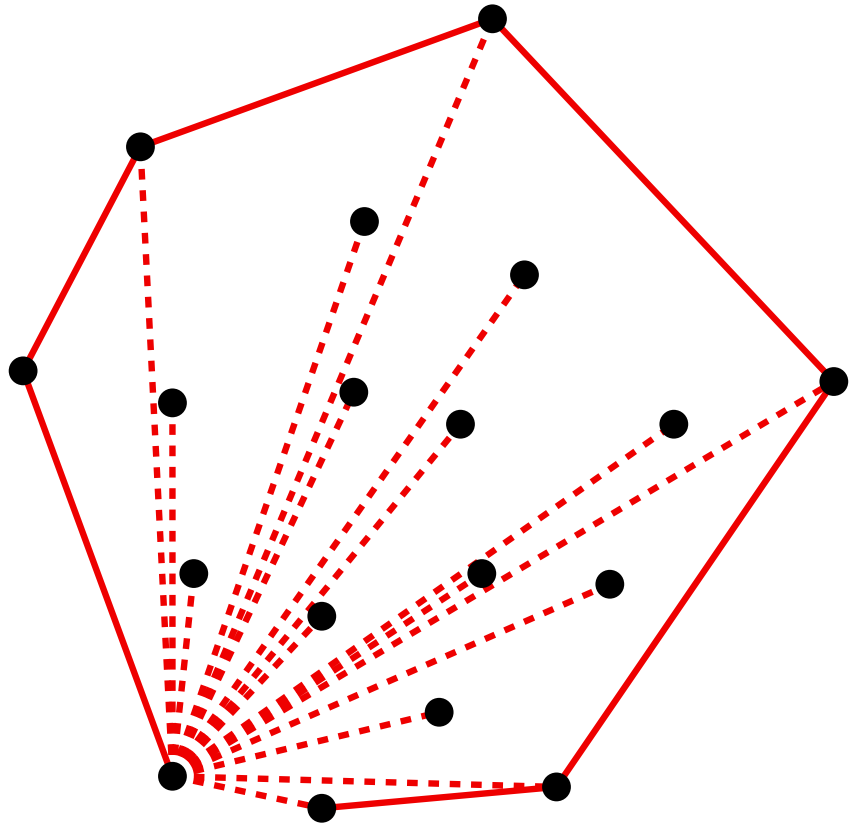

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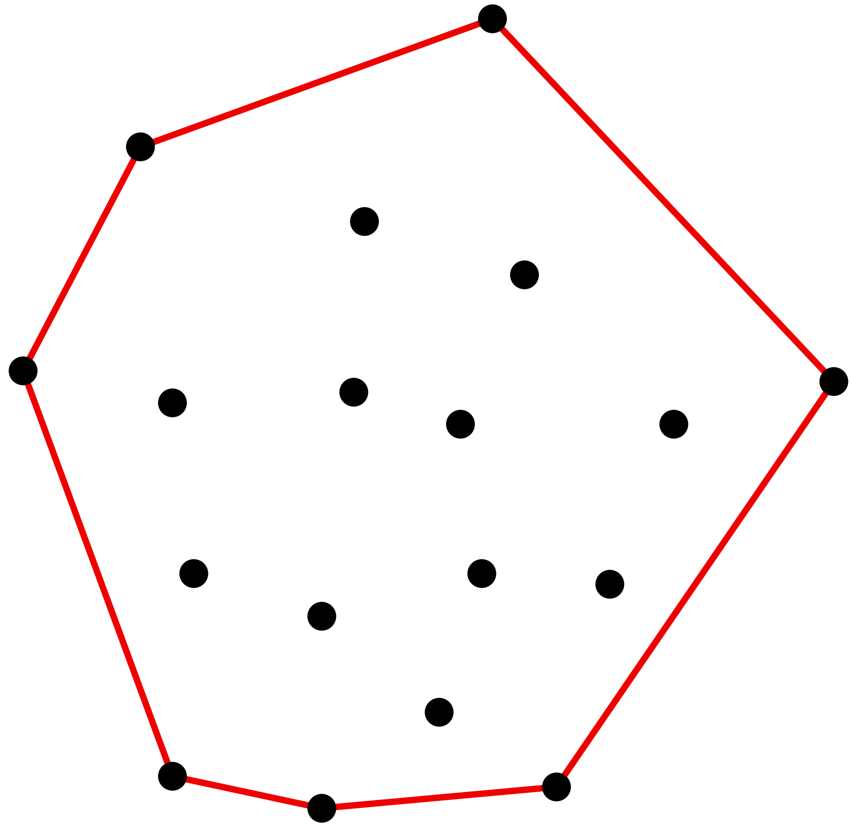

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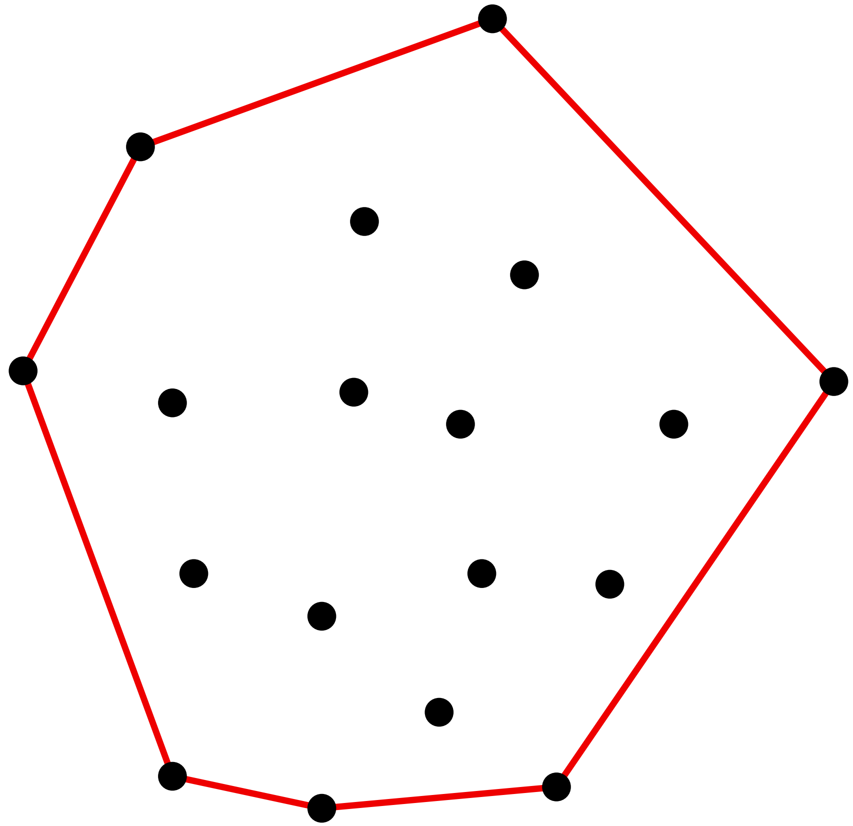


tamaño de la salida → h; tamaño de la entrada → n

Time cost: $\Theta(hn) = O(n^2)$

output sensitive

h = # de vértices del cierre convexo.

QuickHull algorithm (by prune-and-search)

Varios investigadores al final de la década de los 70. lo bautizaron QuickHull por su similitud con QuickSort.

↓

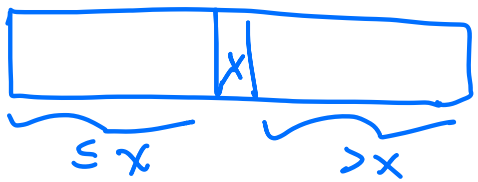
Preparata & Shamos, 1985.

QuickSort

```
QUICKSORT(A[1..n]):
  if (n > 1)
        Choose a pivot element A[p]
        r ← PARTITION(A, p)
        QUICKSORT(A[1..r − 1])     ⟨⟨Recurse!⟩⟩
        QUICKSORT(A[r + 1..n])     ⟨⟨Recurse!⟩⟩
```

```
PARTITION(A[1..n], p):
  swap A[p] ↔ A[n]
  ℓ ← 0                ⟨⟨#items < pivot⟩⟩
  for i ← 1 to n − 1
        if A[i] < A[n]
              ℓ ← ℓ + 1
              swap A[ℓ] ↔ A[i]
  swap A[n] ↔ A[ℓ + 1]
  return ℓ + 1
```

Figure 1.8. Quicksort



QuickHull algorithm (by prune-and-search)

Initialization

1. Find the extreme points in the horizontal and vertical directions.
2. Compute the convex hull of these (between 2 and 8) points.
3. Test all the remaining points, and classify them according to their position (NE, SE, SW, NW) or eliminate them if they lie in the interior.

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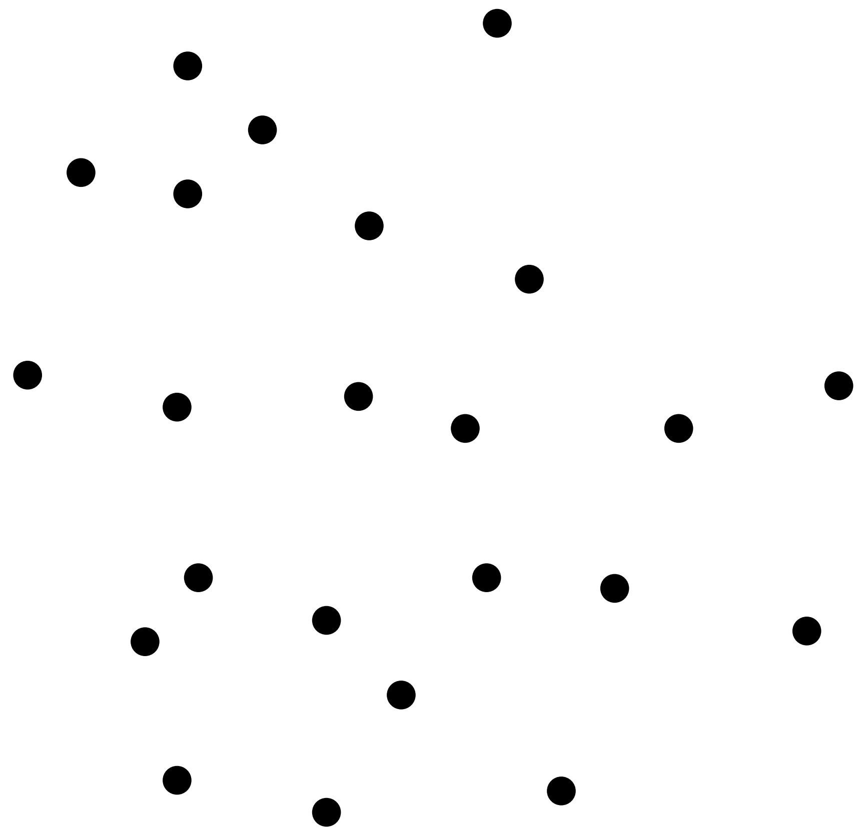

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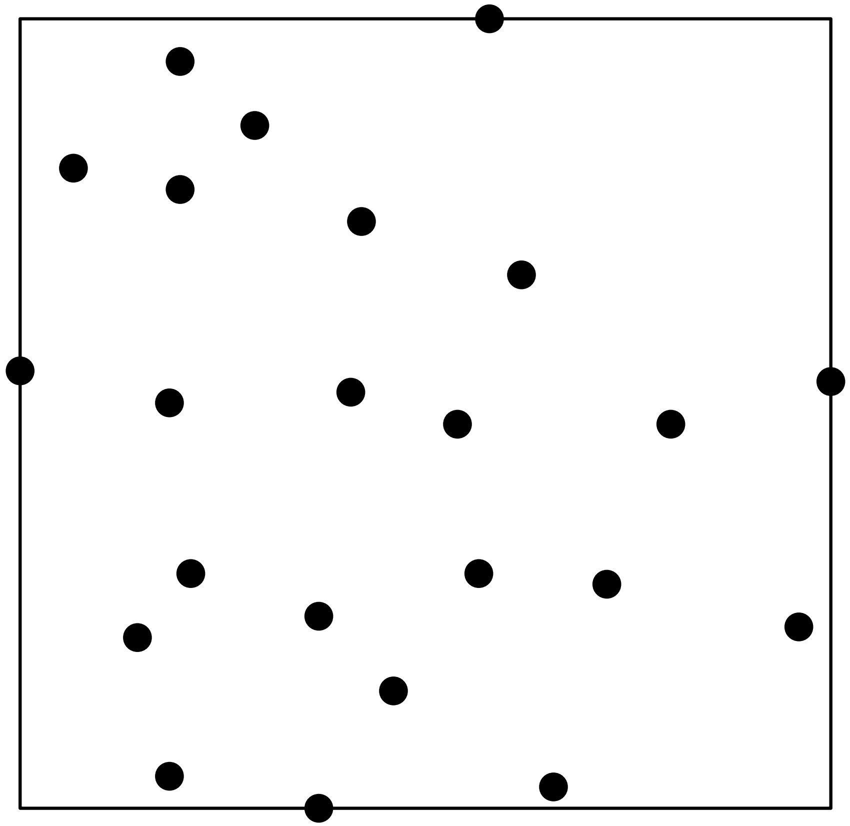

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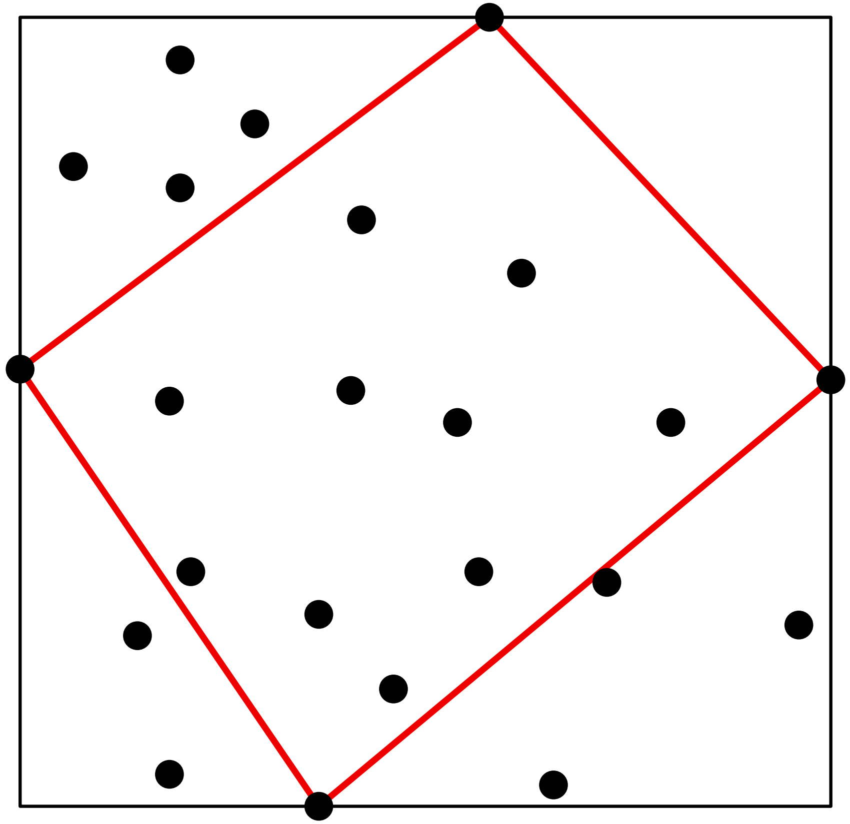

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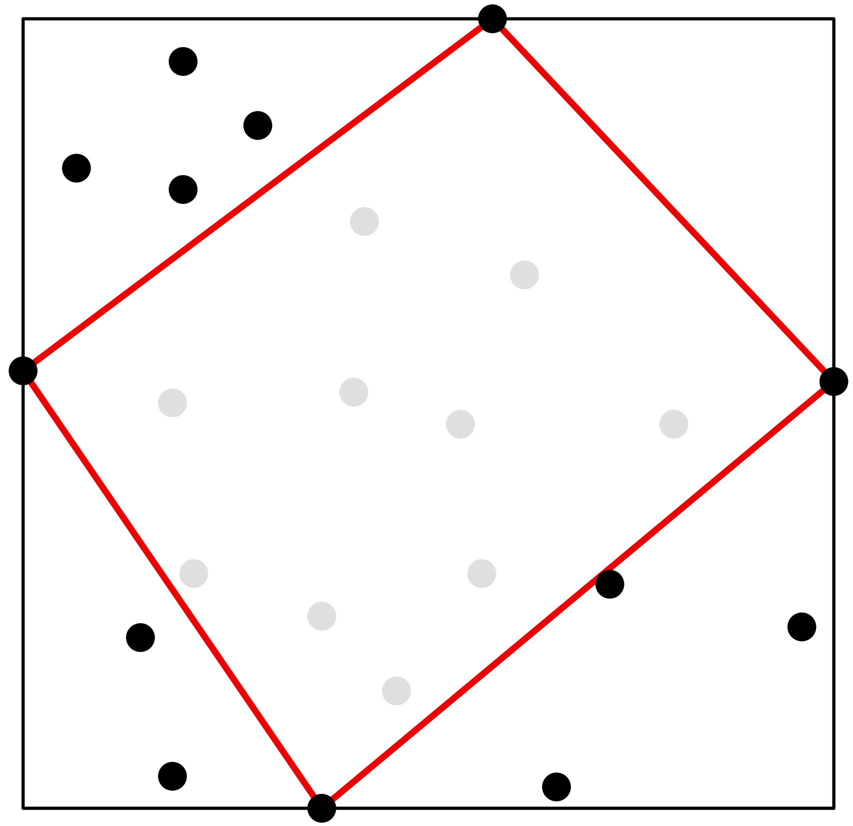

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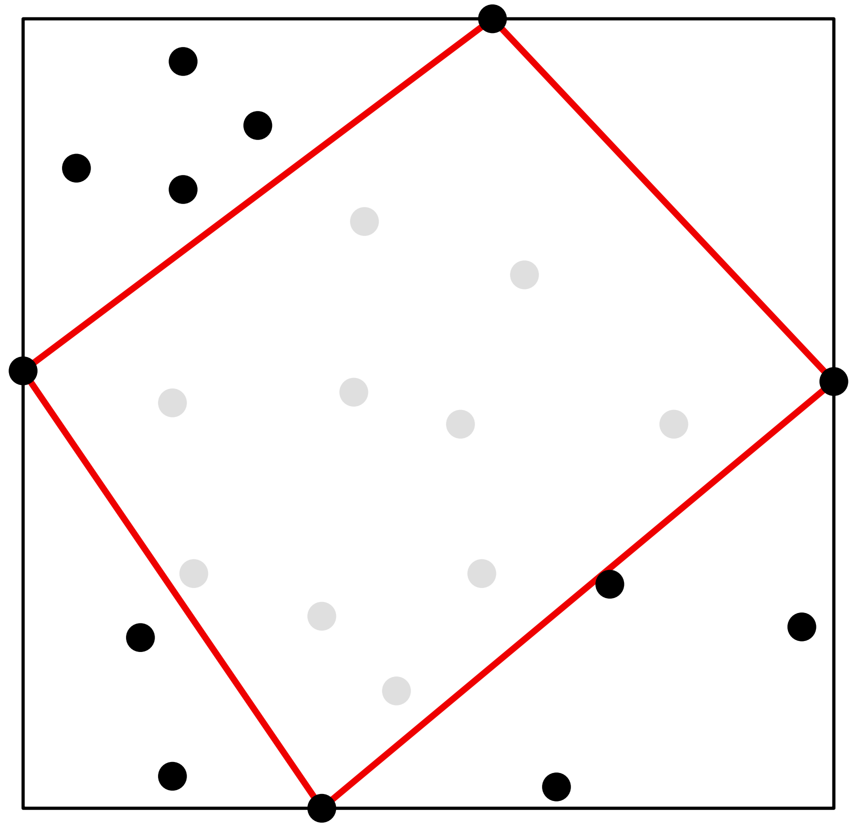


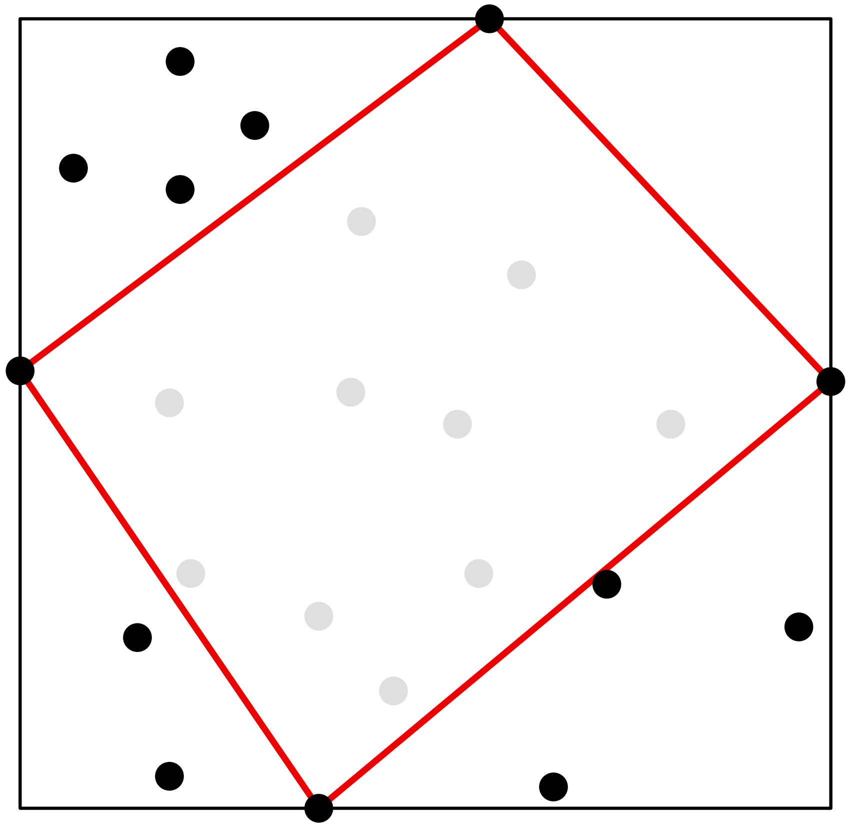
Running time of this step: $O(n)$

QuickHull algorithm (by prune-and-search)

Advance

Recursively, do:

1. Among all points lying in each region, find the extreme point in the direction orthogonal to the edge that determines the region.
2. Connect the extreme point with te endpoints of the edge, and update the convex hull.
3. Test all the remaining points of each region, and classify them according to their position (left or right) or eliminate them if they lie in the interior of the newly created triangle.



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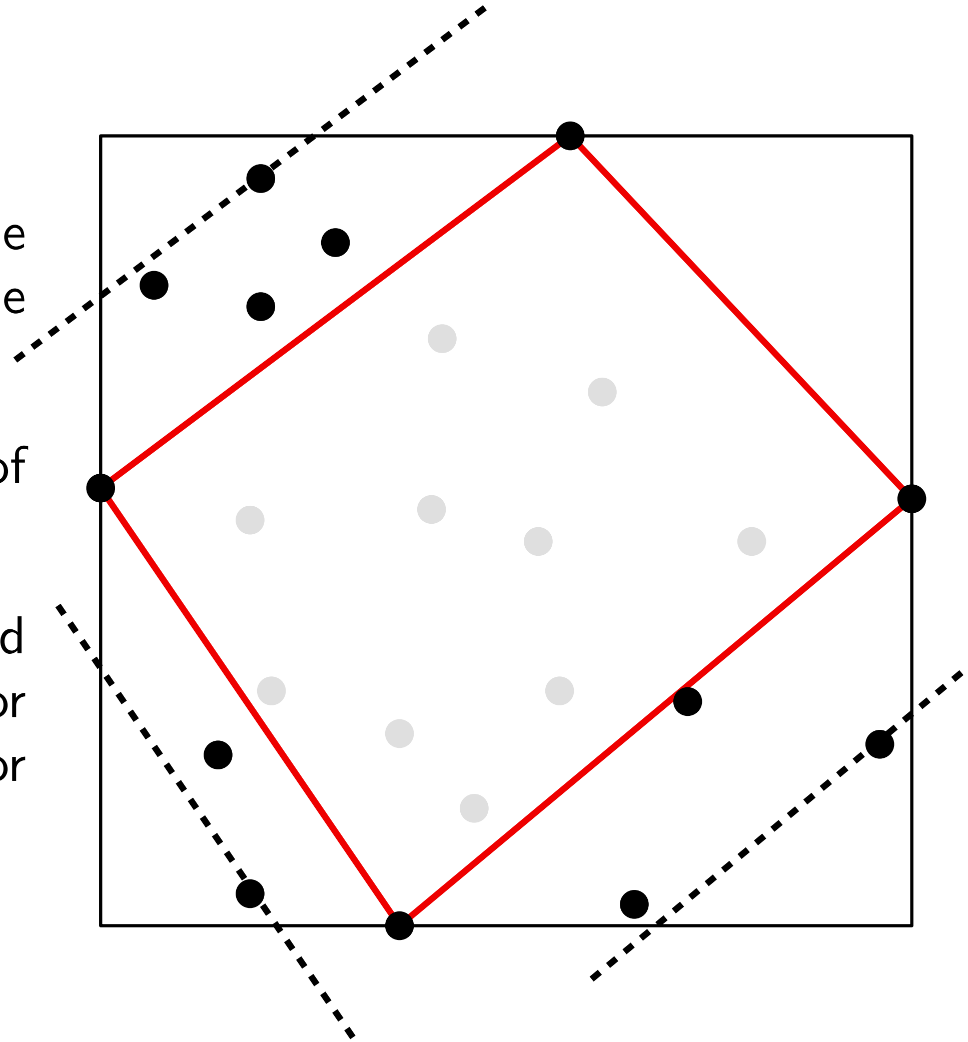

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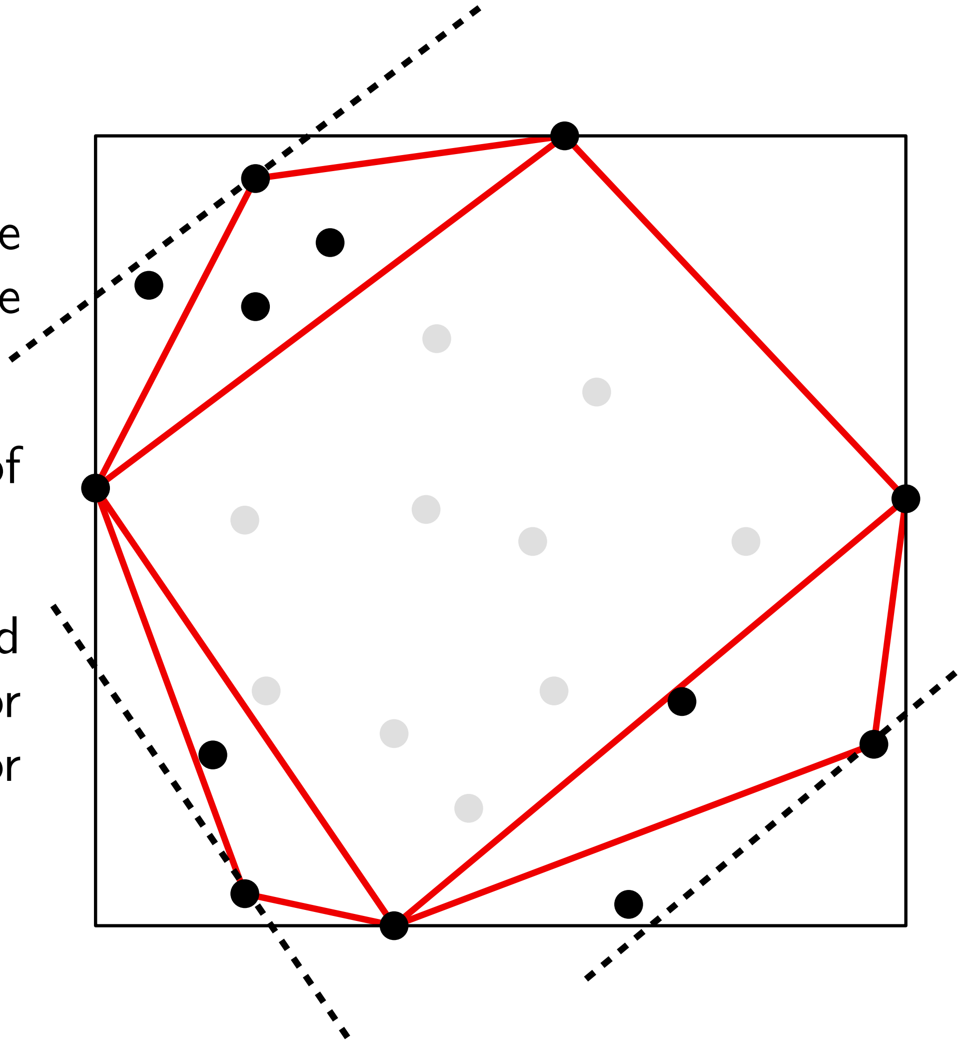

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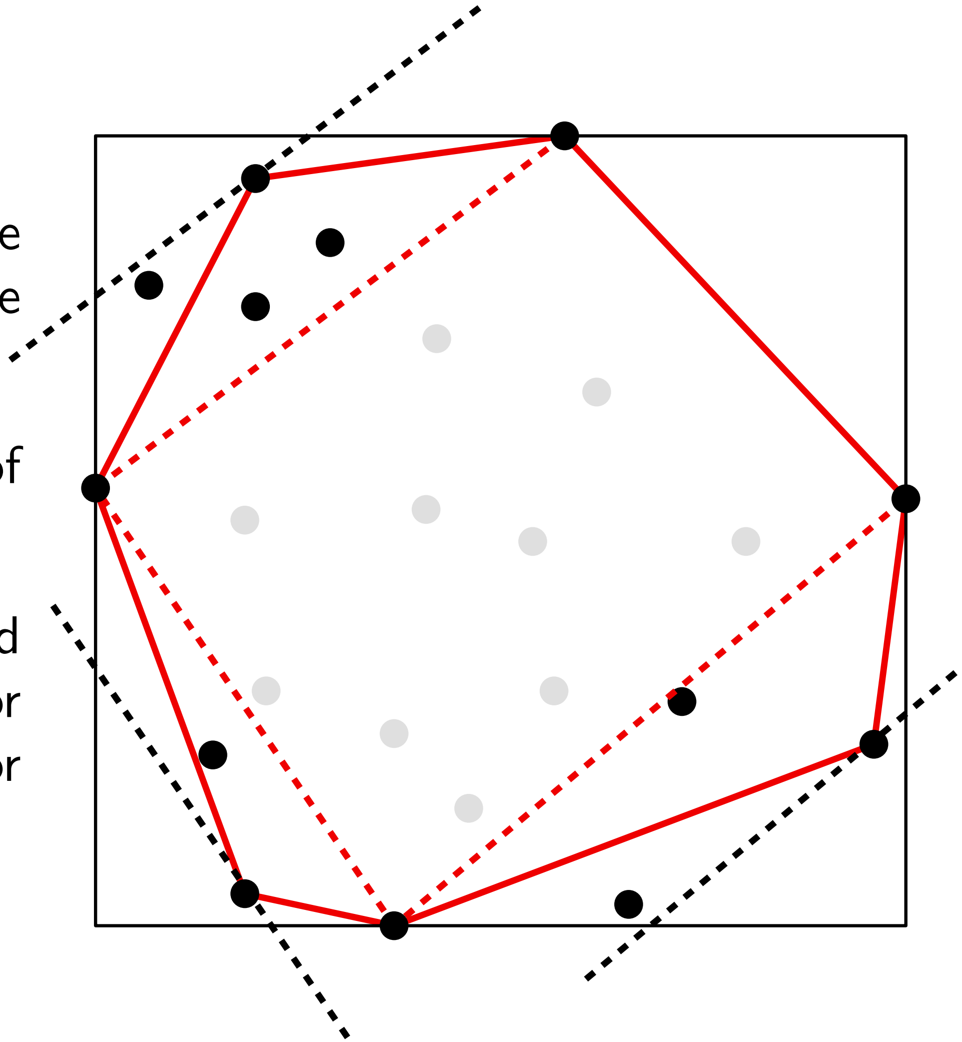

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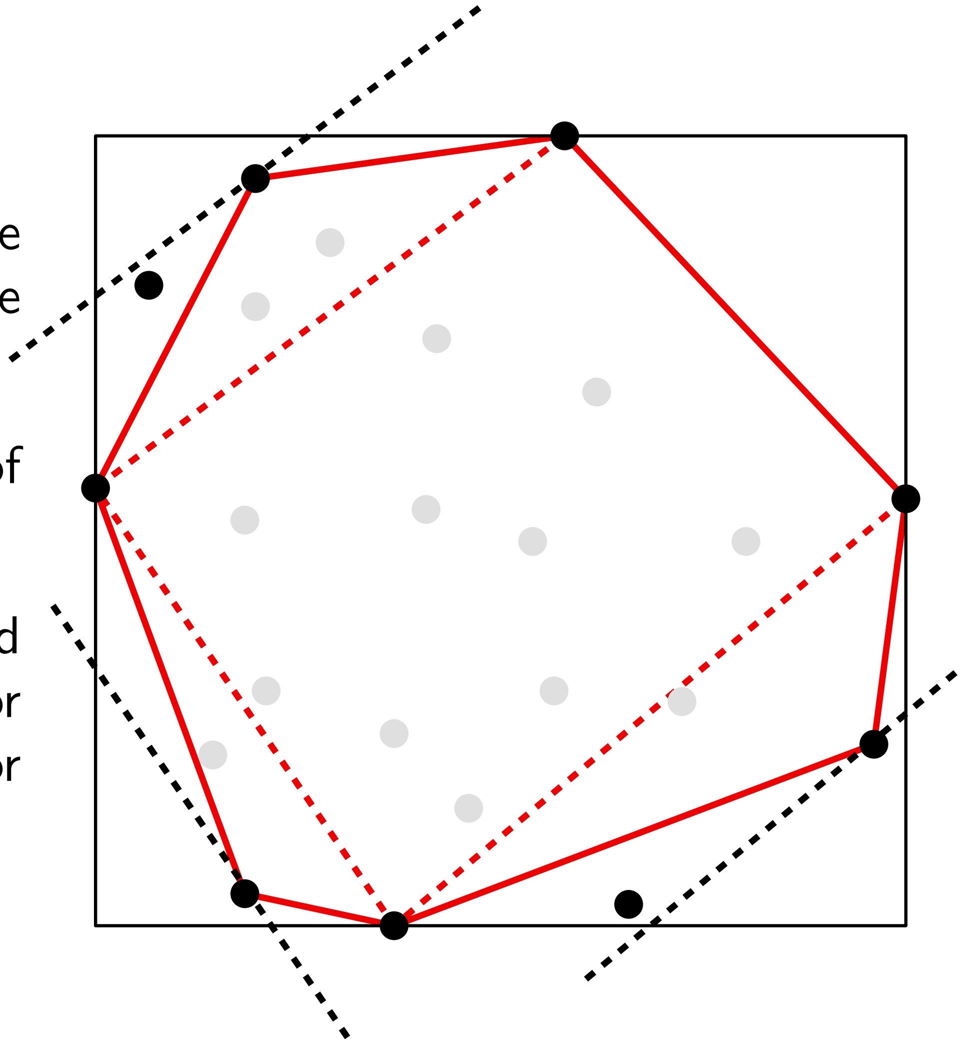

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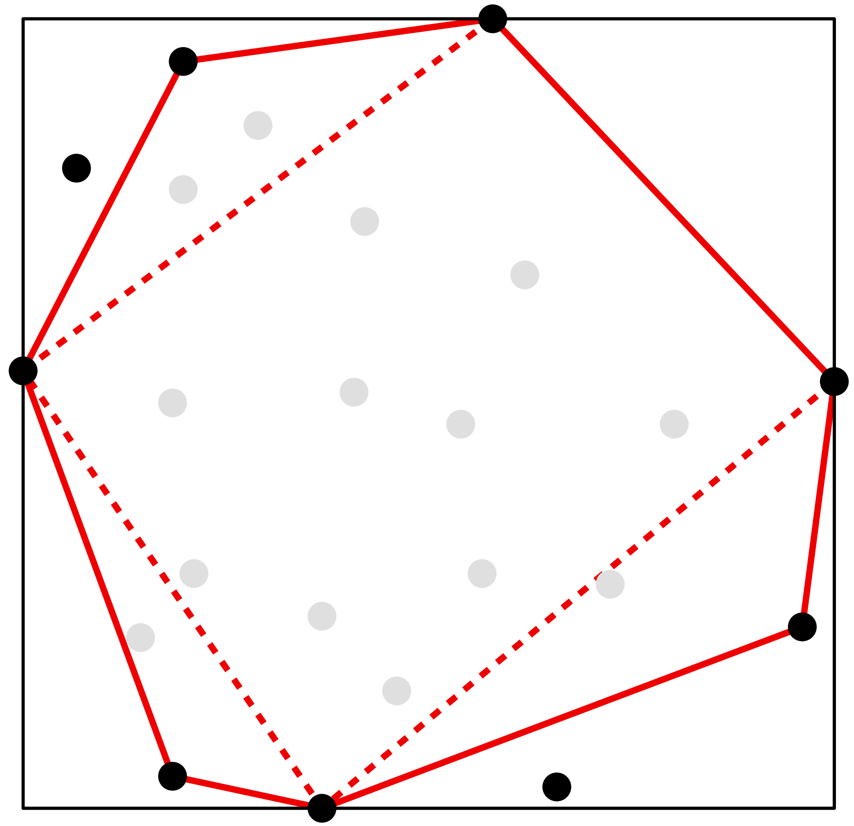

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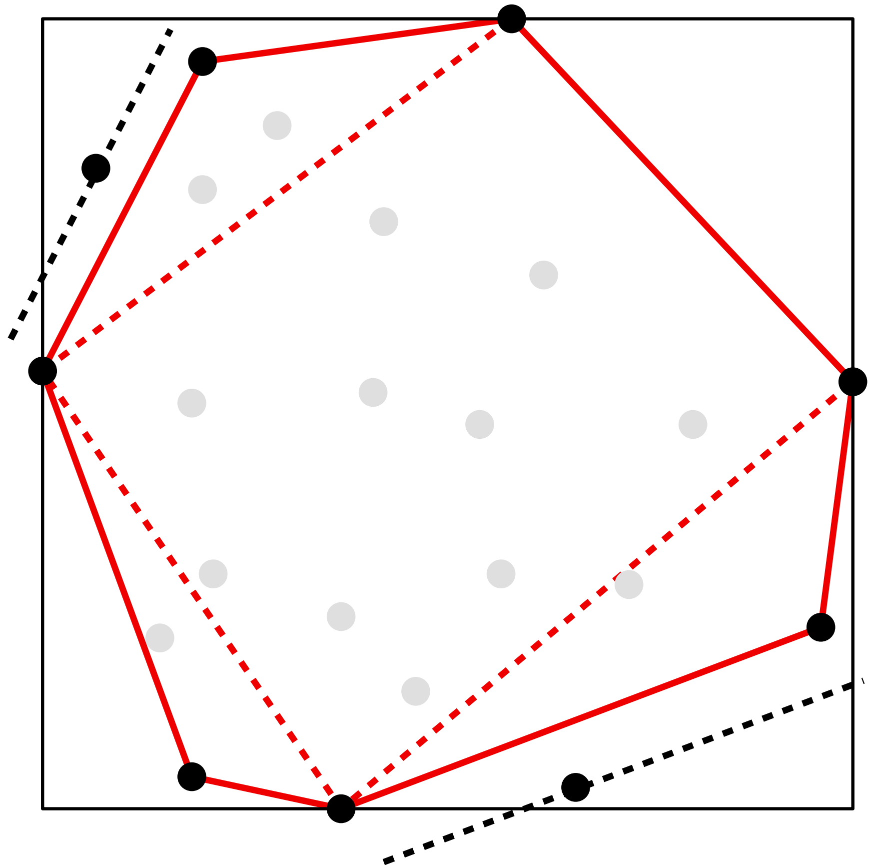

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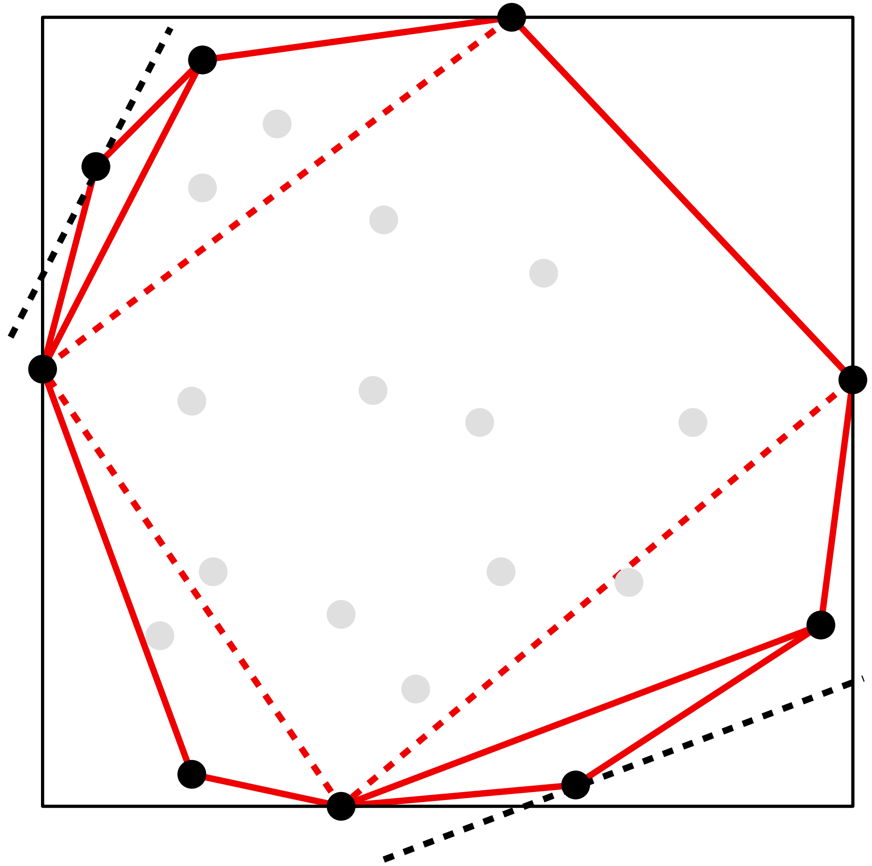

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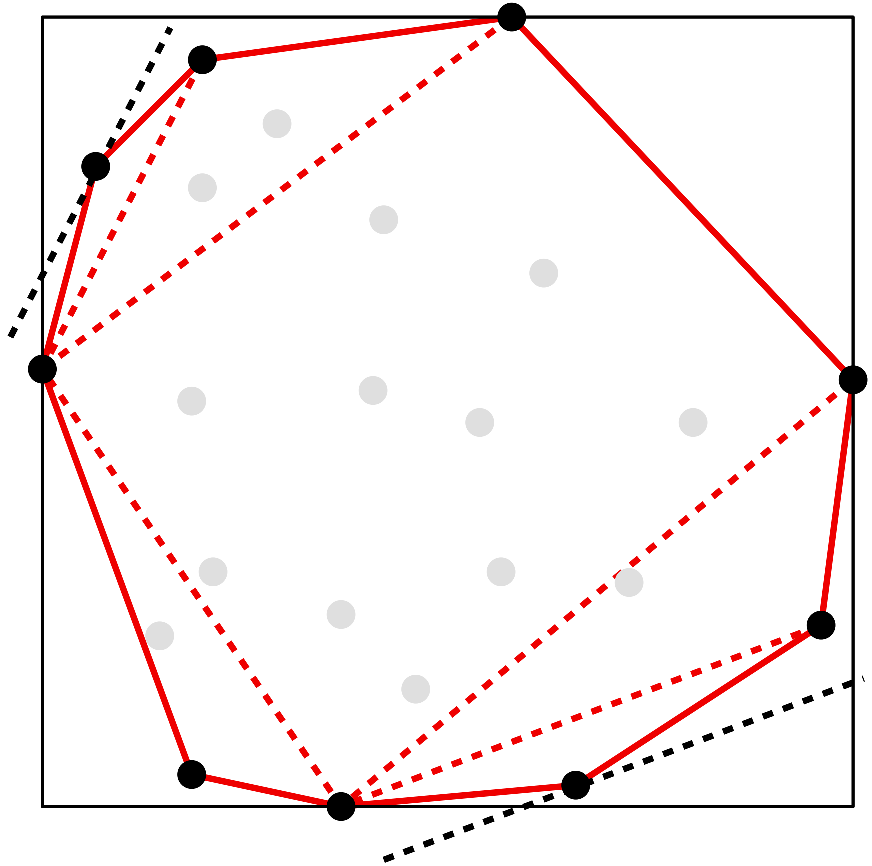

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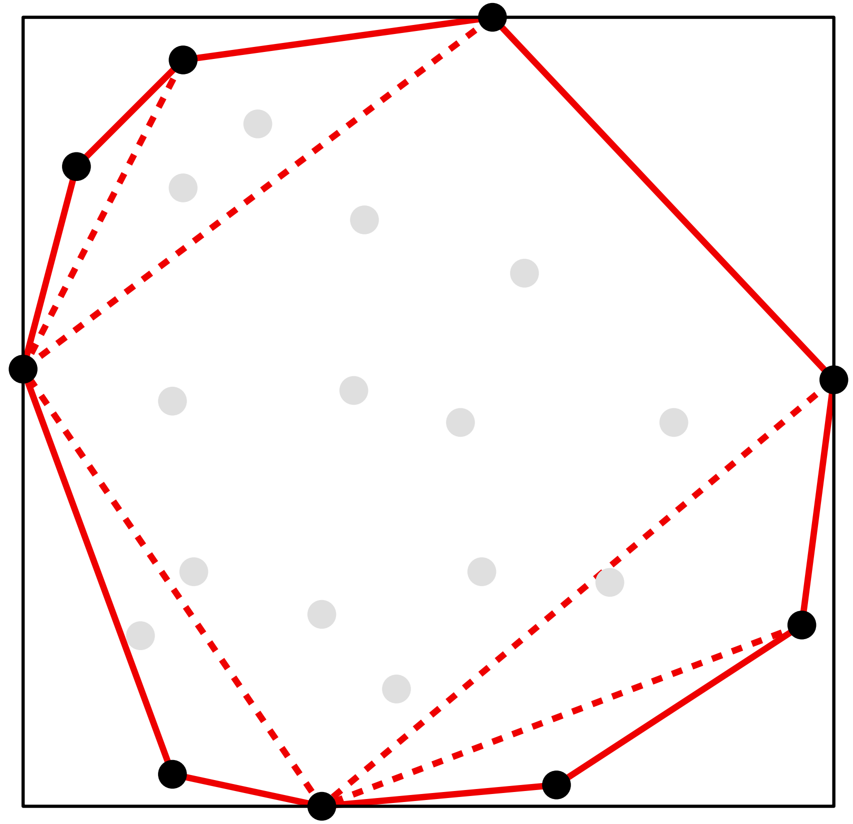

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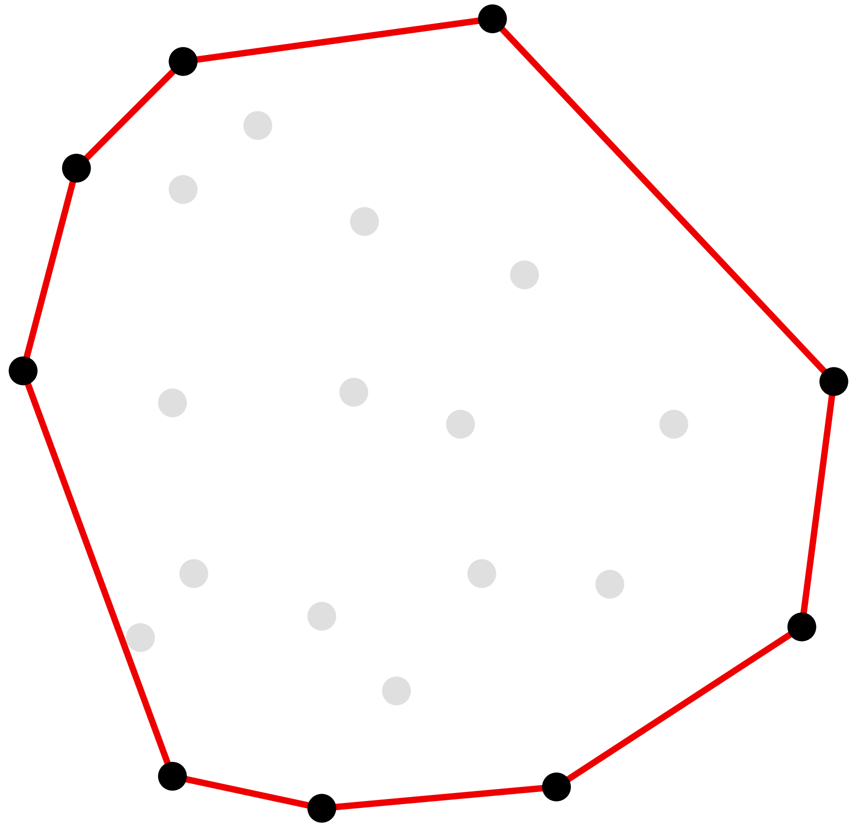

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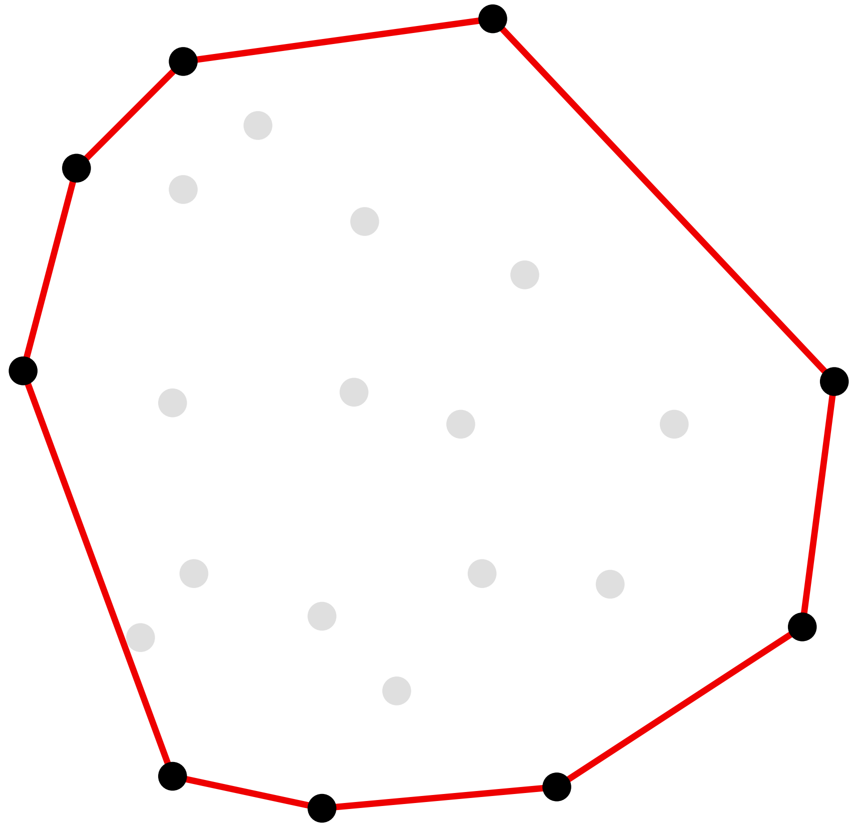
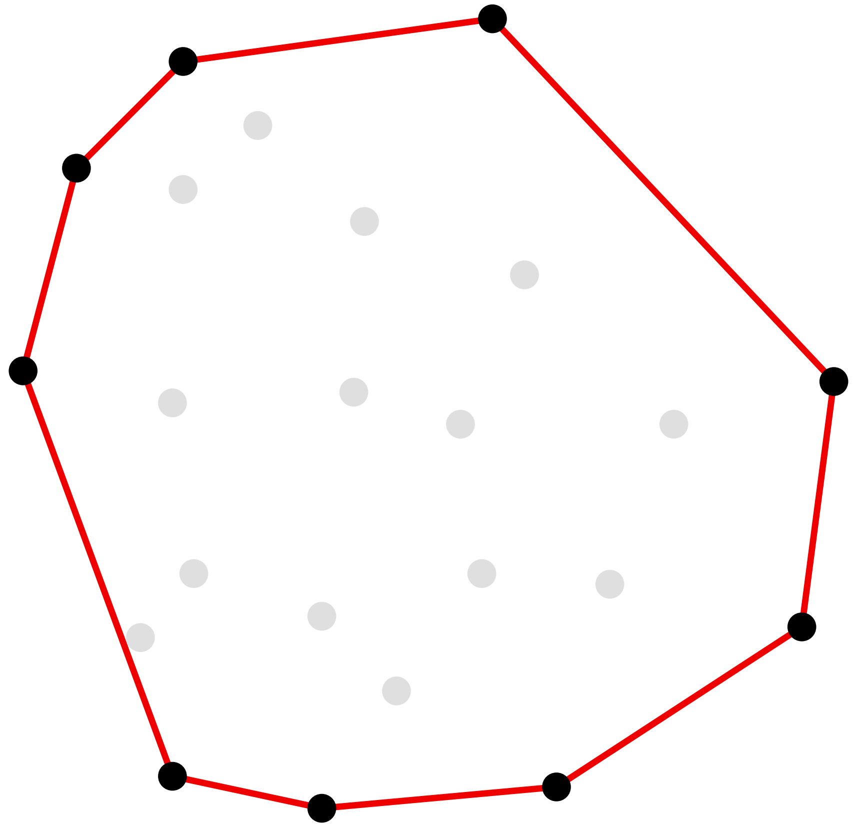


Running time of this step: $O(n^2)$

QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$

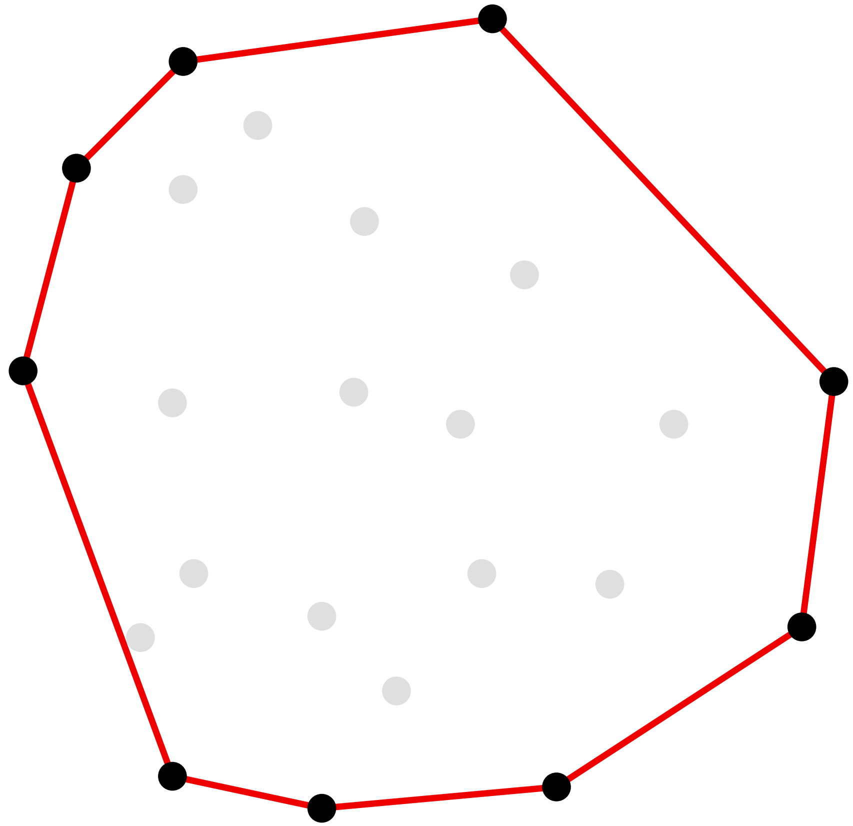


QuickHull algorithm (by prune-and-search)

Overall running time: $O(n^2)$

Nevertheless, the running time of this algorithm depends on the position of the input points.
For example:

- If the input points are in convex position, the running time is $\Theta(n^2)$.
- If the points are such that each prune step eliminates half of the current points, then the algorithm runs in $\Theta(n \log n)$ time.
- If the convex hull is triangular, the algorithm runs in $\Theta(n)$ time.



Graham's algorithm

Graham's algorithm

Initialization

- Find a vertex v of $ch(P)$, push it in l and delete it from P
- Angularly sort the points around v
- Push the first point in l and delete if from P

Advance

While there exist points $p_i \in P$ to be explored, do:

$p = \text{top}(l)$

$p^- = \text{previous}(\text{top}(l))$

- If $p^- p p_i$ is a left turn:
    - Push p_i in l
    - Advance i
- Else:
    - Pop p from l

Return l

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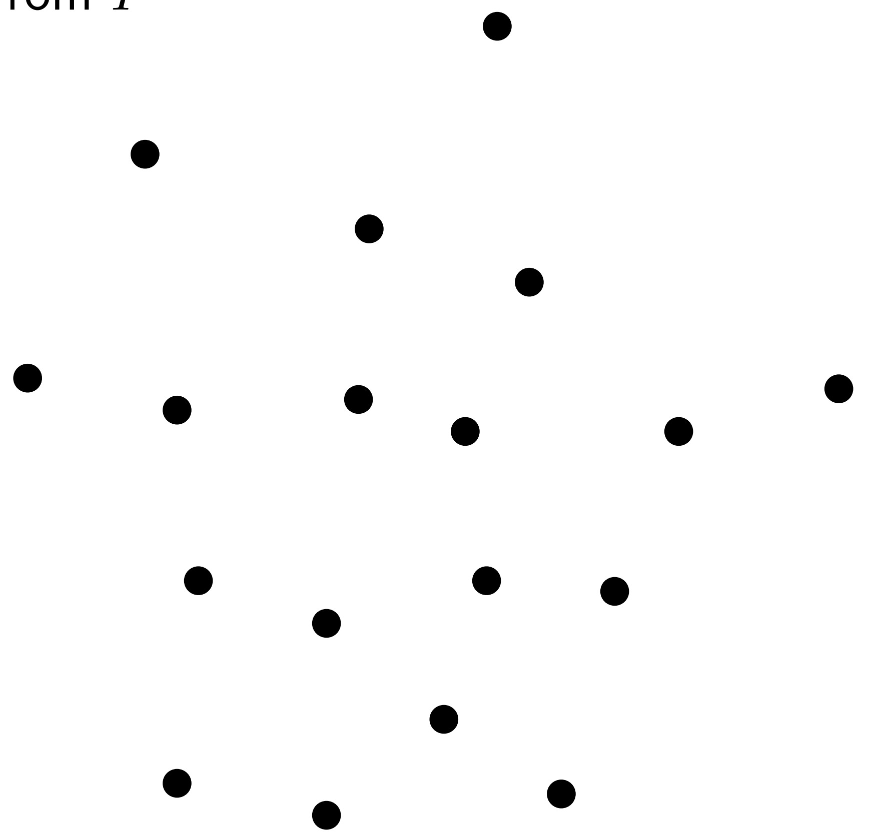

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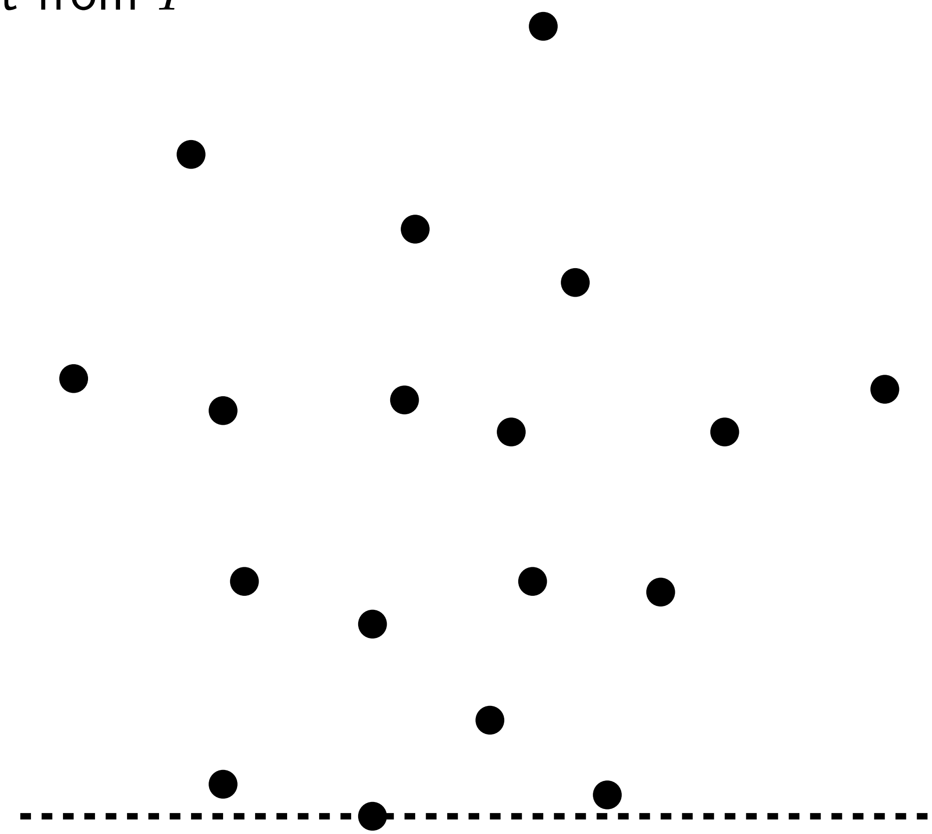

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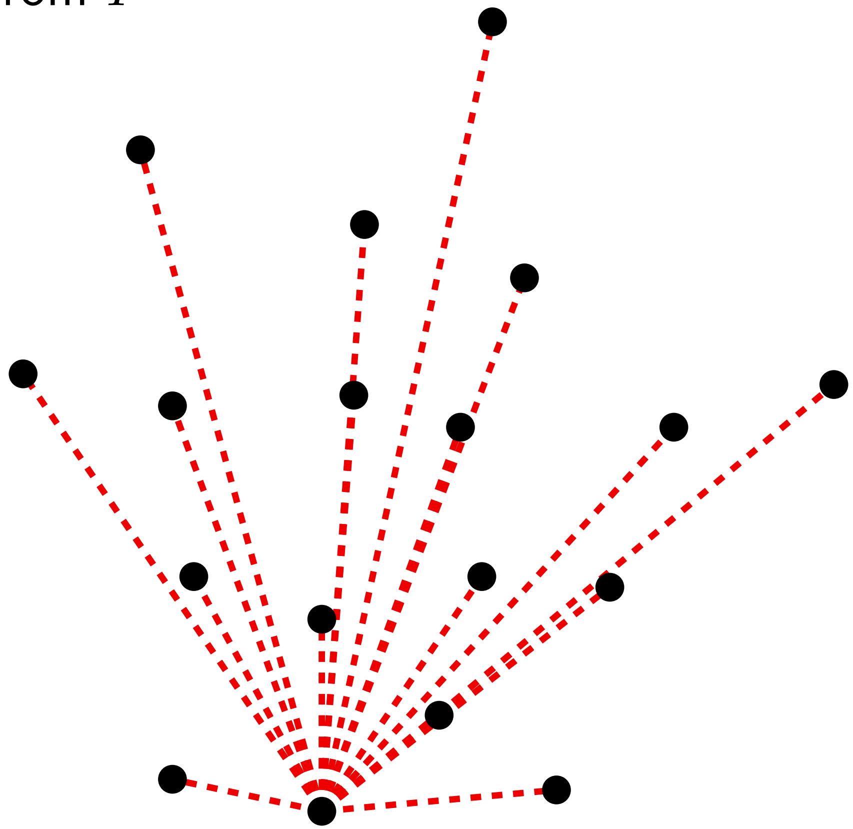

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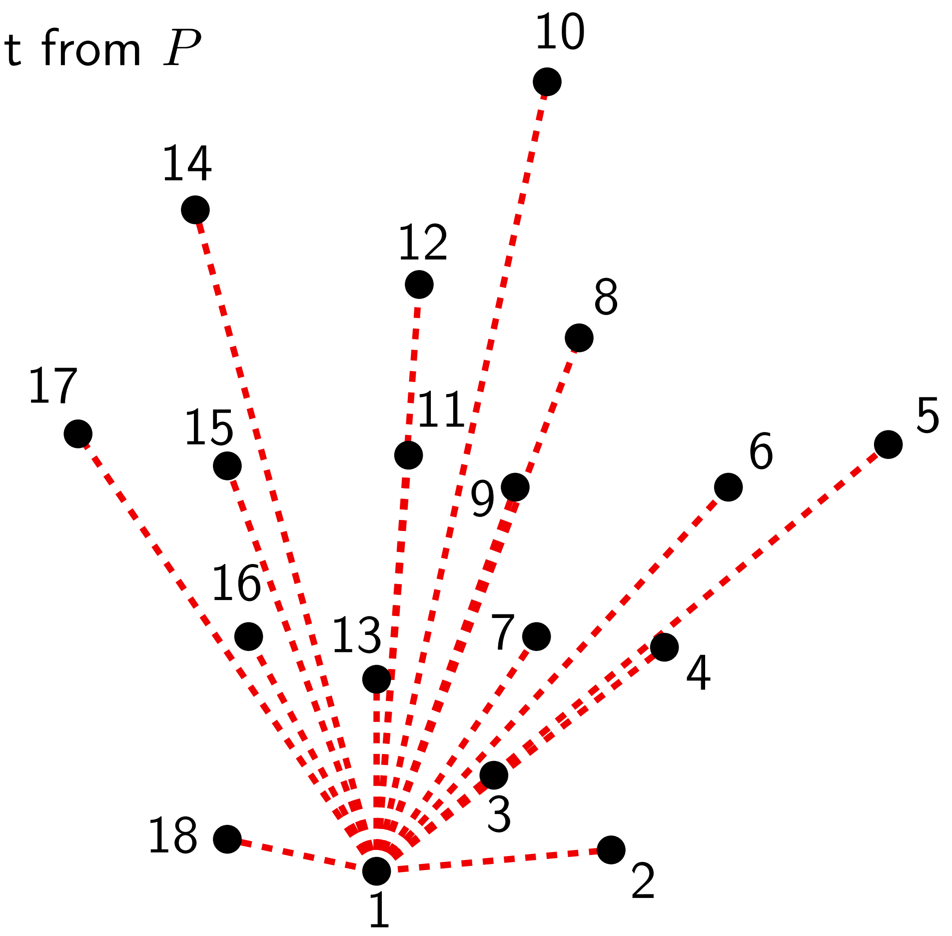

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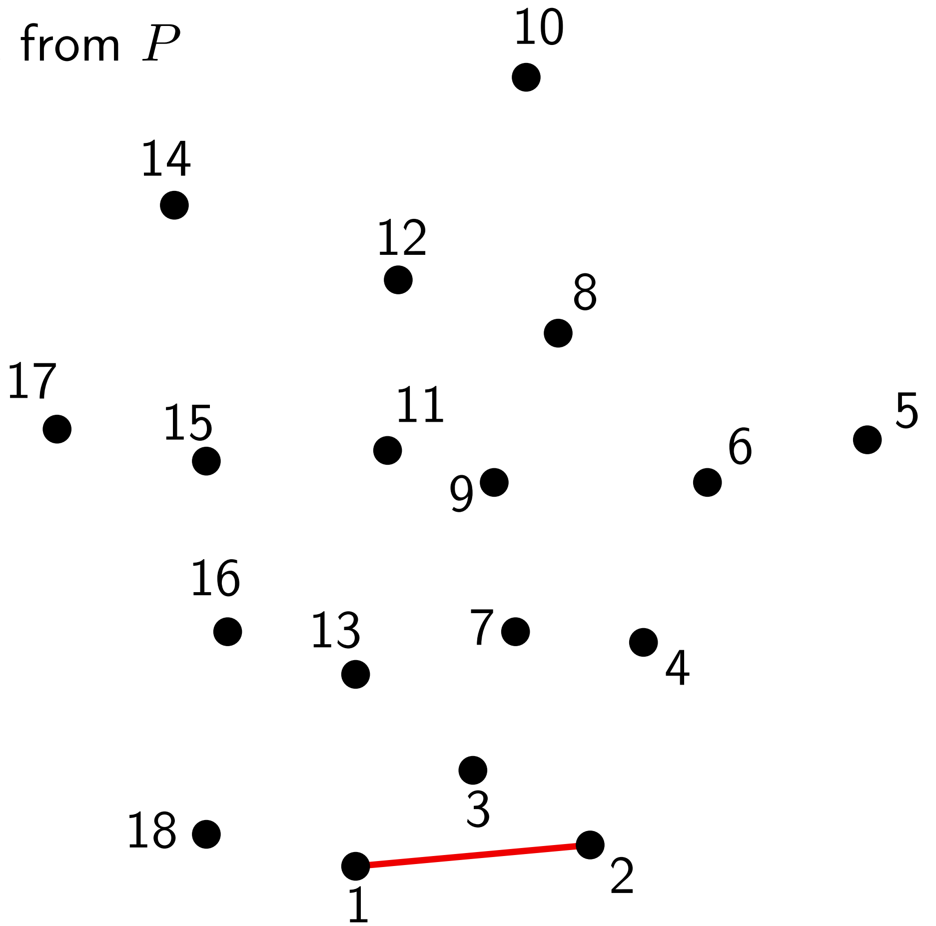

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While there exist points $p_i \in P$ to be explored, do:
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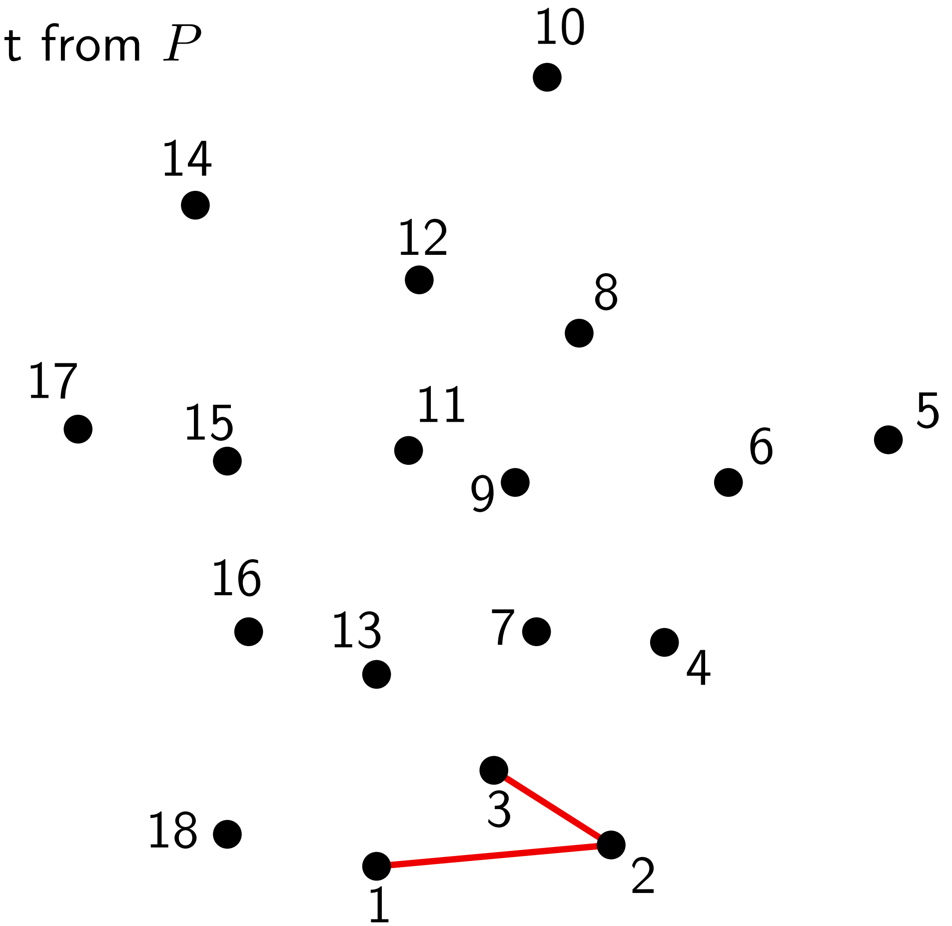
Return l



Graham's algorithm

Initialization

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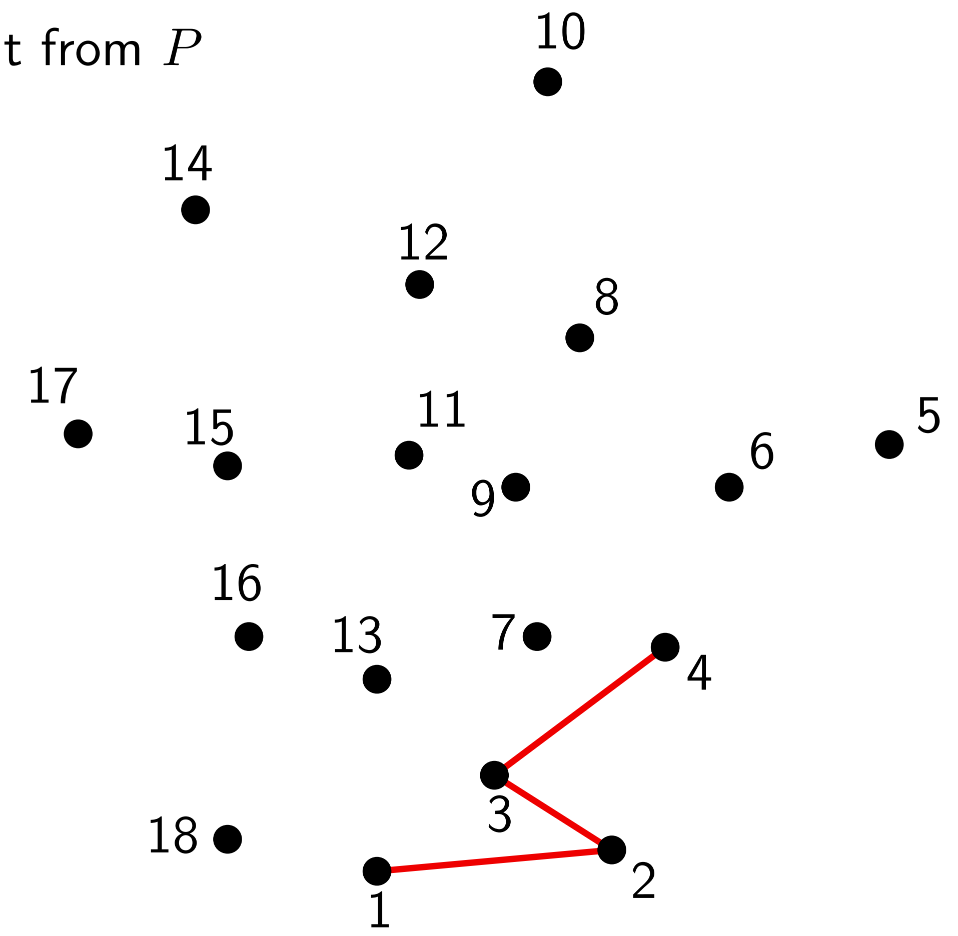

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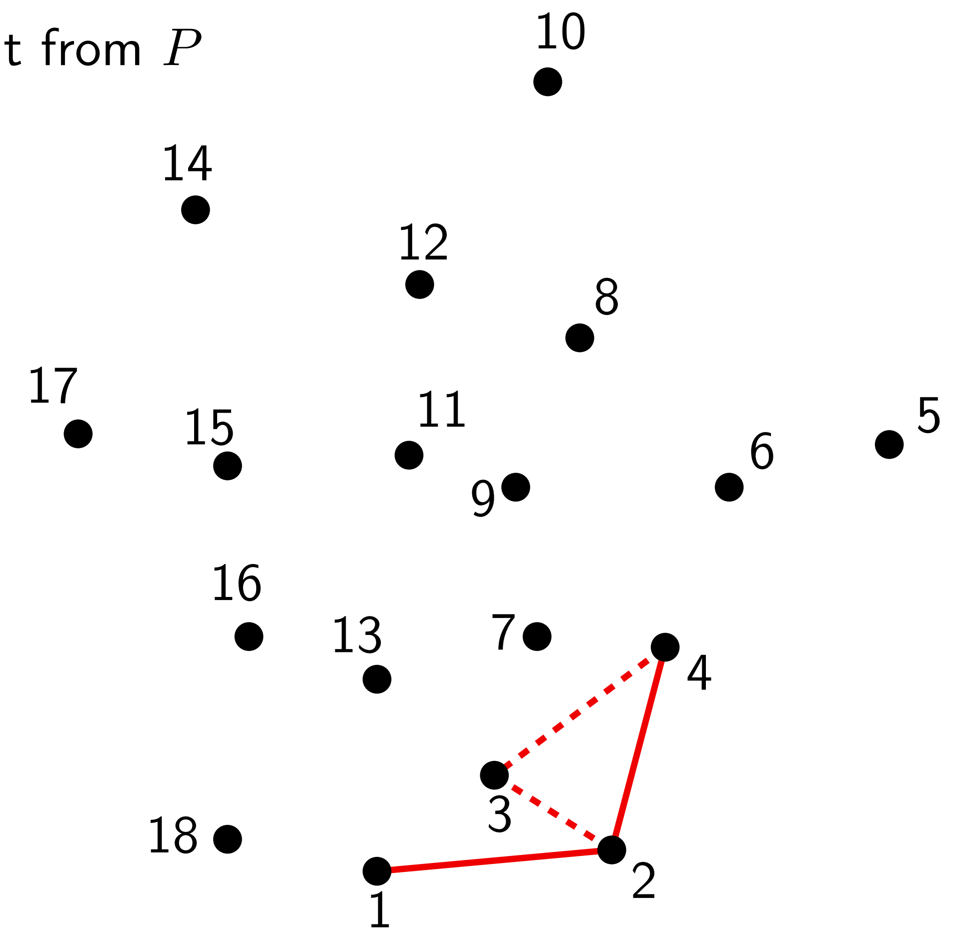

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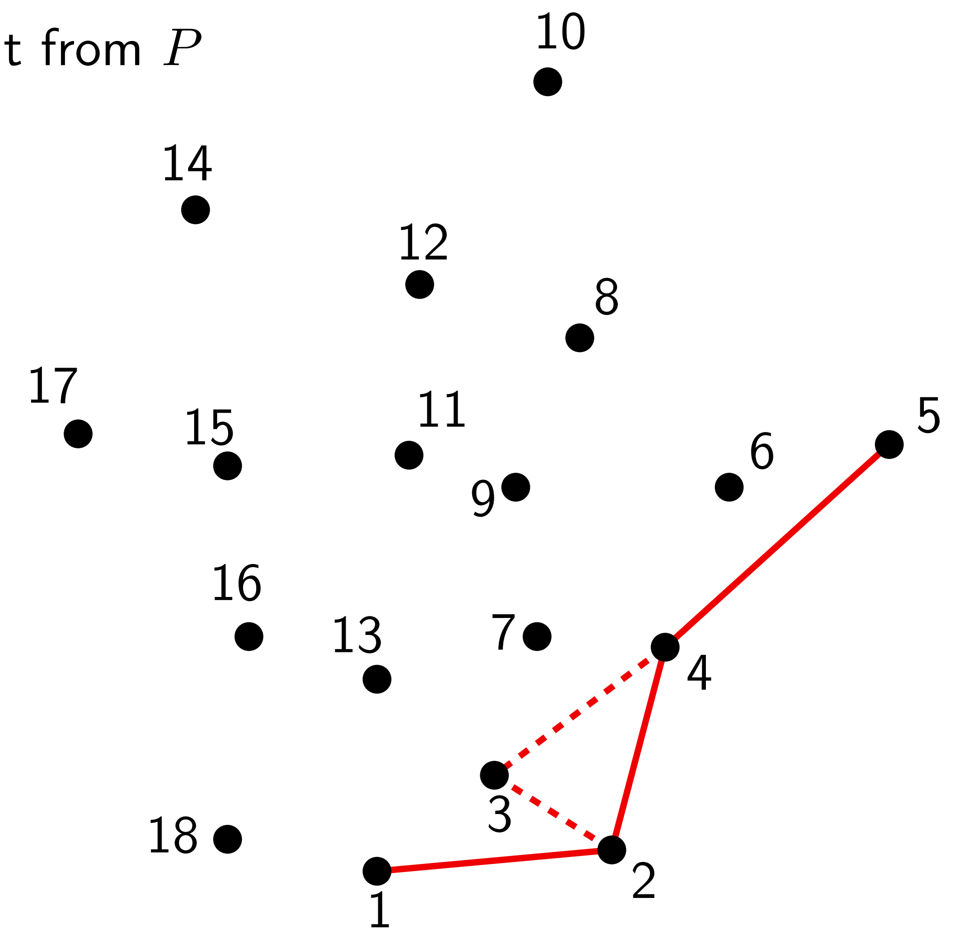

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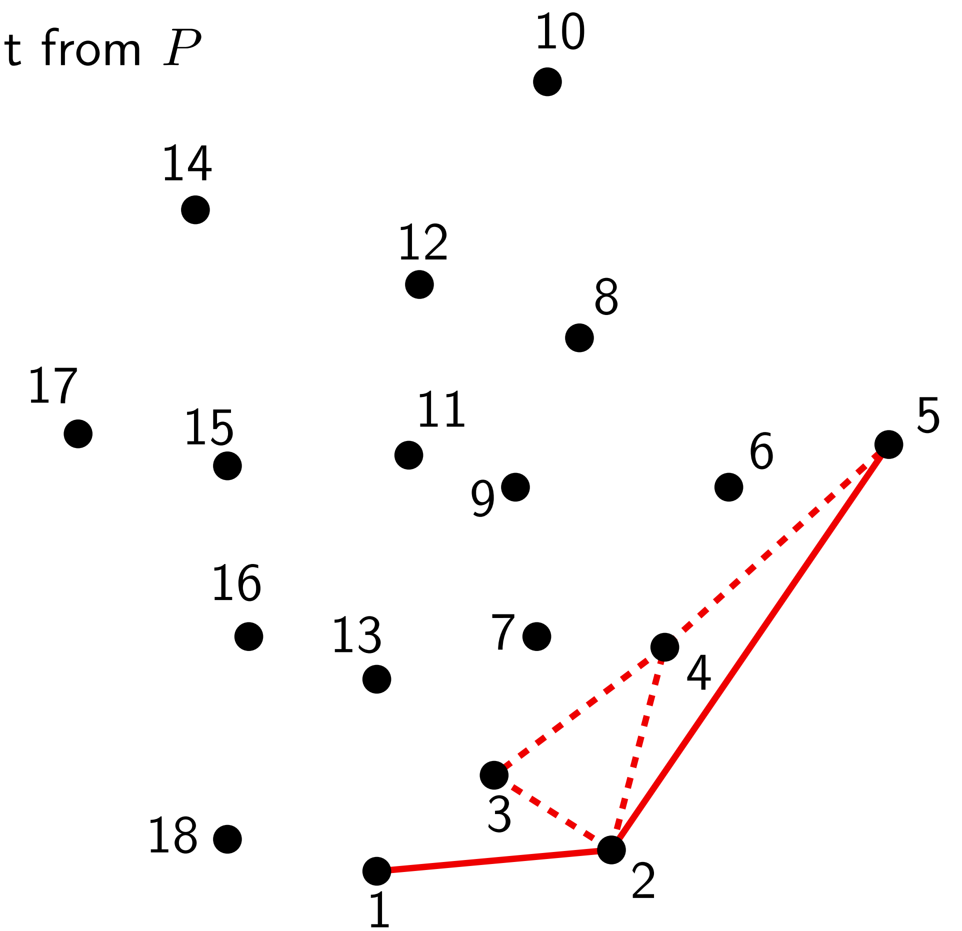

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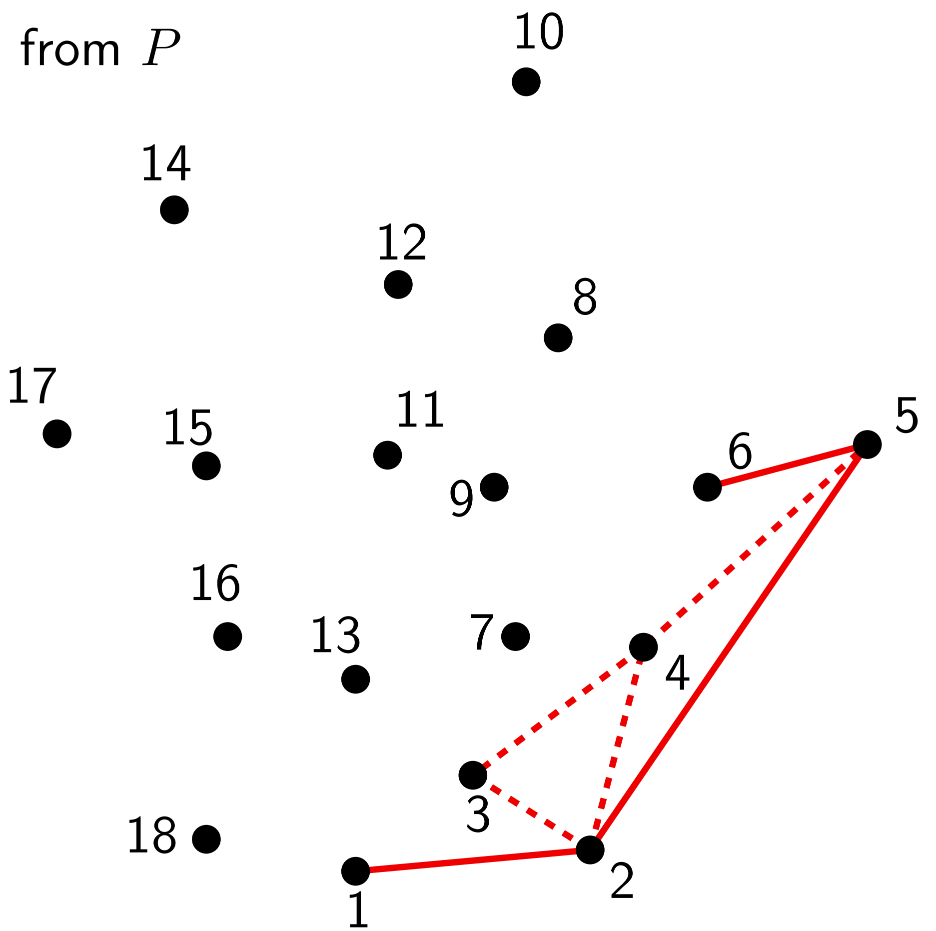

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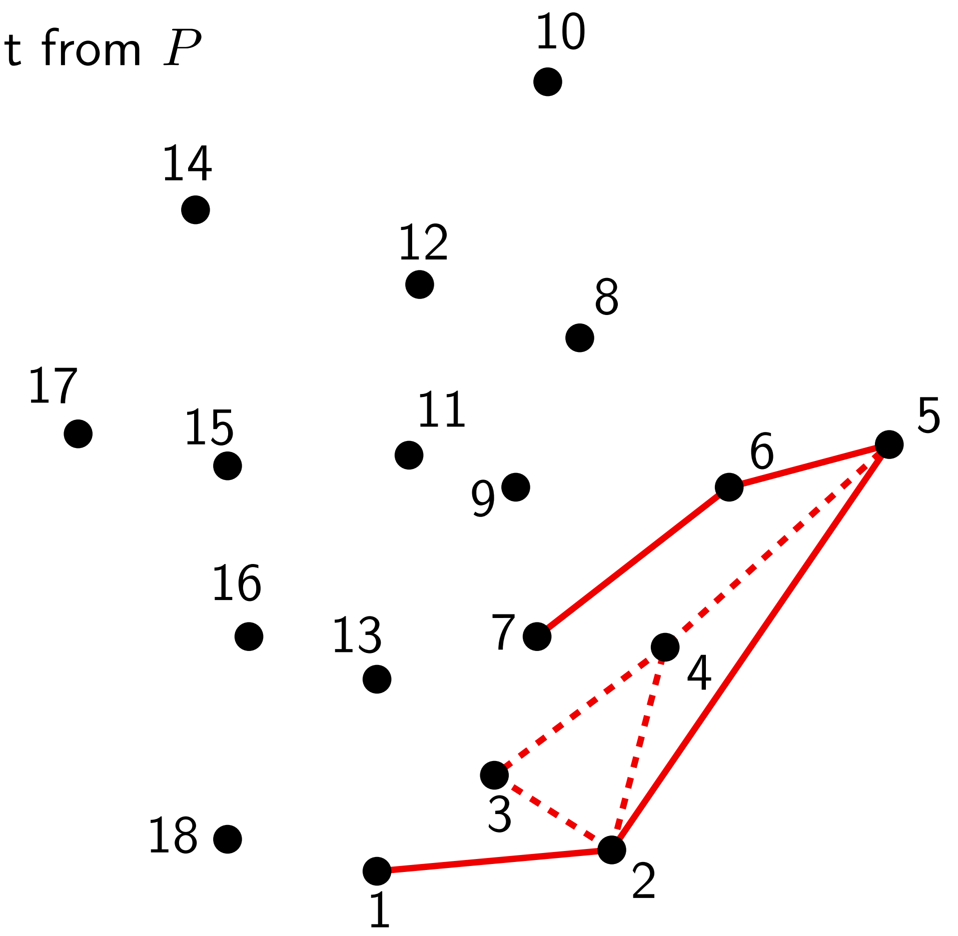

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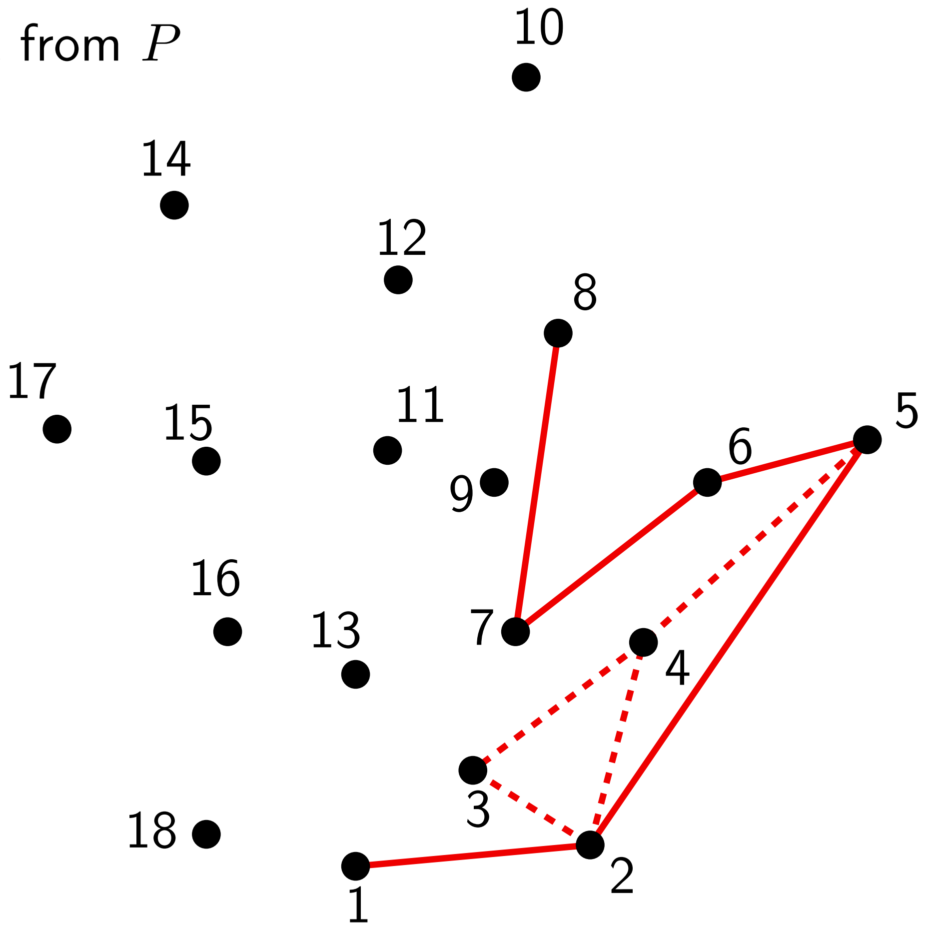

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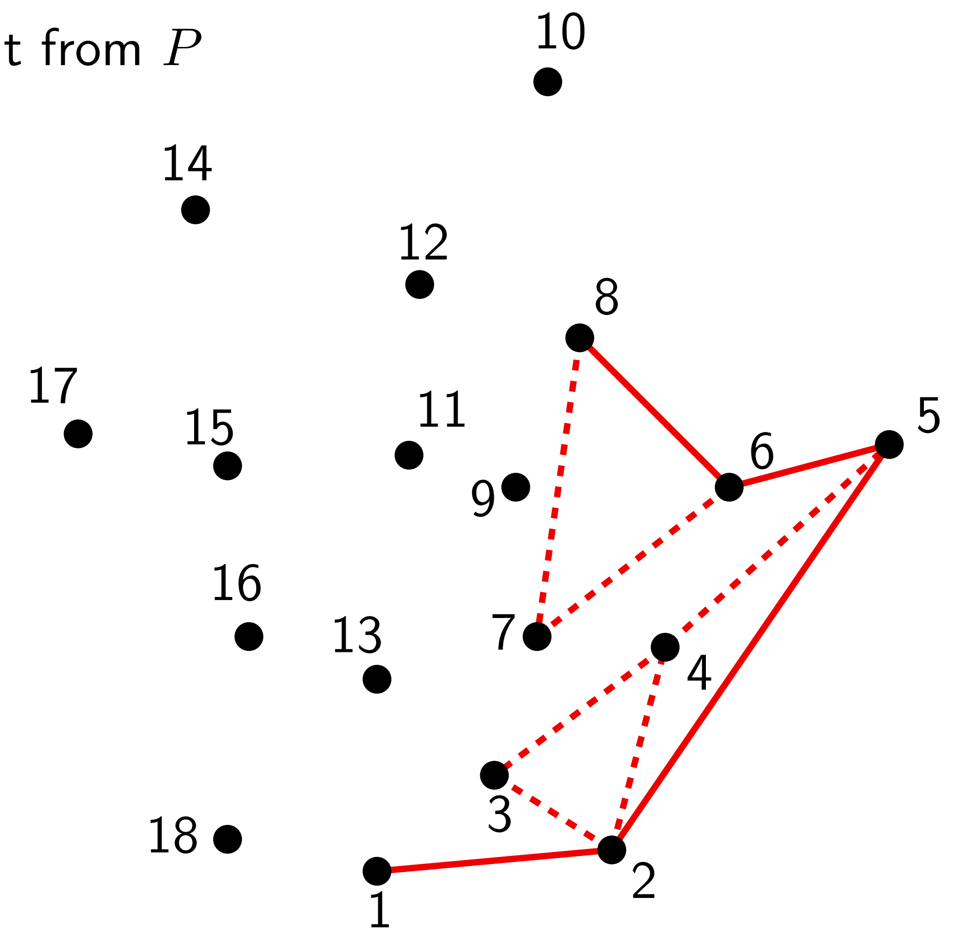

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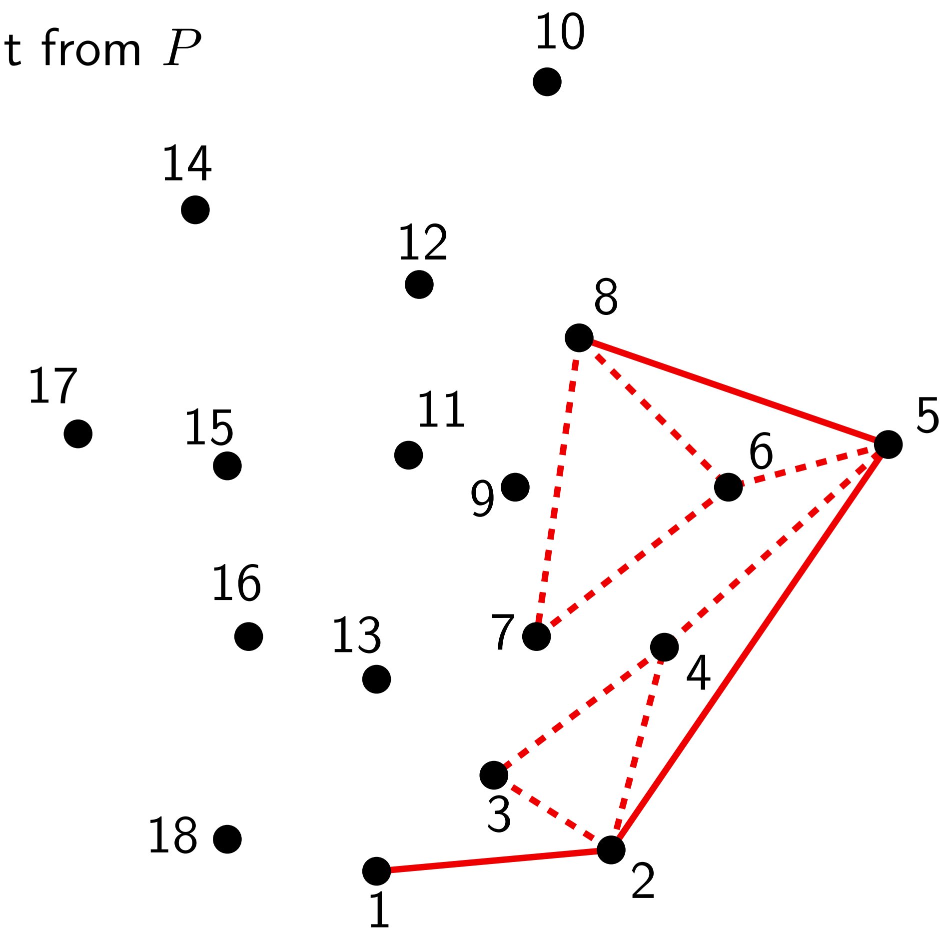

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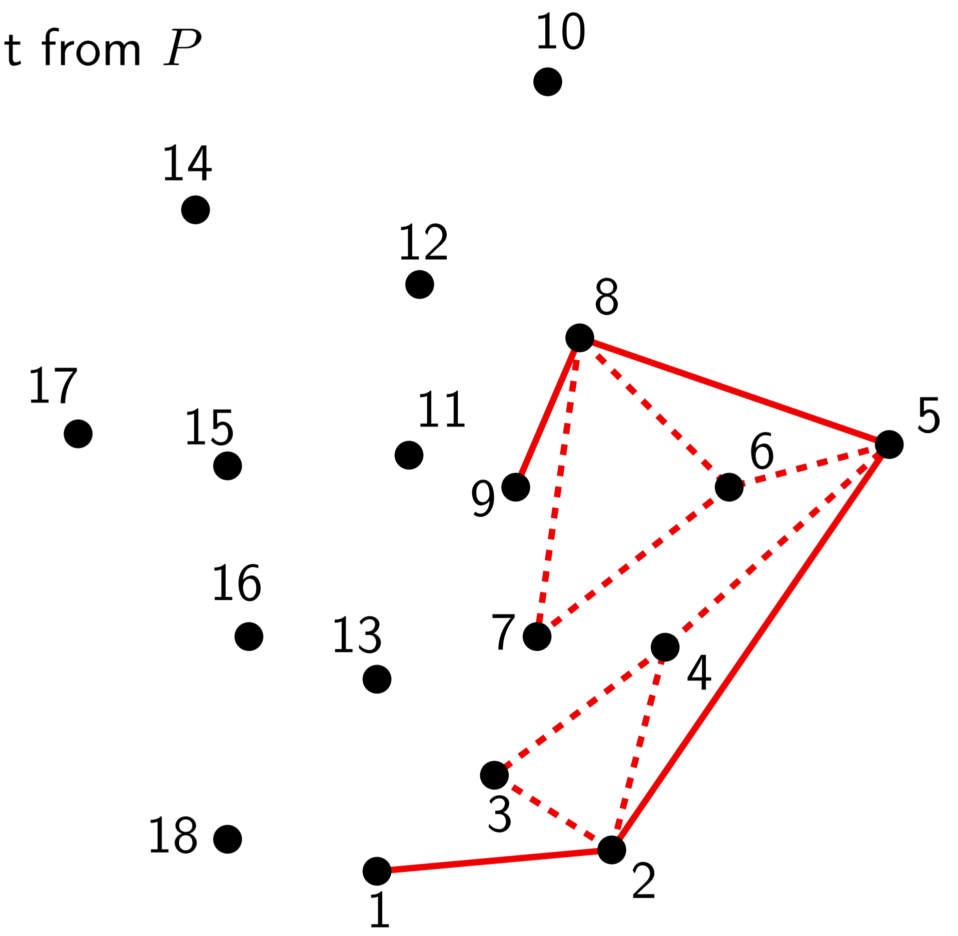

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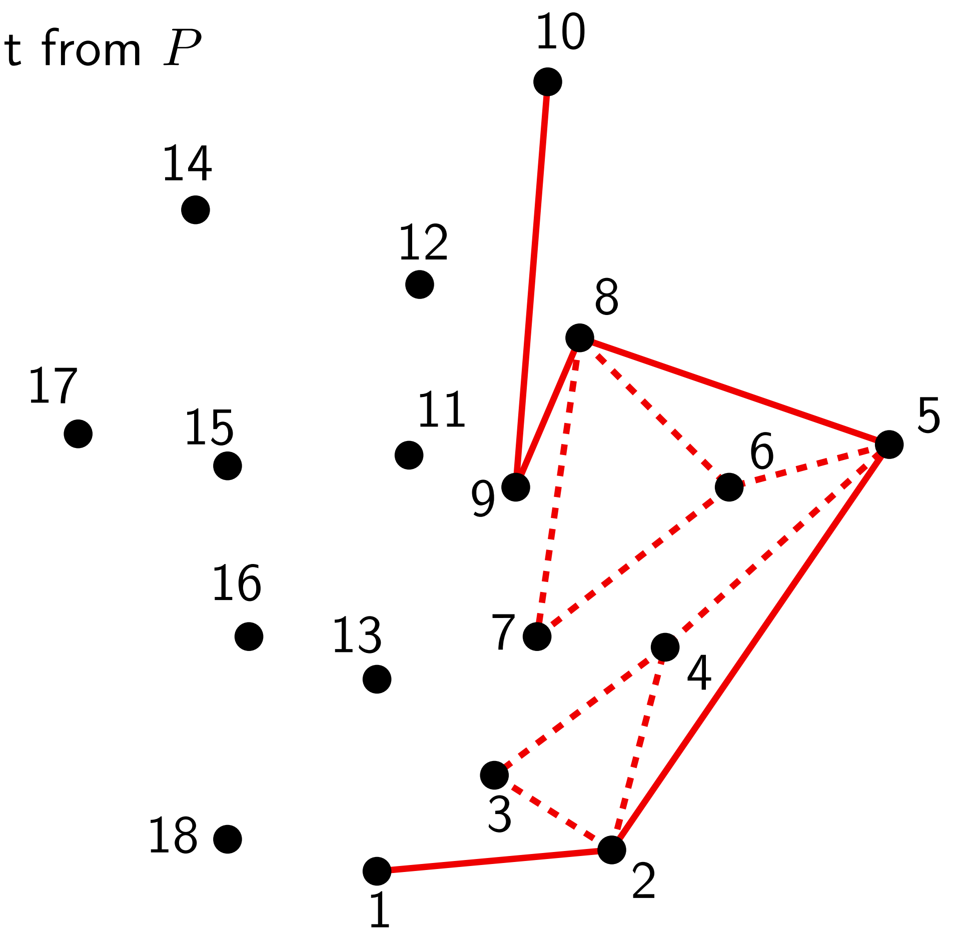

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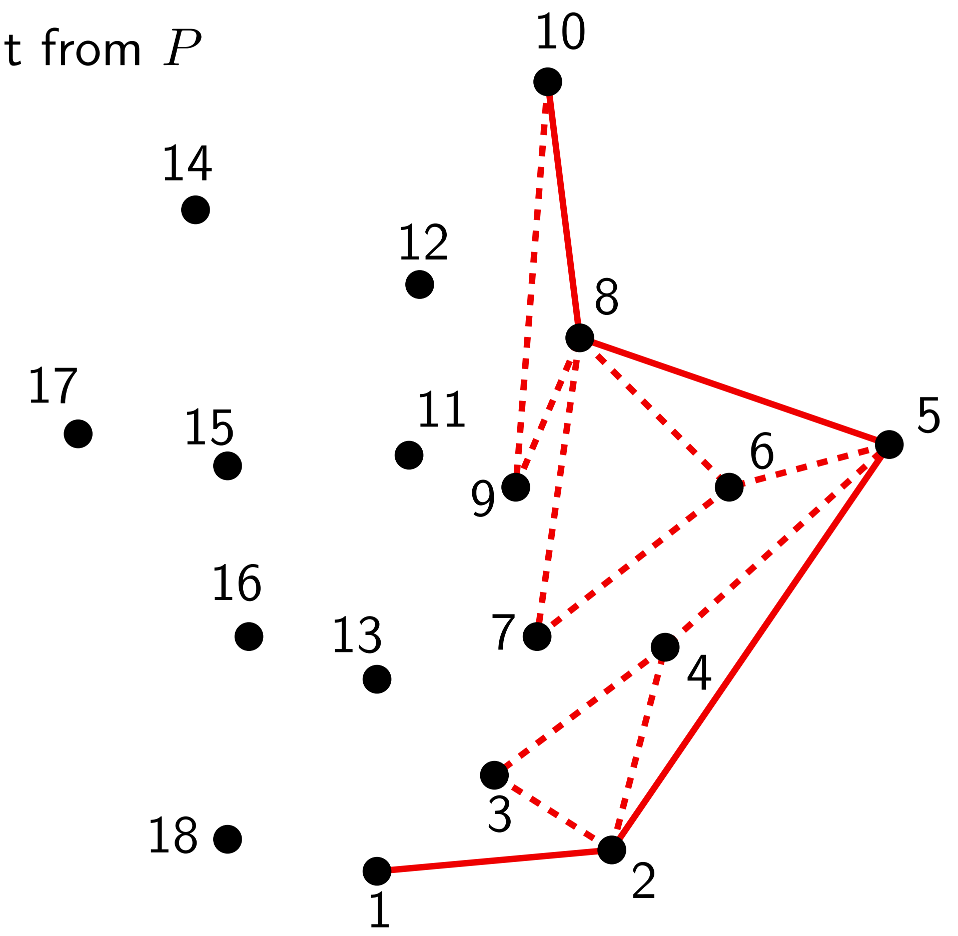

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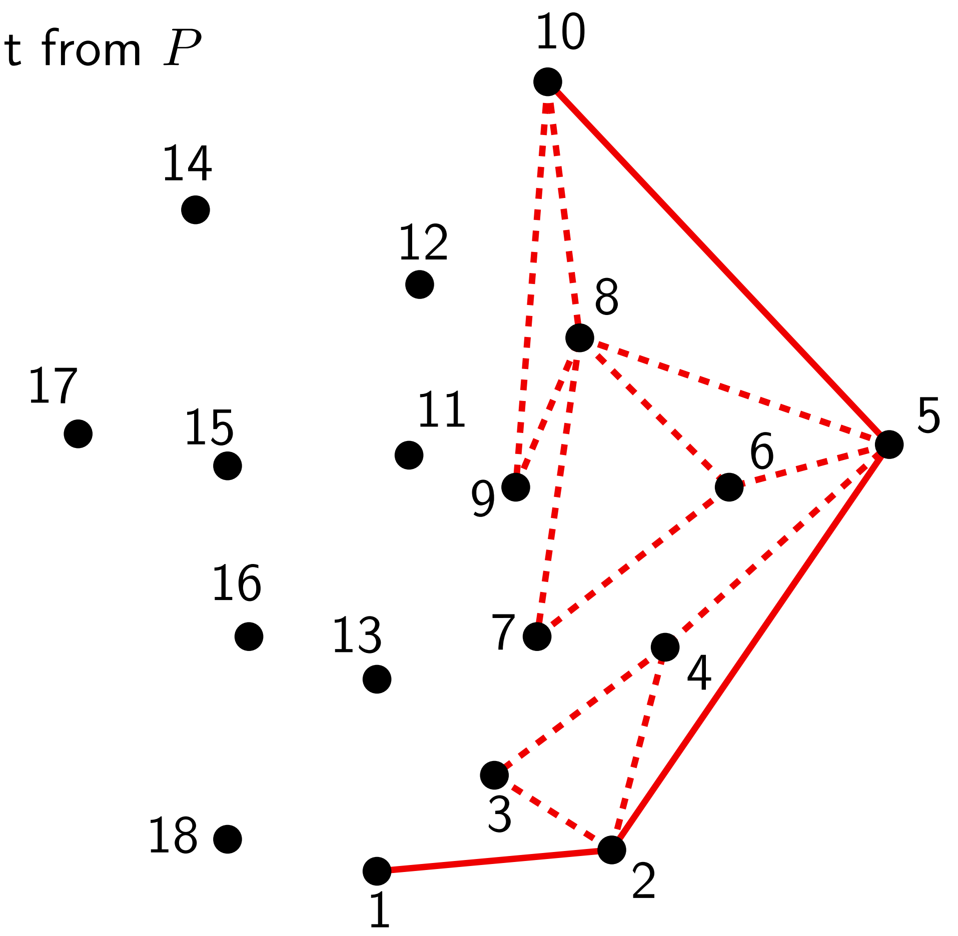

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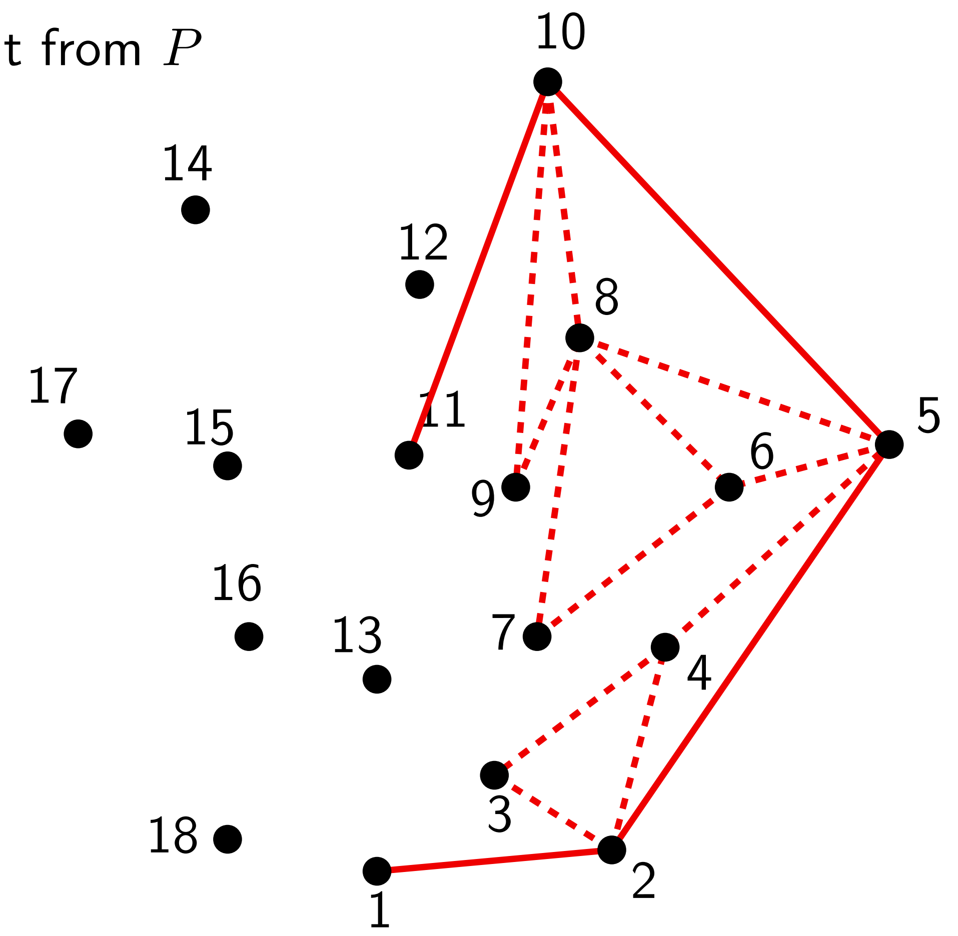

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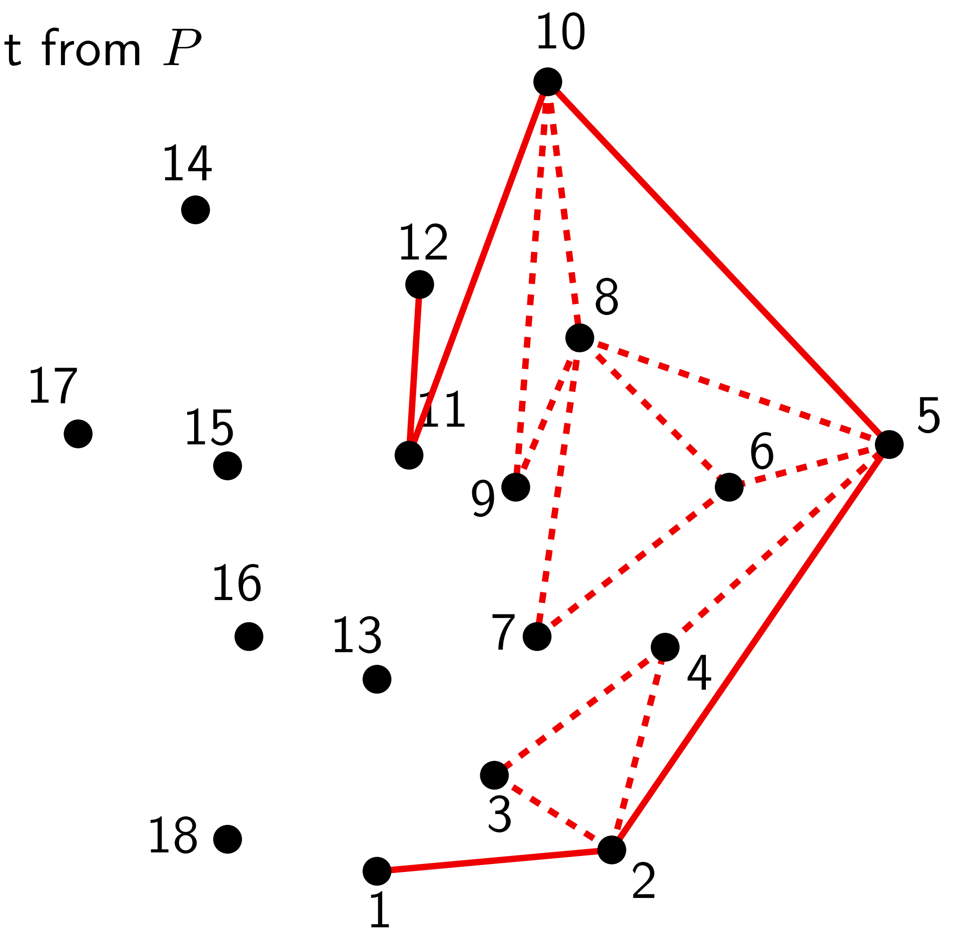

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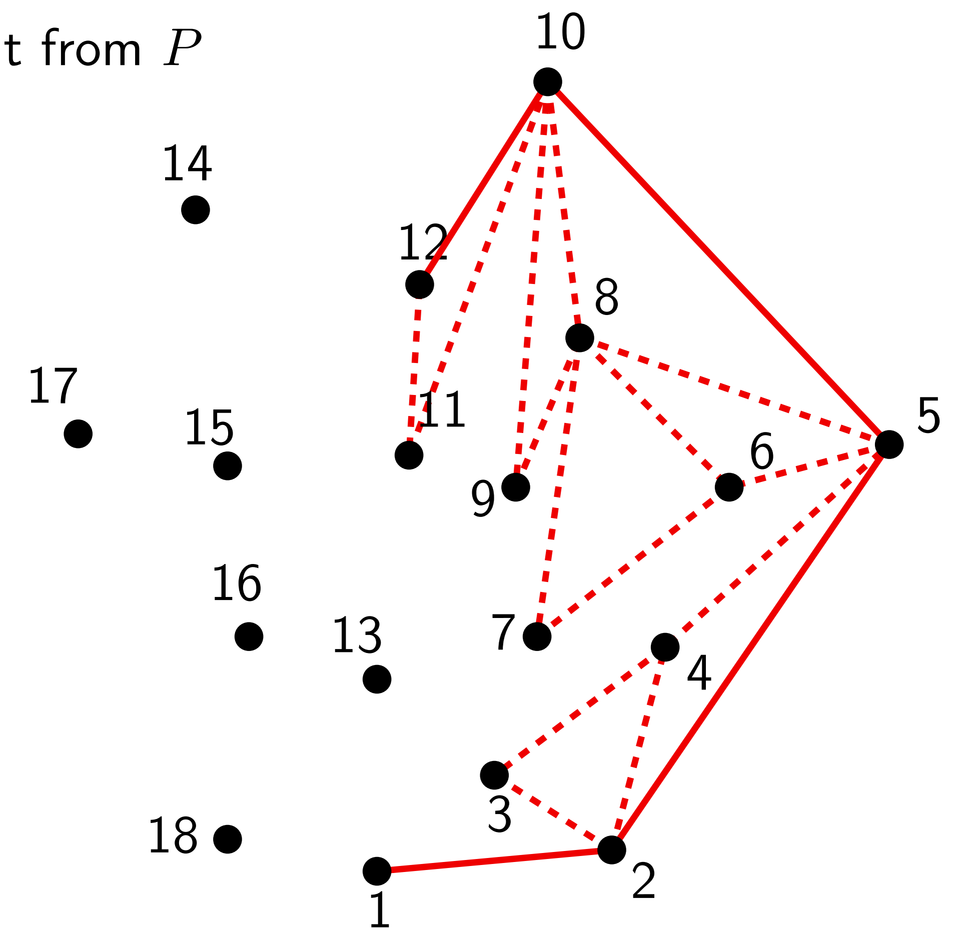

CONVEX HULL

Graham's algorithm

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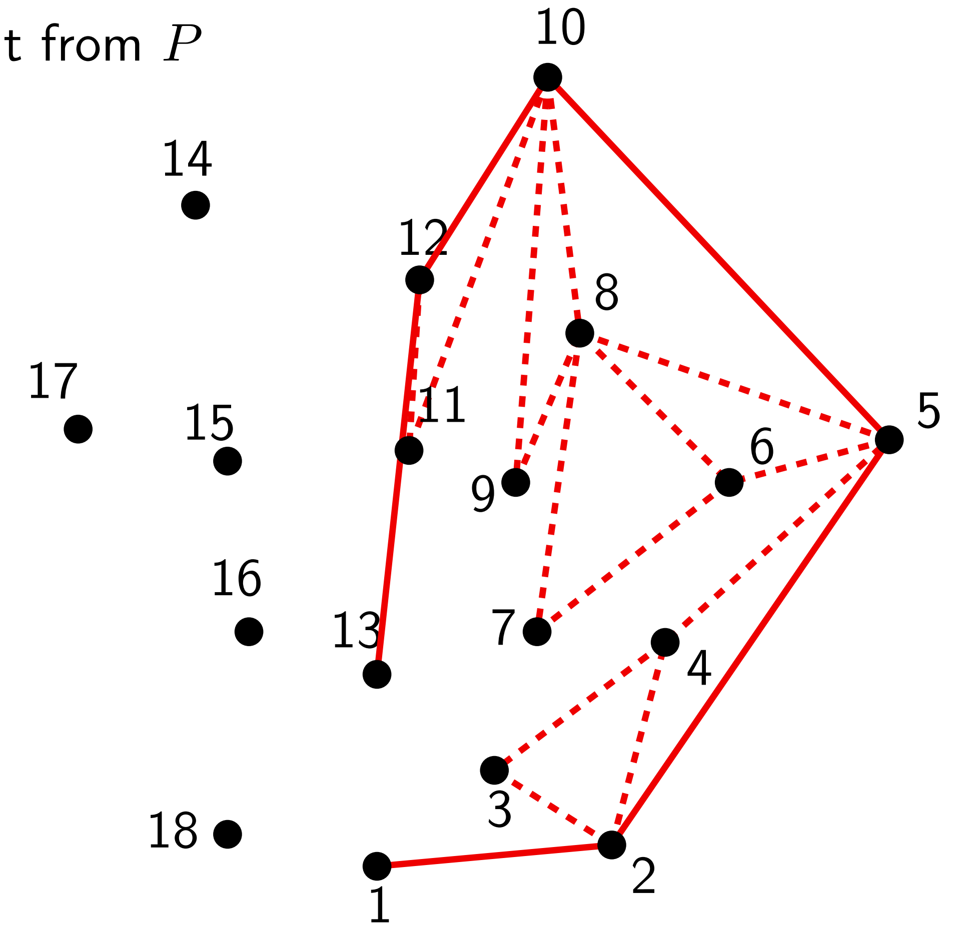

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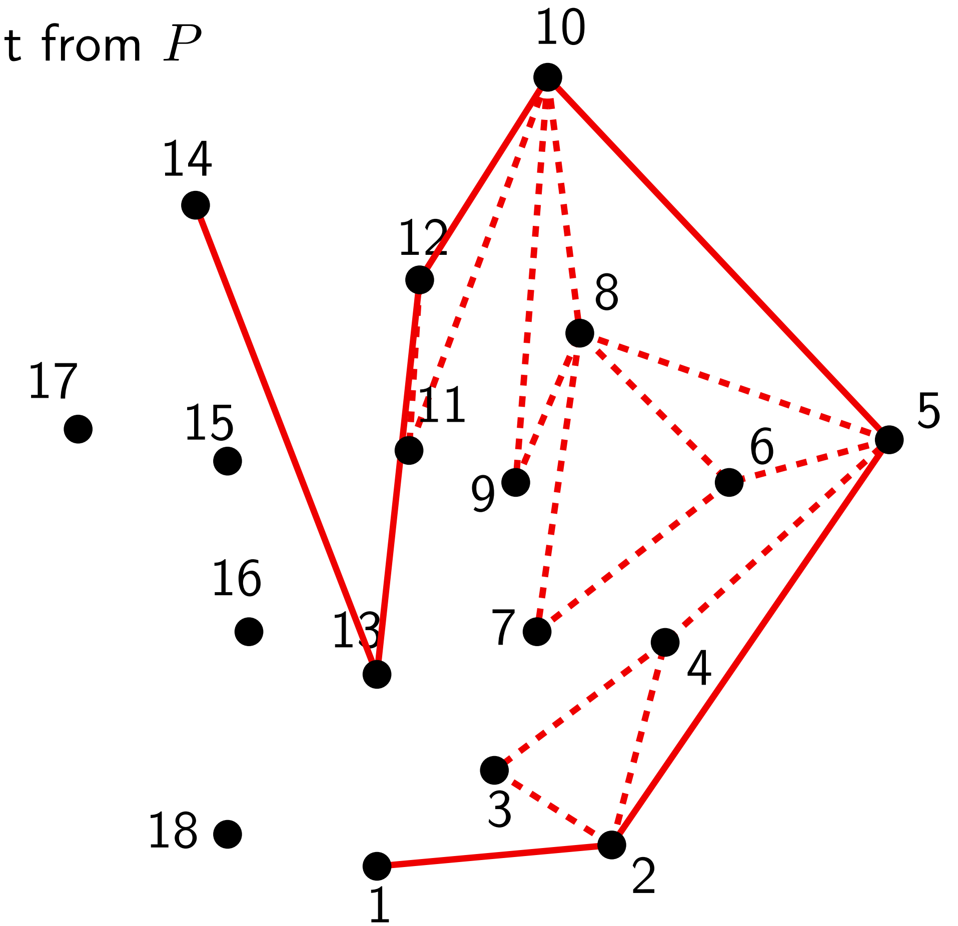

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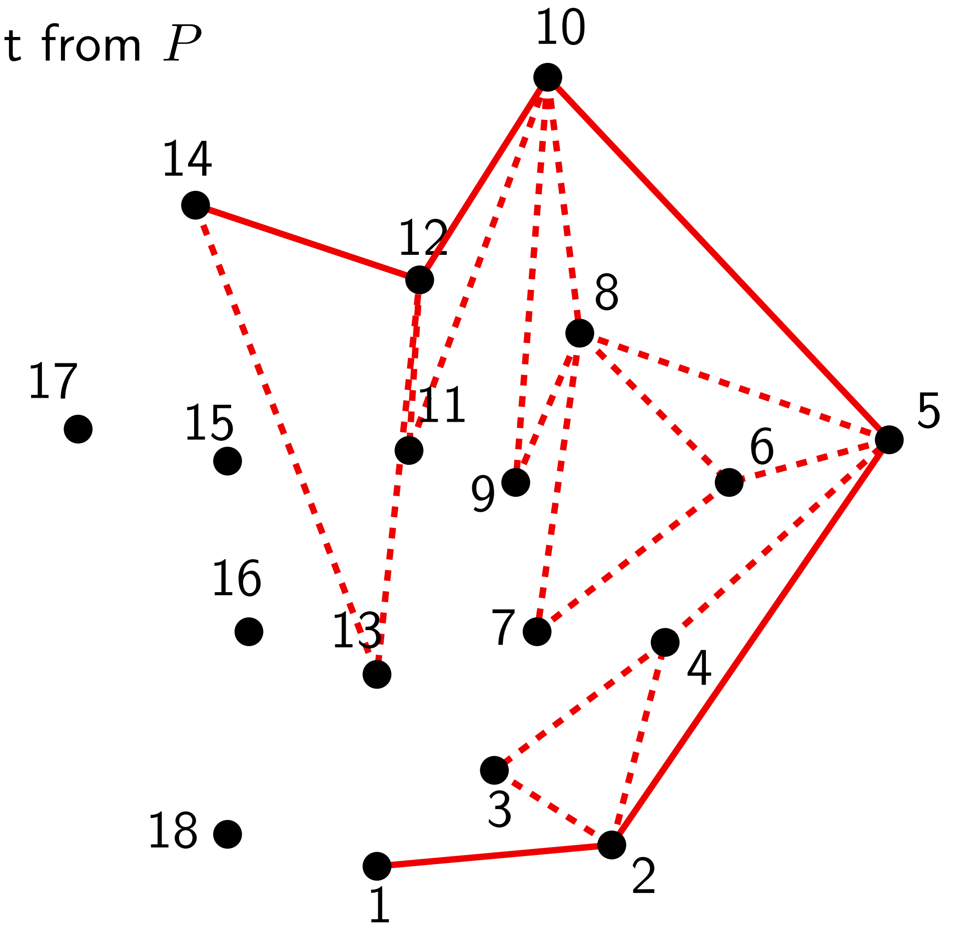

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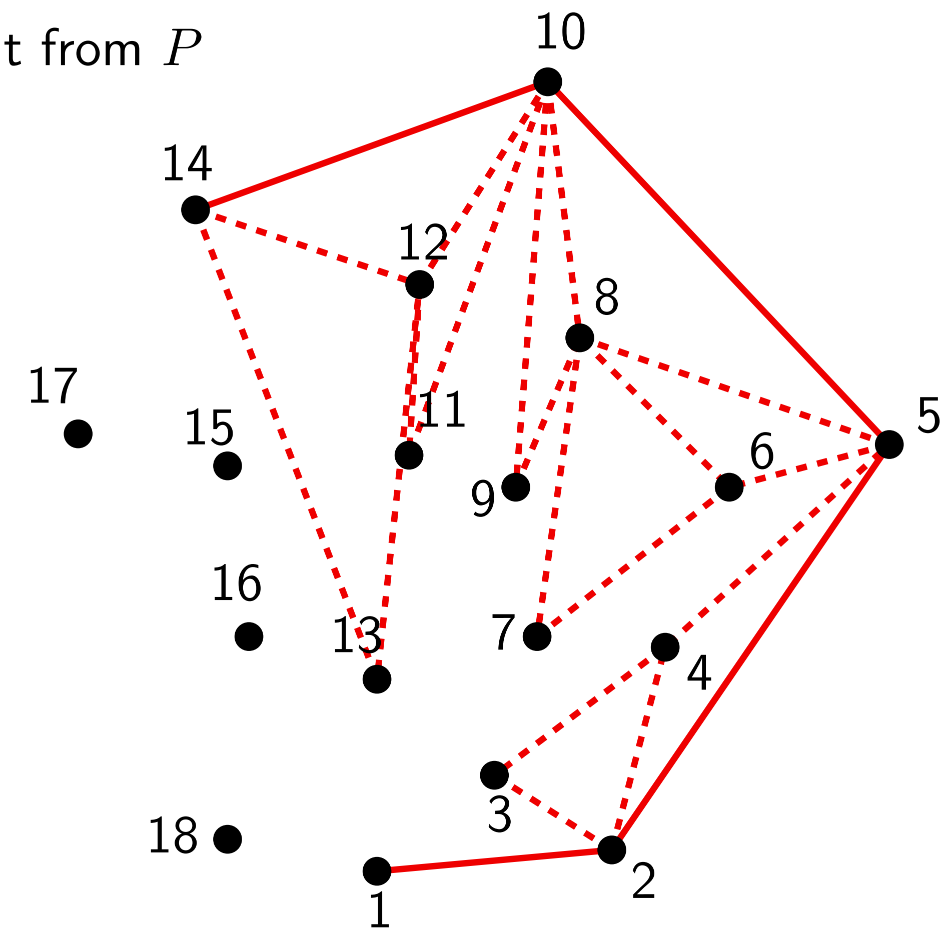

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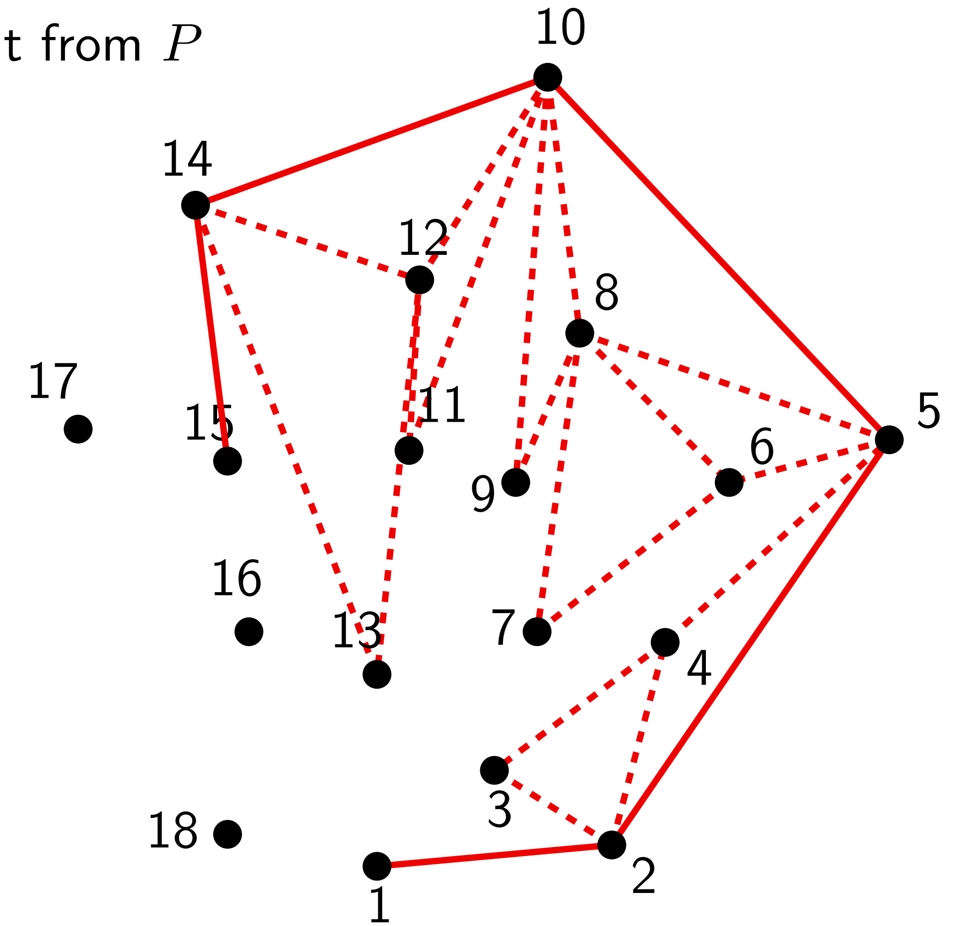

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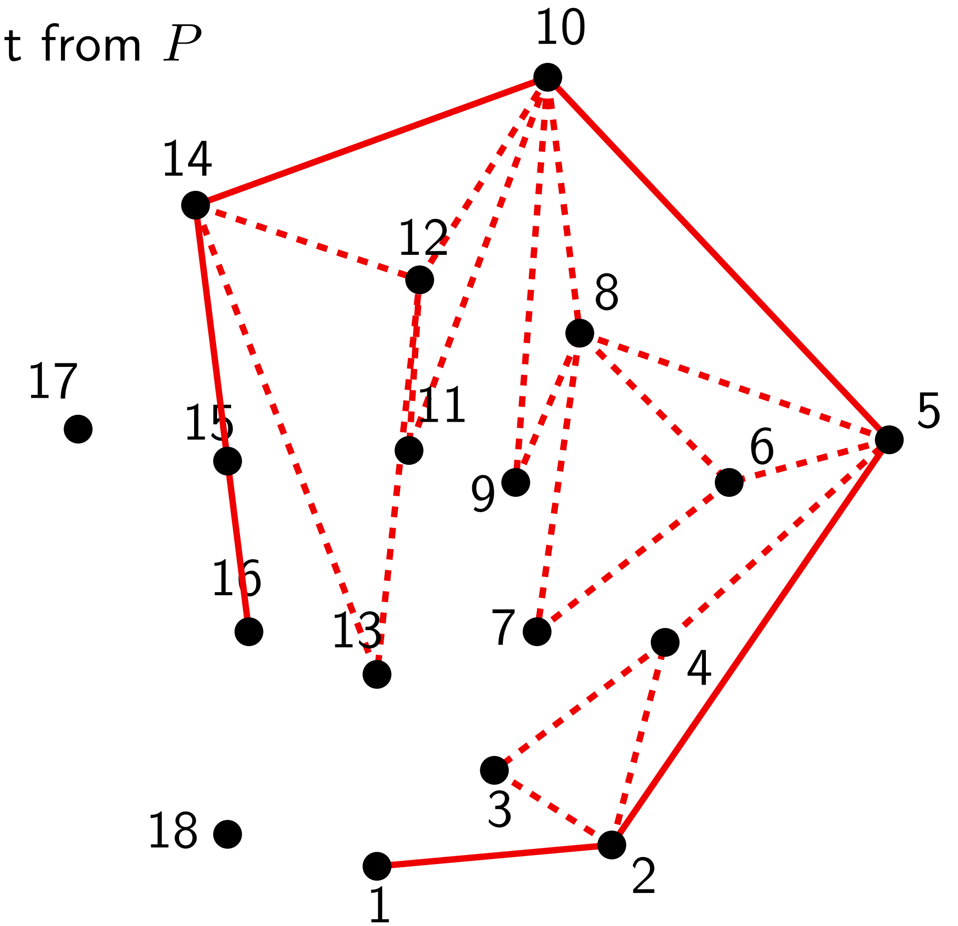

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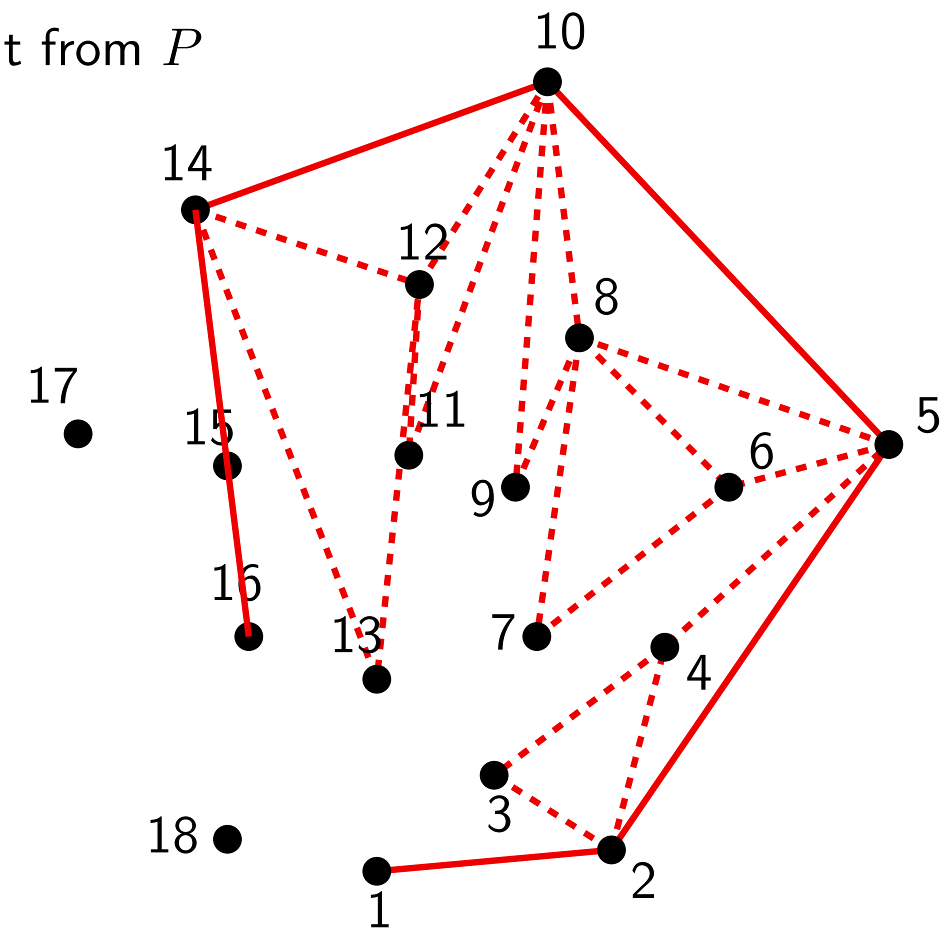

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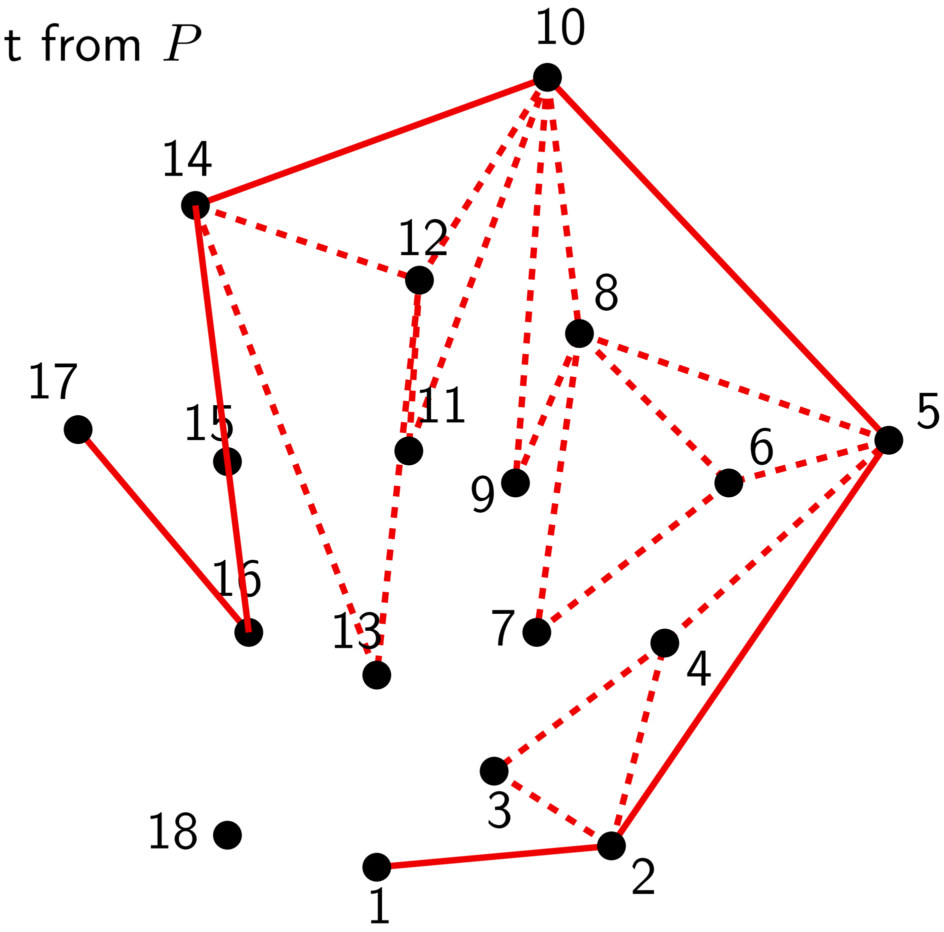

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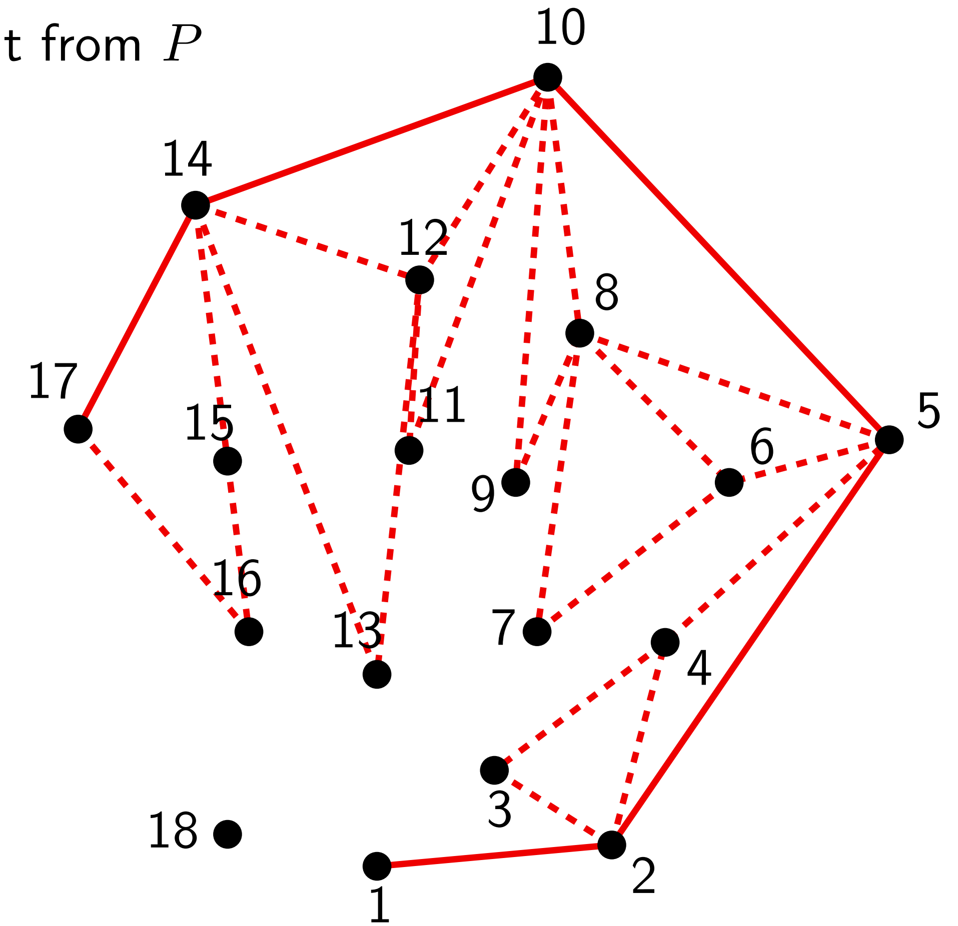

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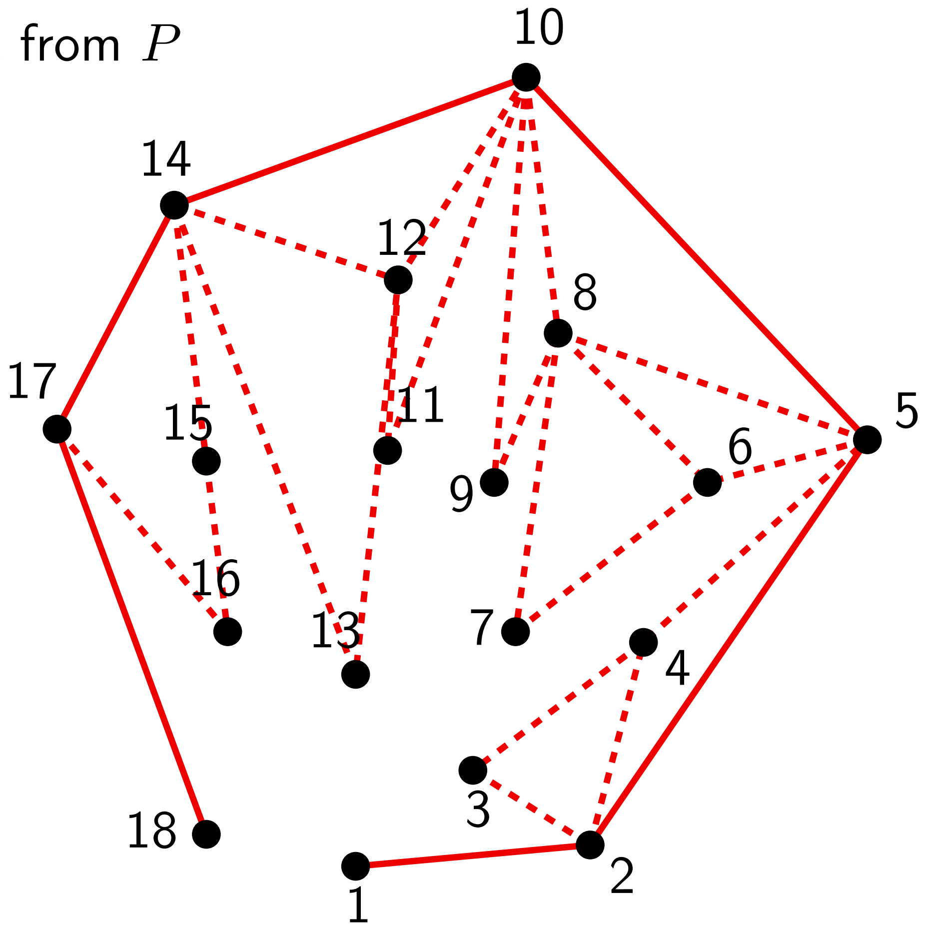

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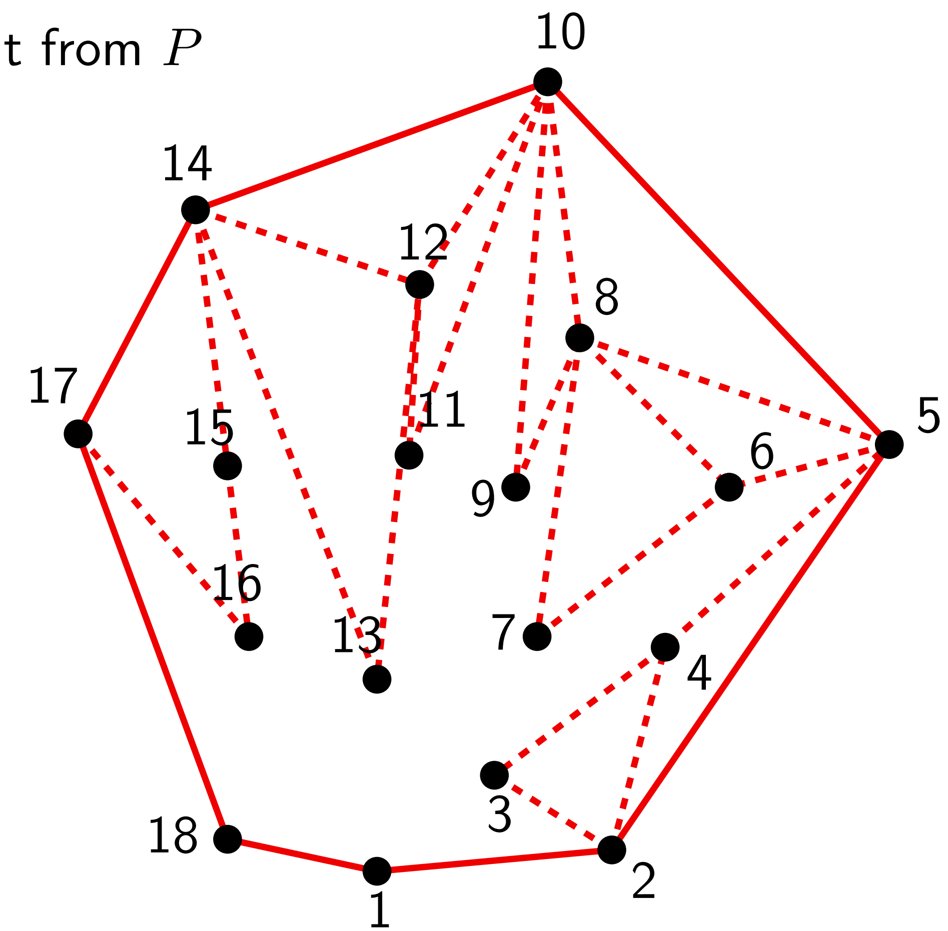

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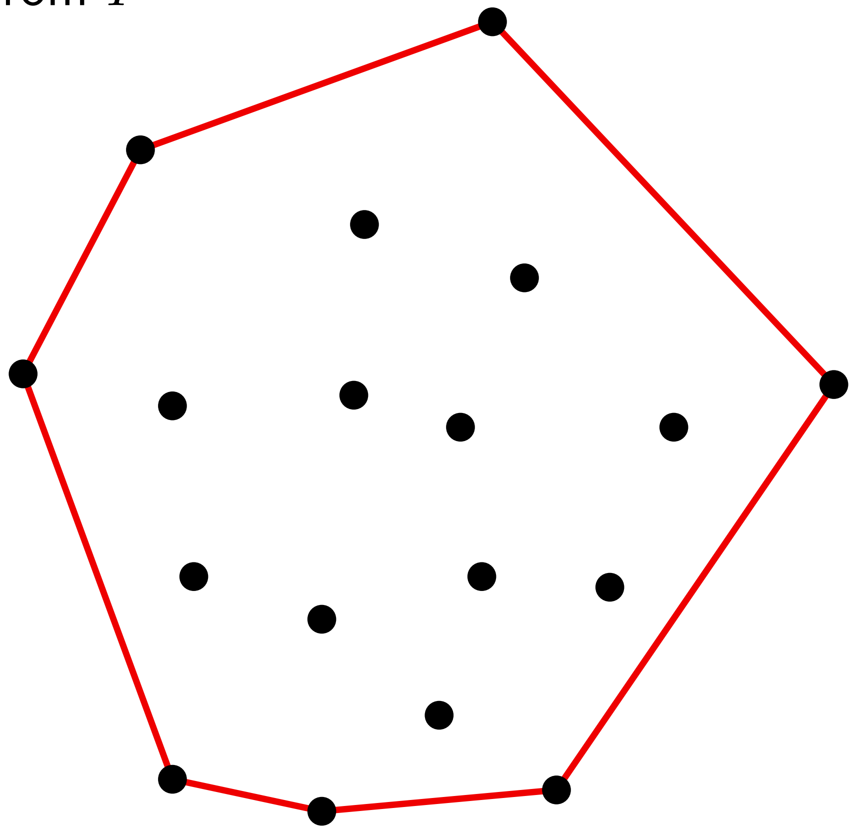

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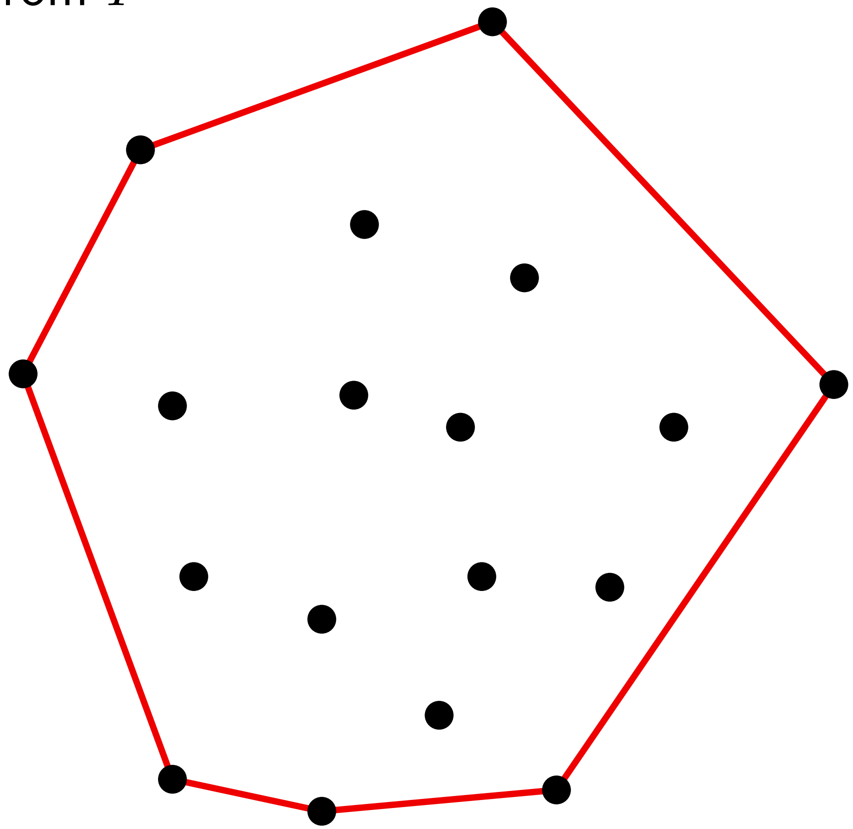

Running time: $O(n \log n)$

Incremental algorithm

Incremental algorithm

Initialization

$l = p_1, p_2, p_3$

Advance

From $i = 4$ to n, do:

If p_i lies in the exterior of the polygon defined by l:

- Compute the points p_l and p_r defining the supporting lines from p_i to the polygon
- Replace the chain $p_l, \ldots, p_r$ in l with the chain p_l, p_i, p_r

Return l

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$l = p_1, p_2, p_3$

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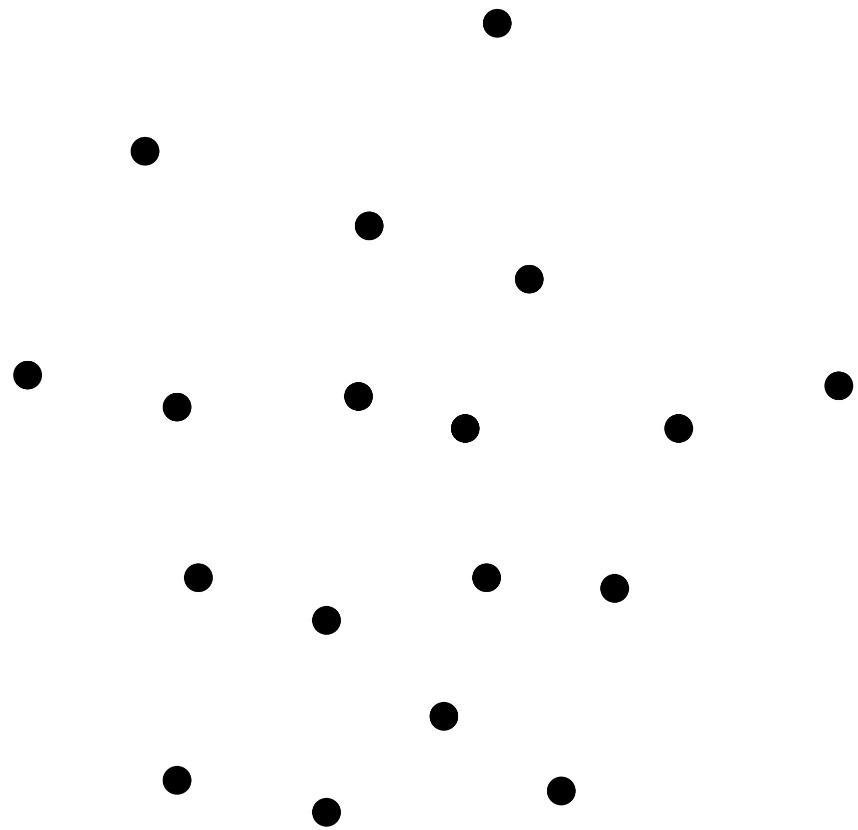

Incremental algorithm

Initialization

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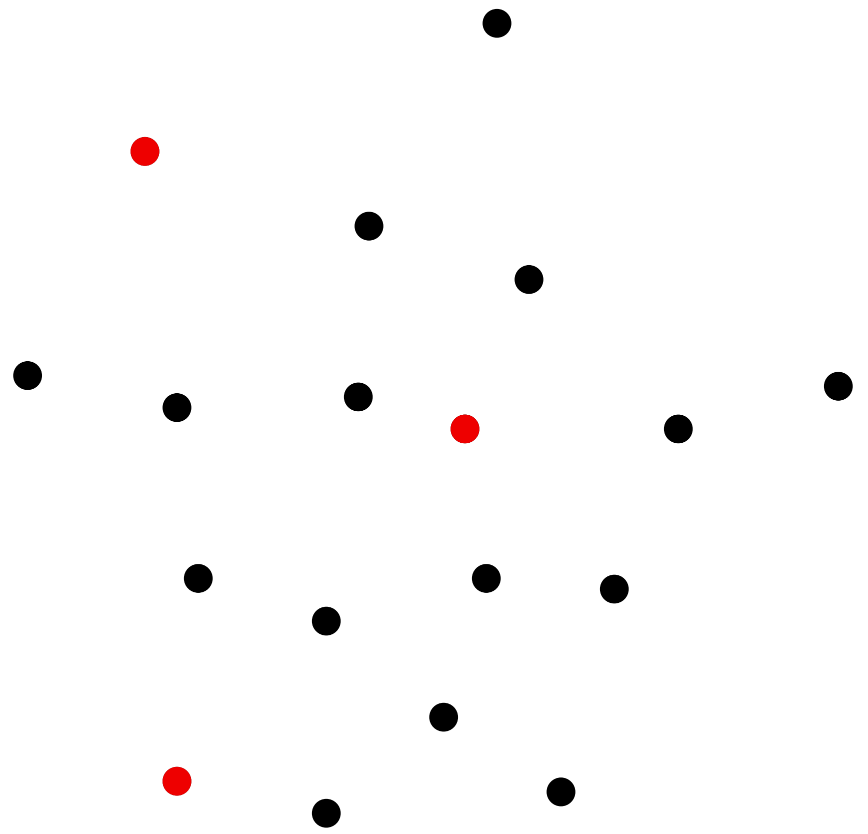

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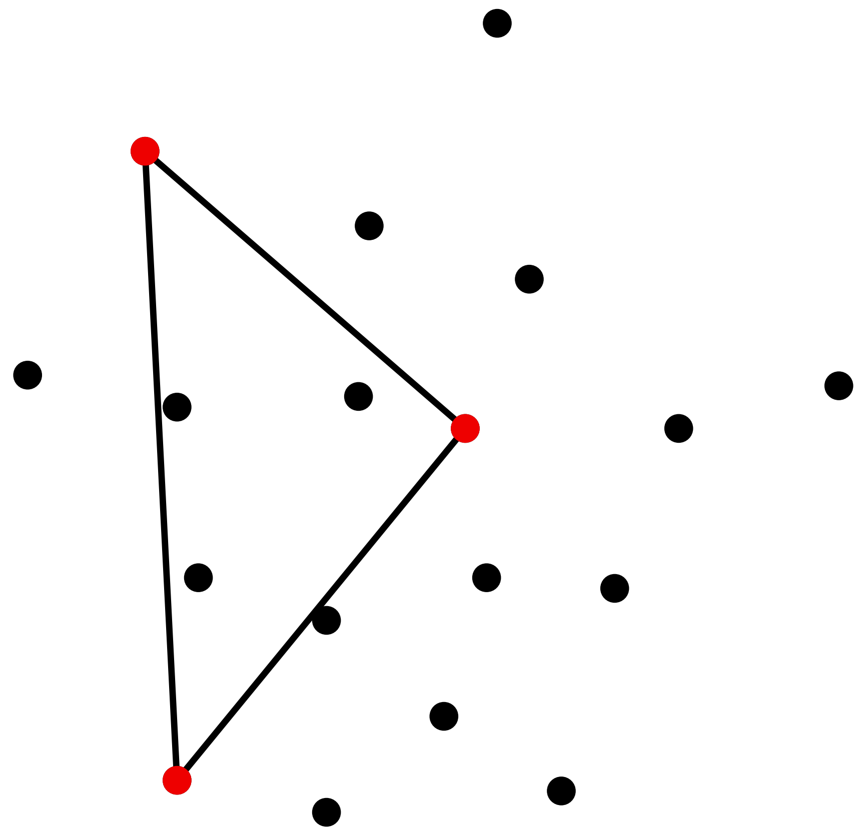

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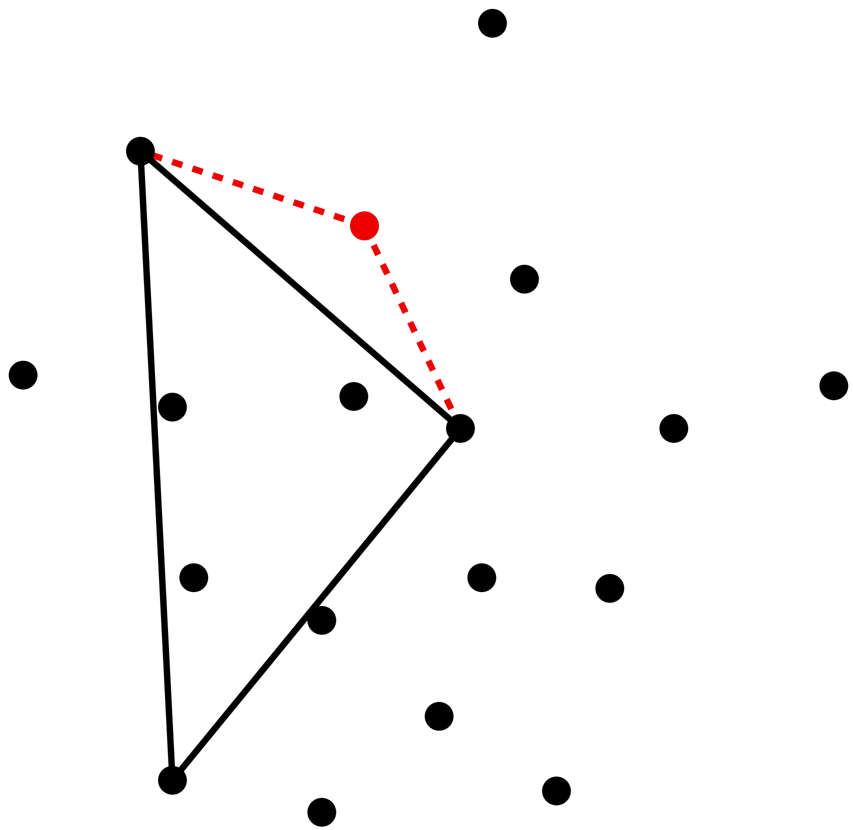

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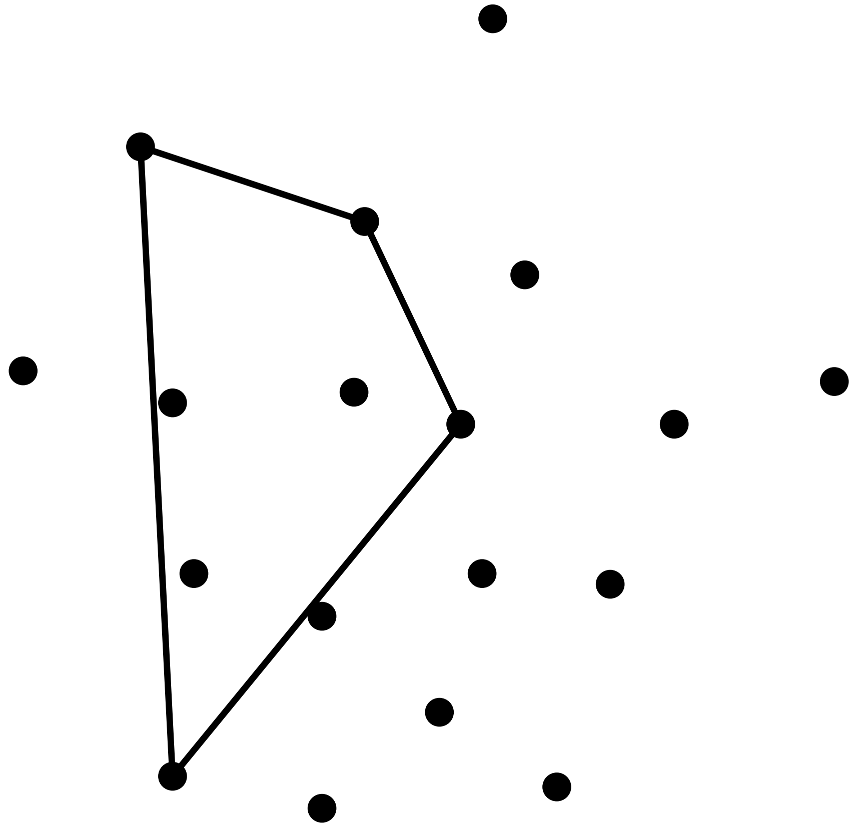

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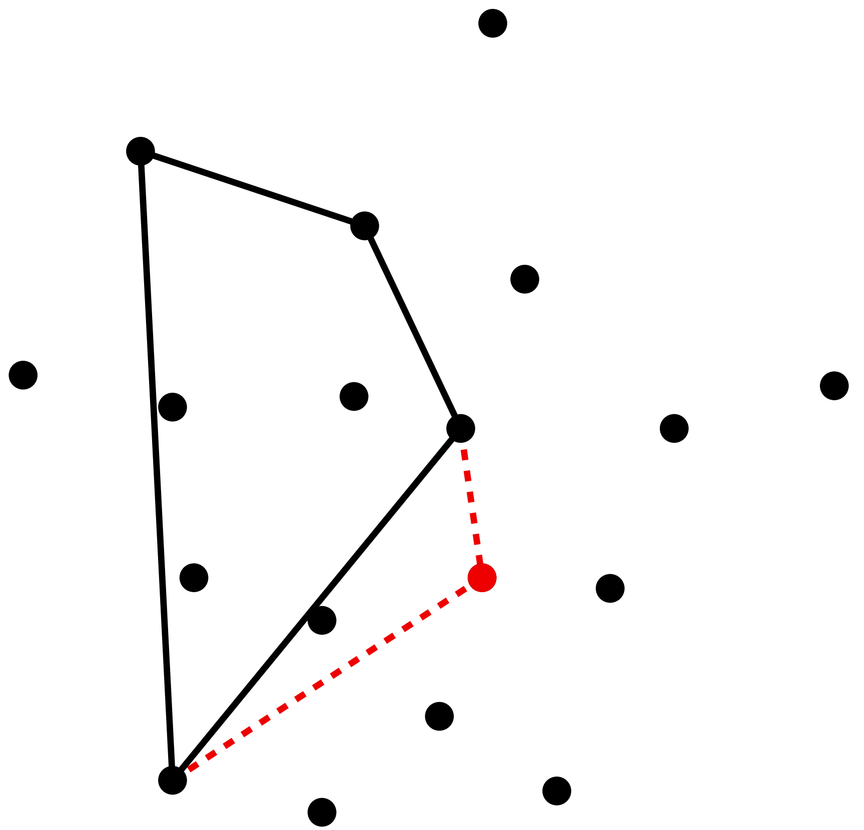

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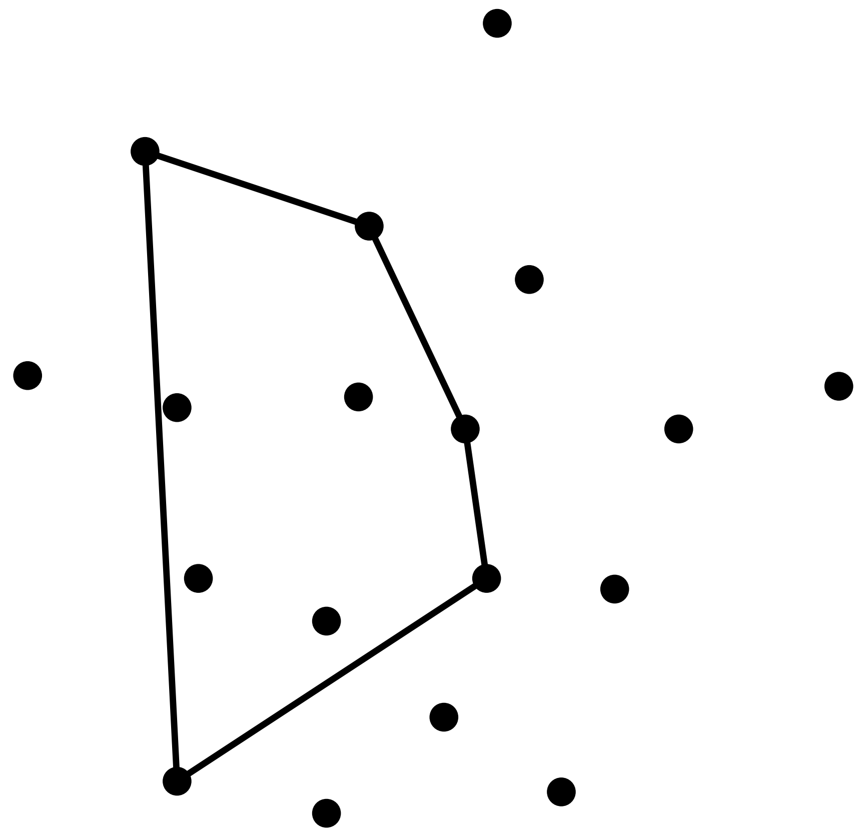

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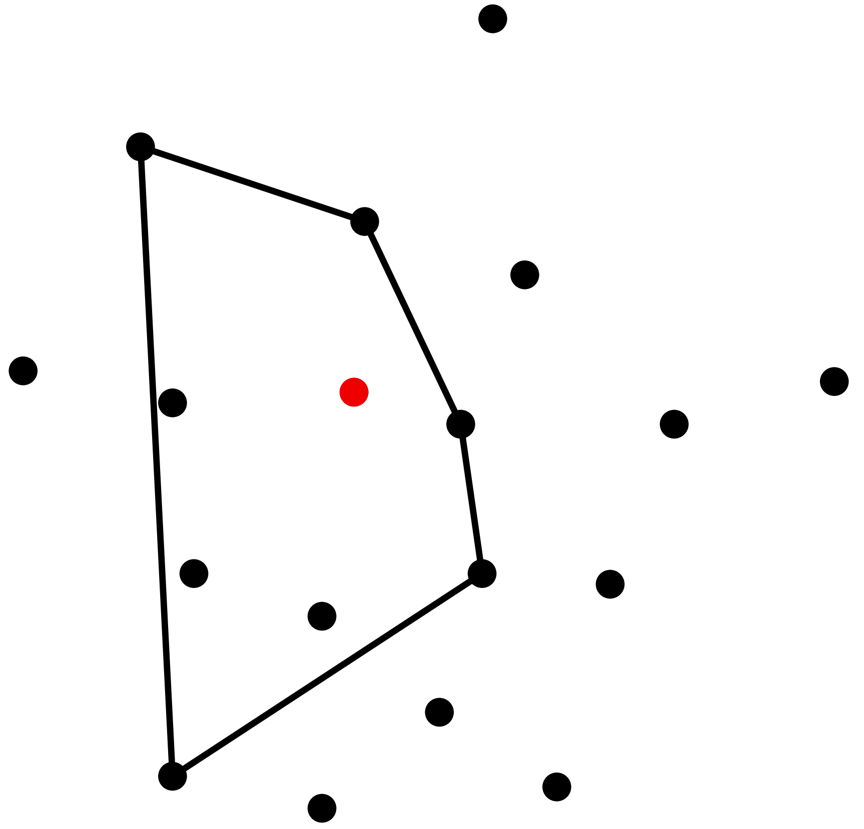

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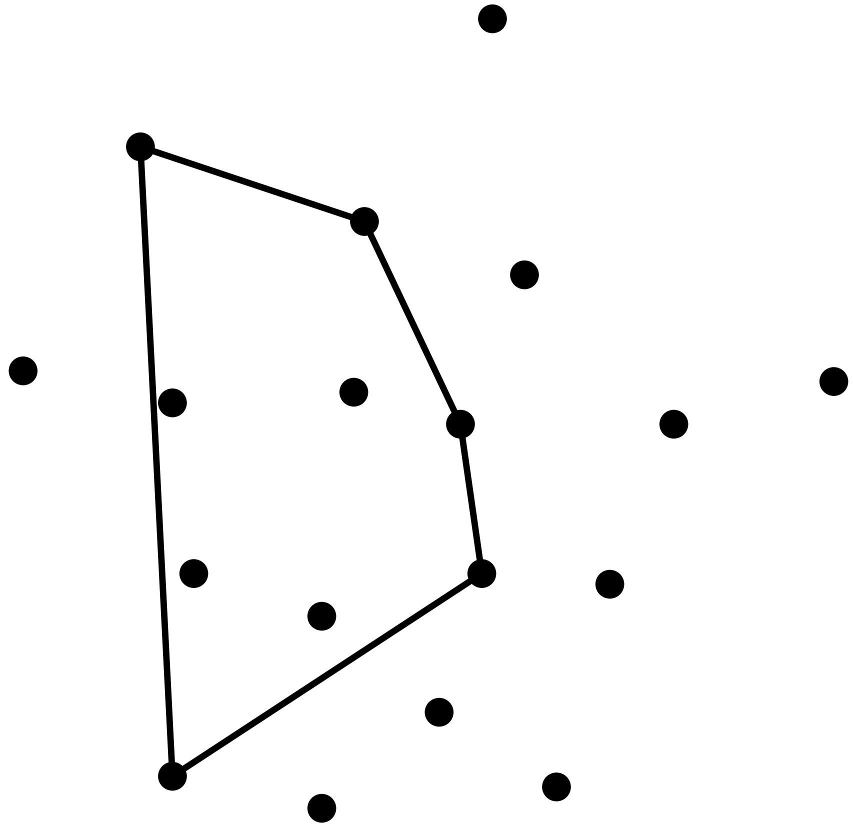

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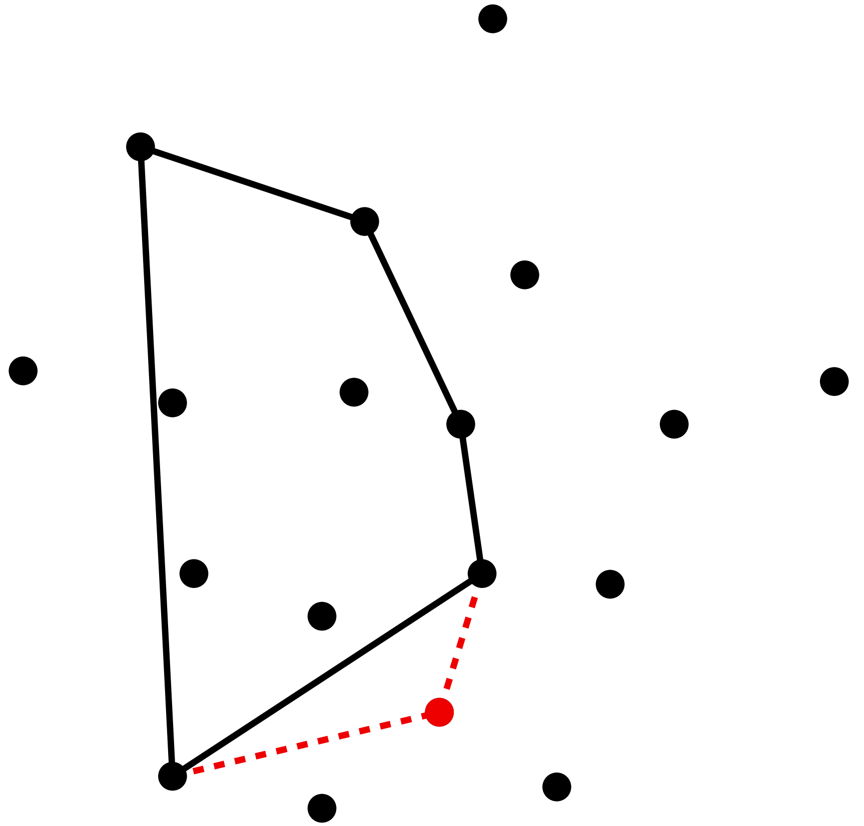

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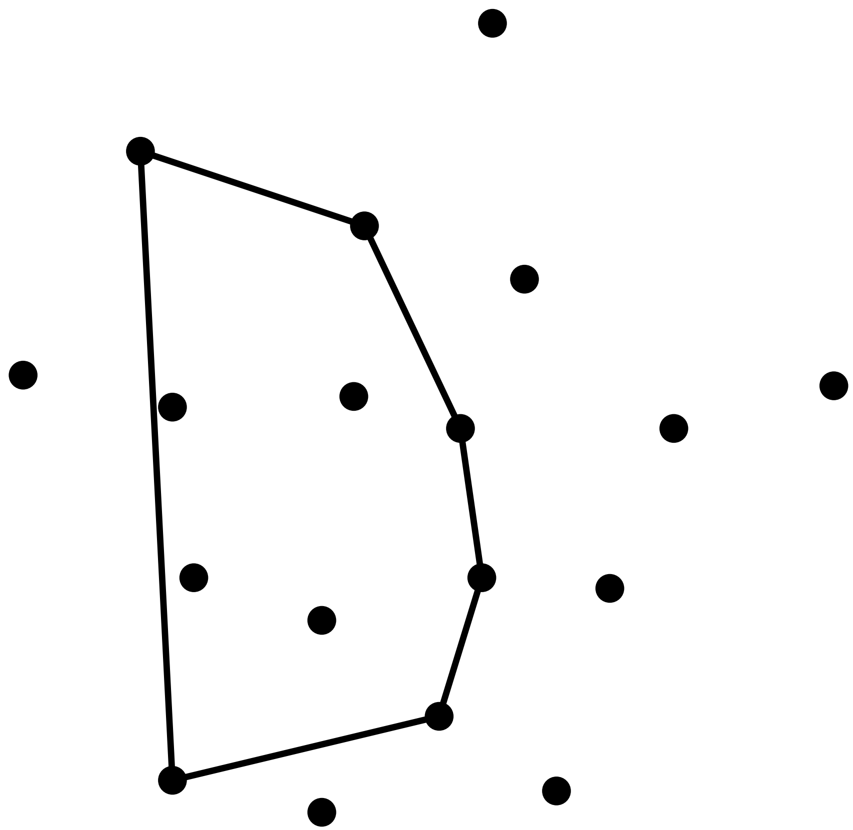

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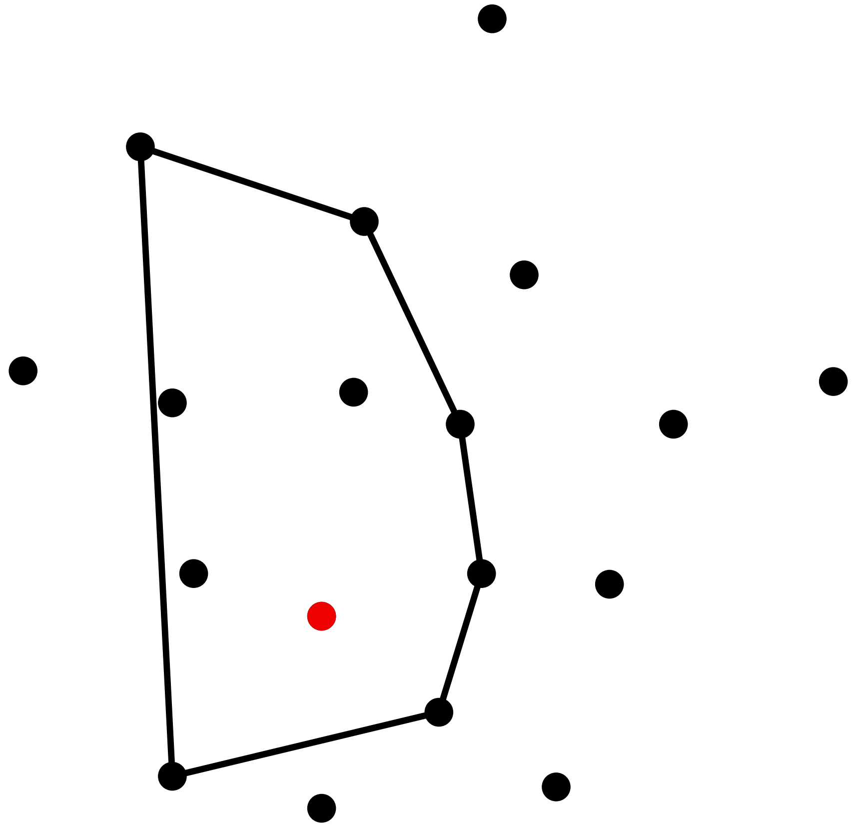

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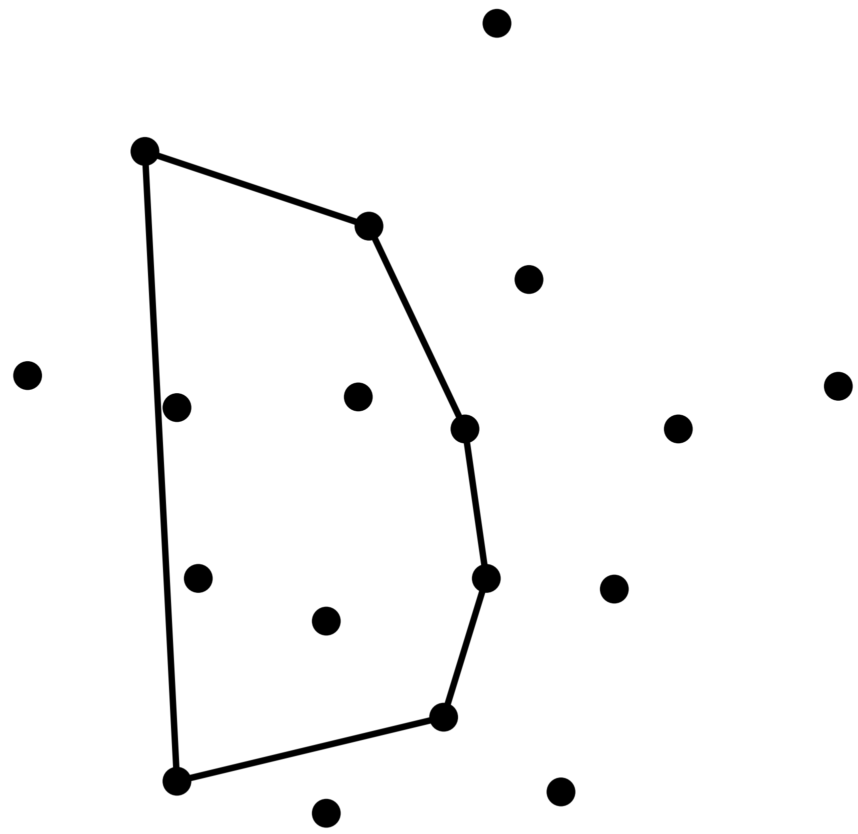

CONVEX HULL

Incremental algorithm

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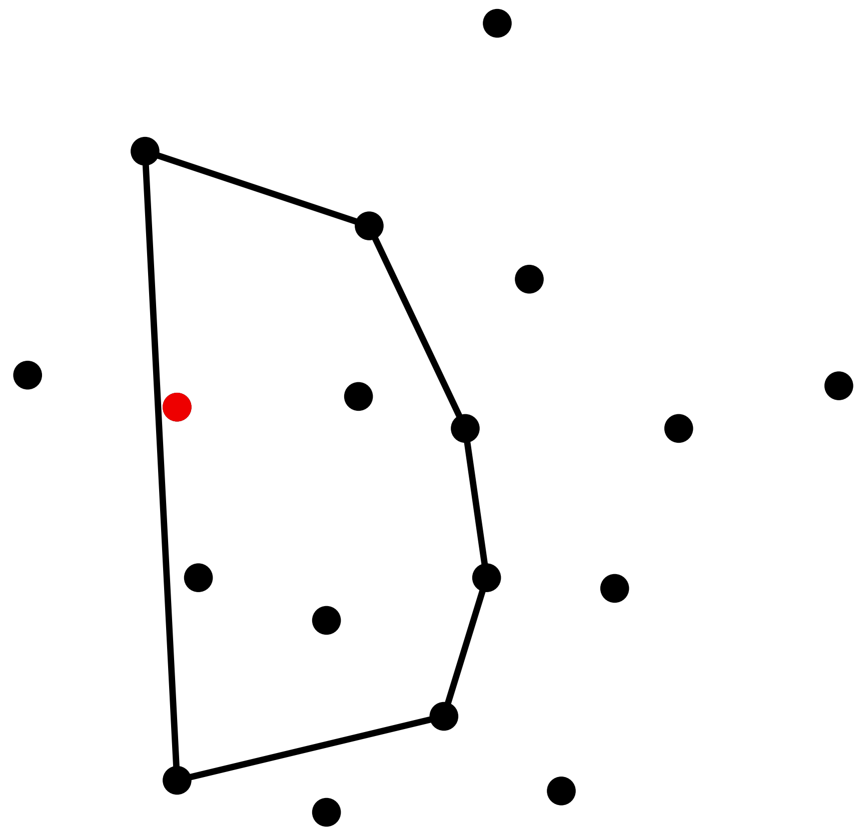

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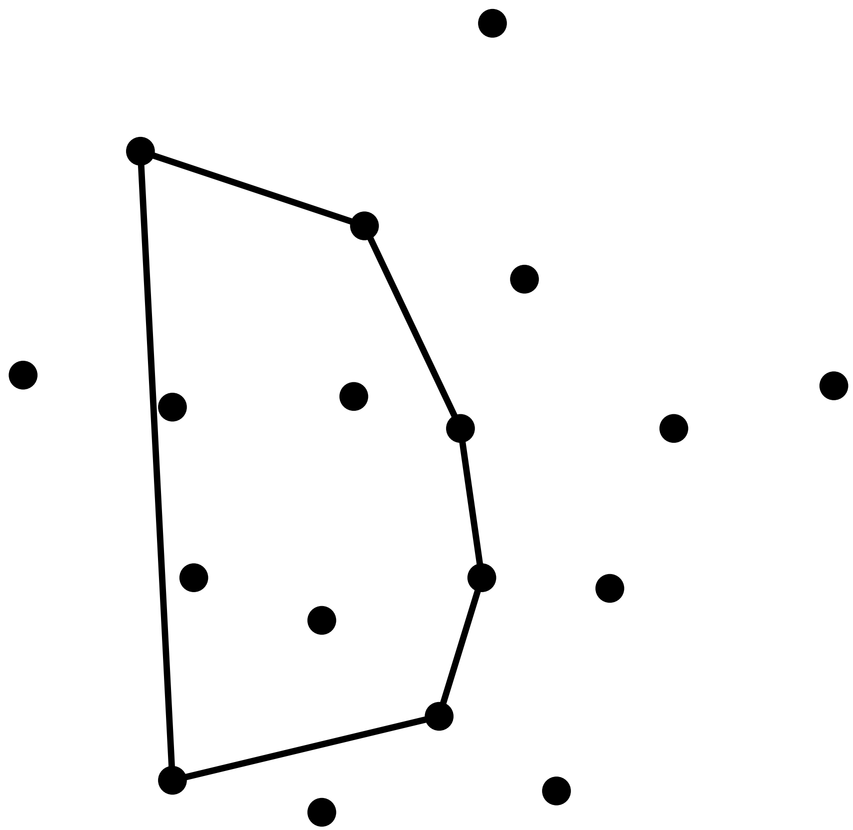

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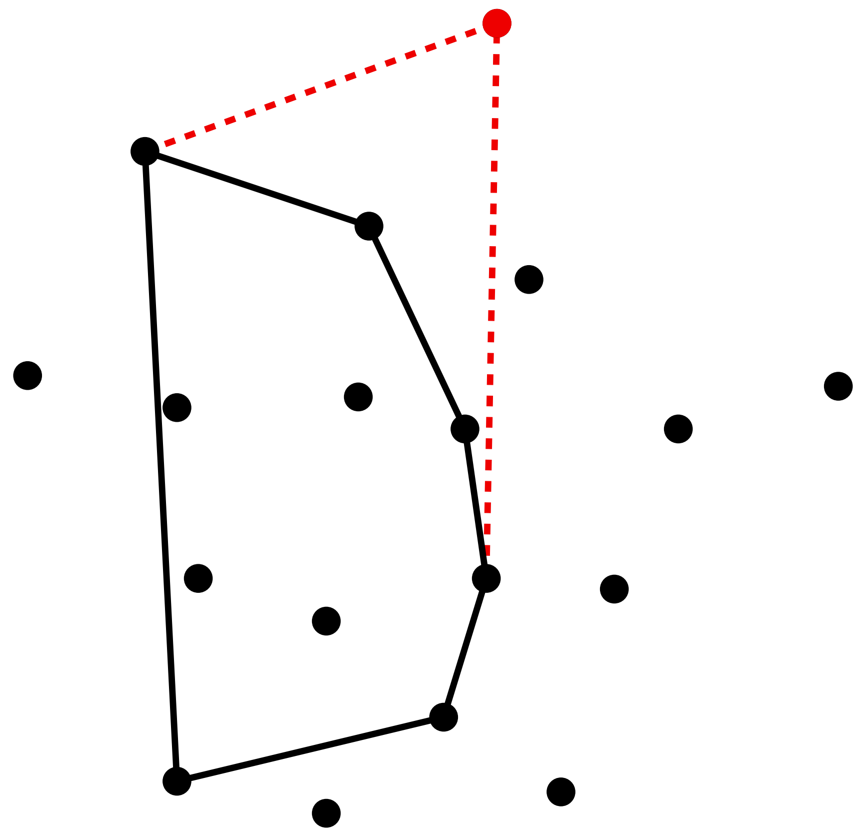

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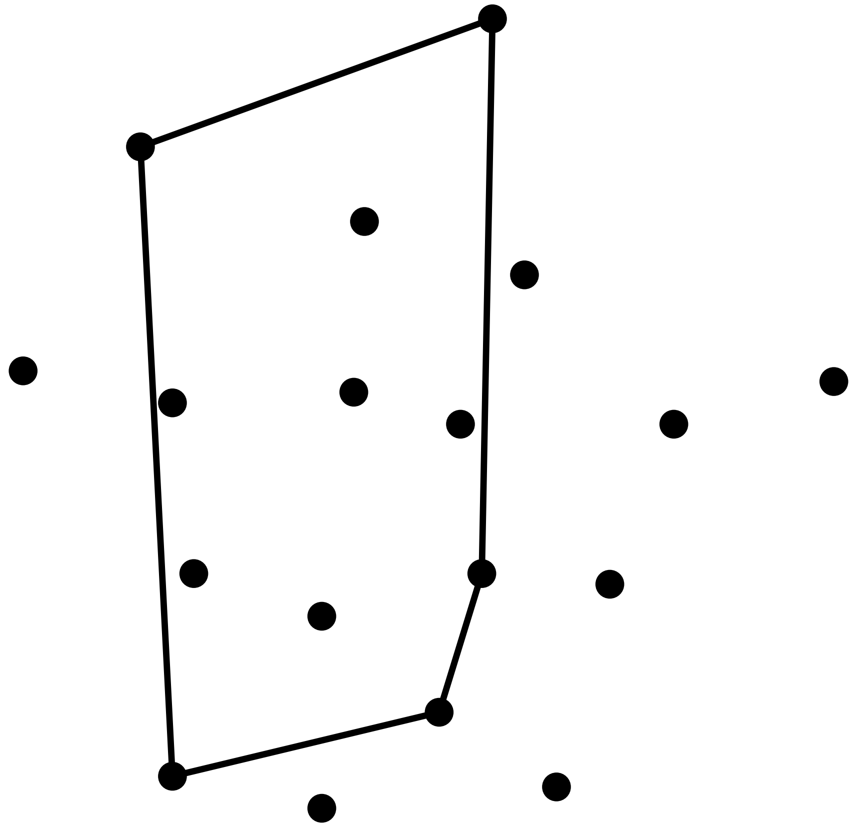

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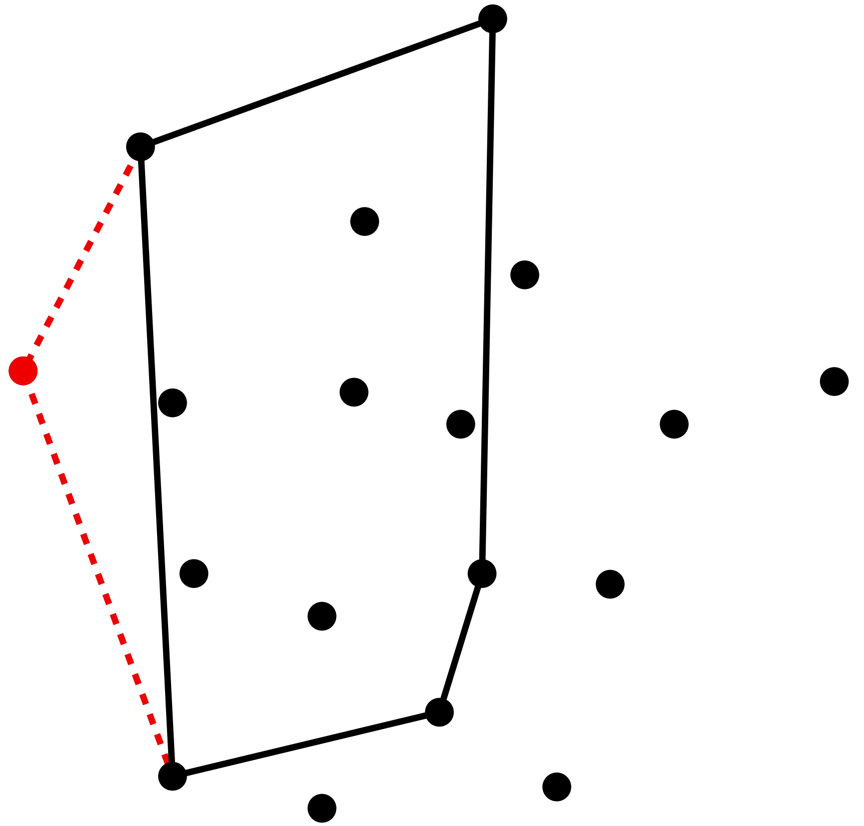

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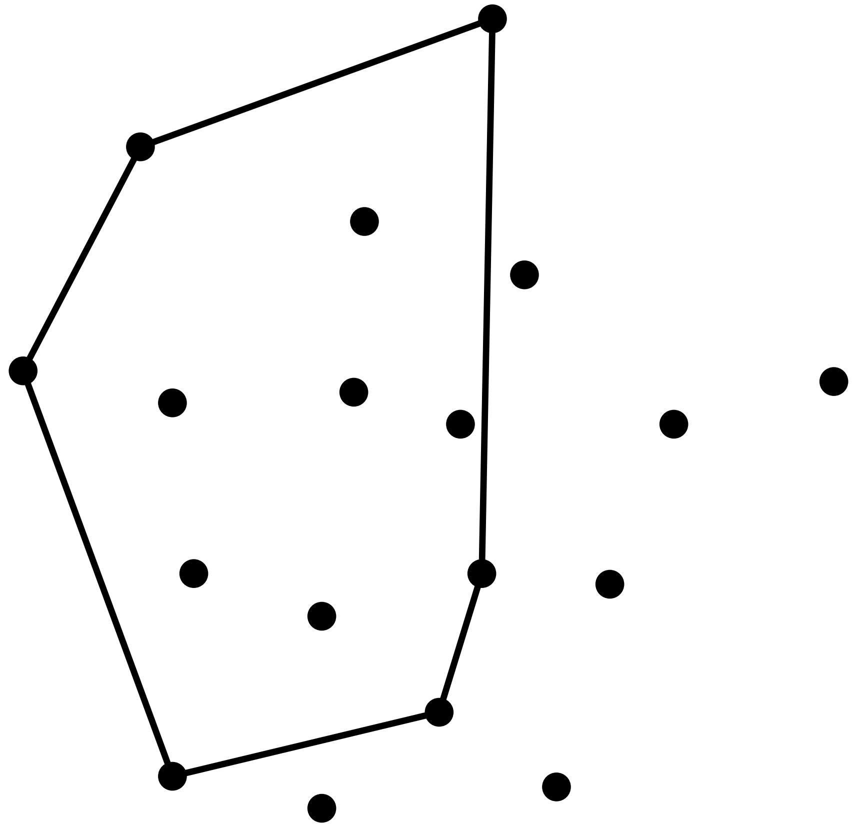

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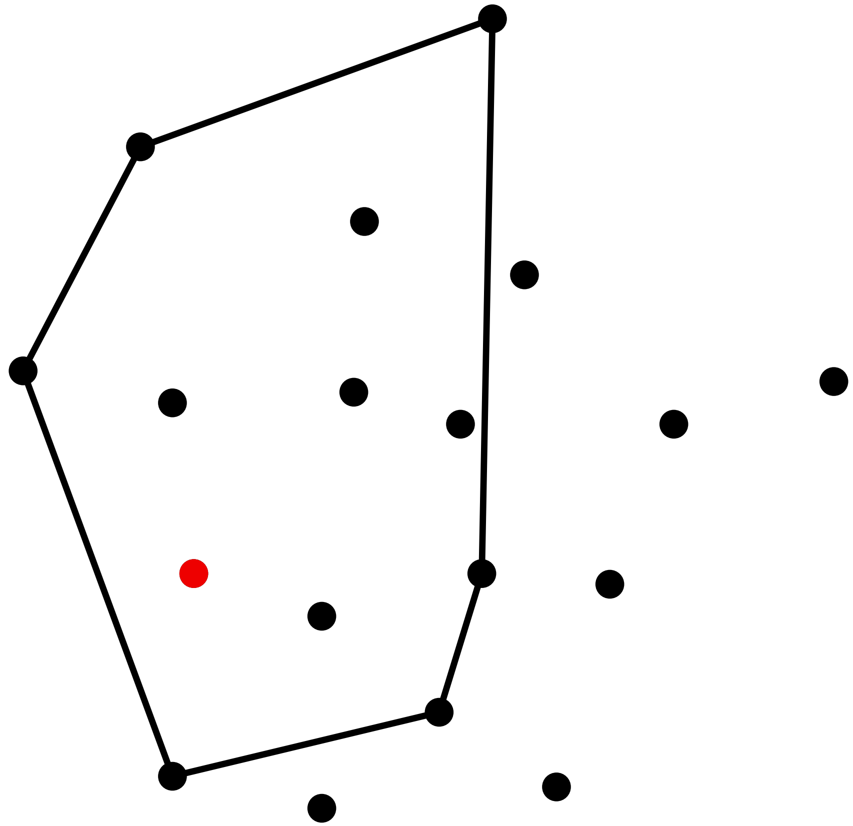

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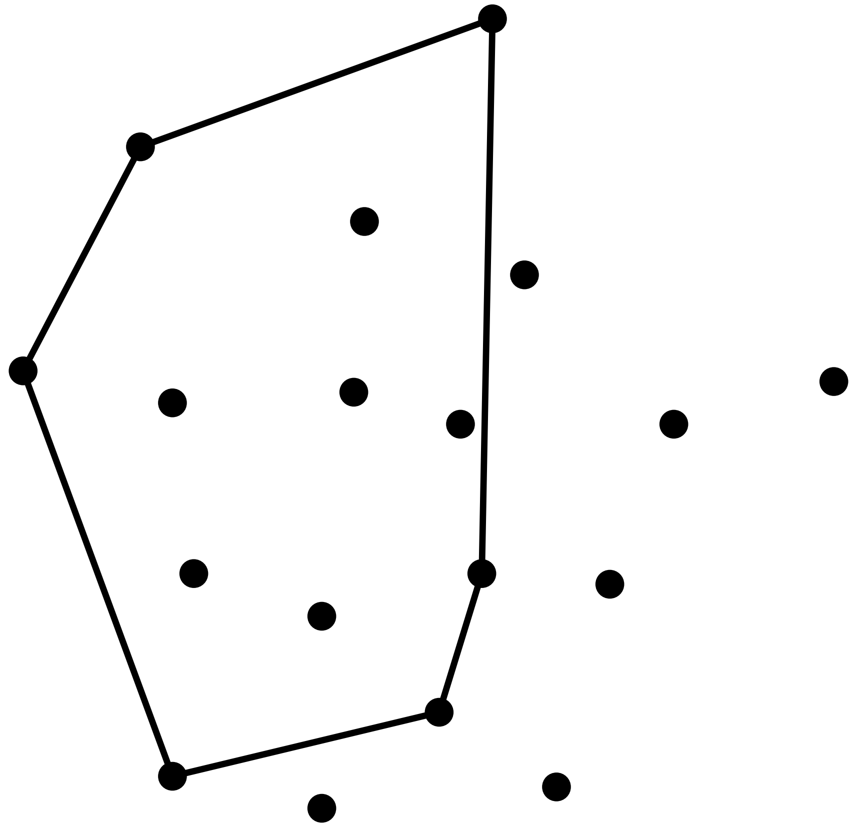

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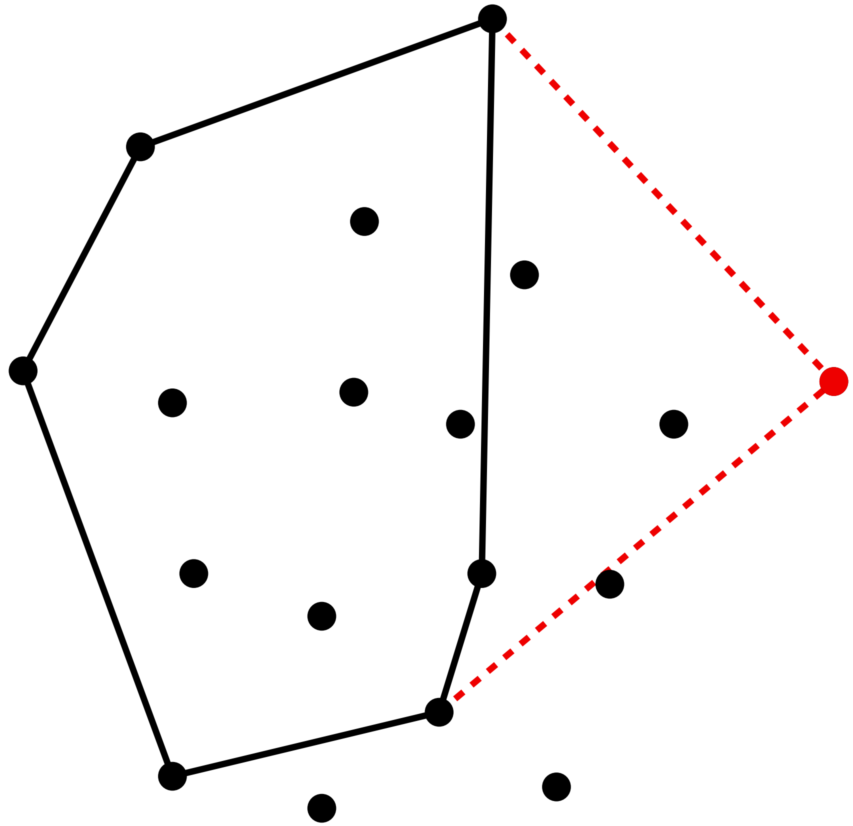

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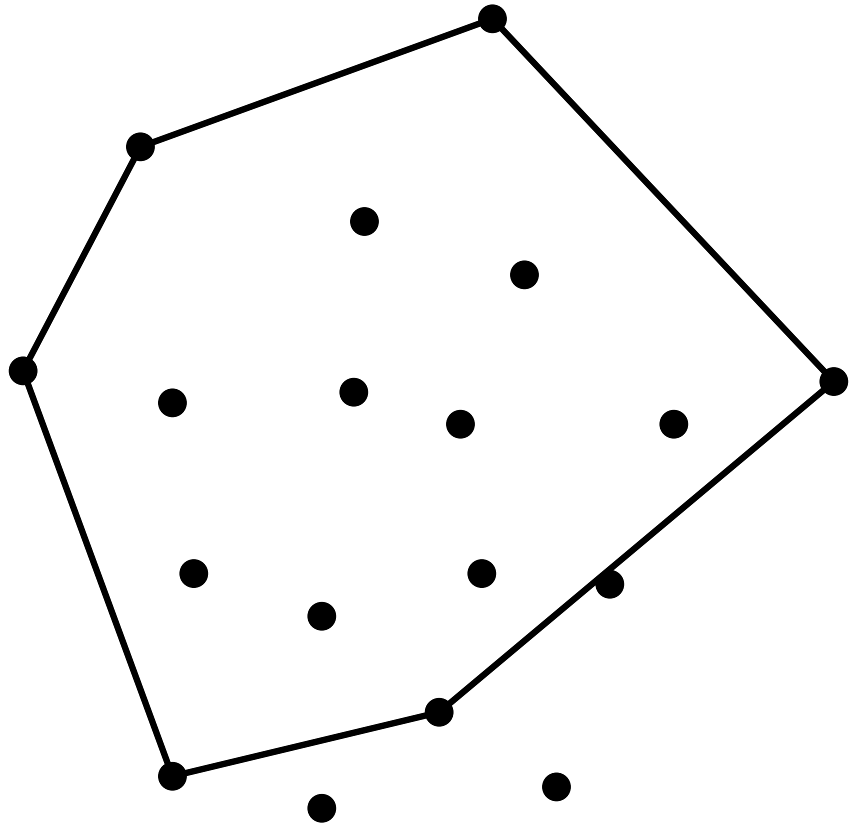

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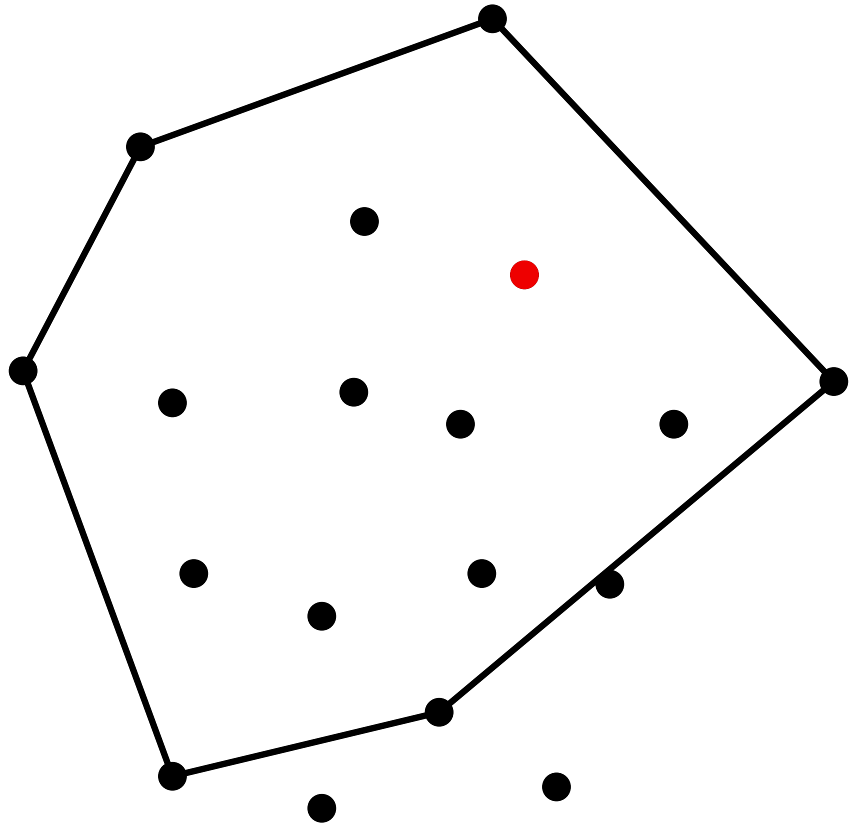

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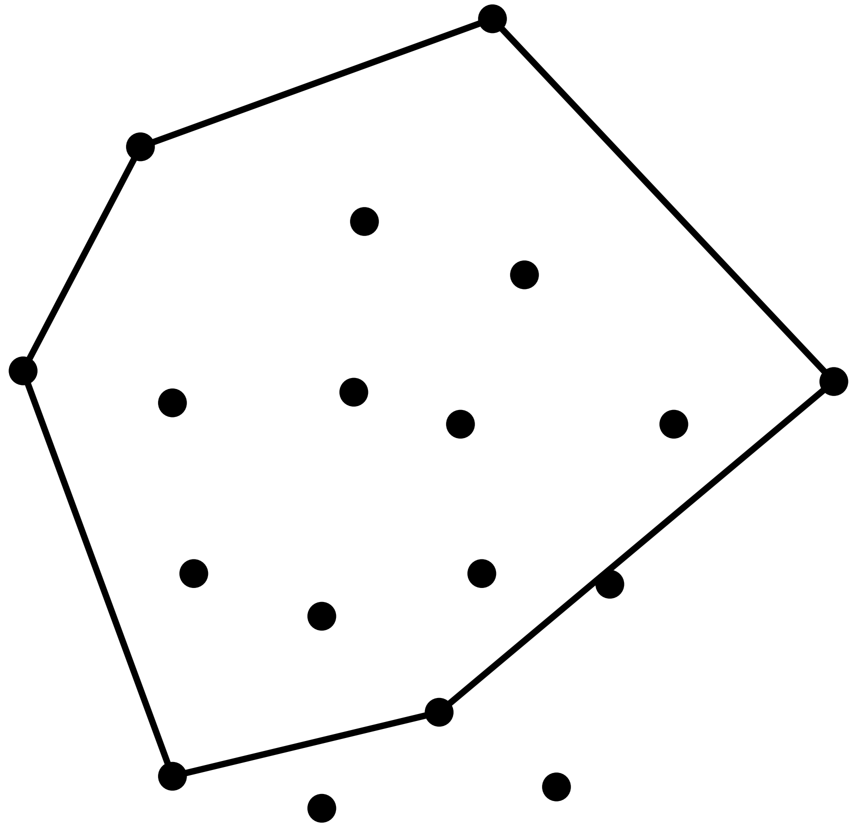

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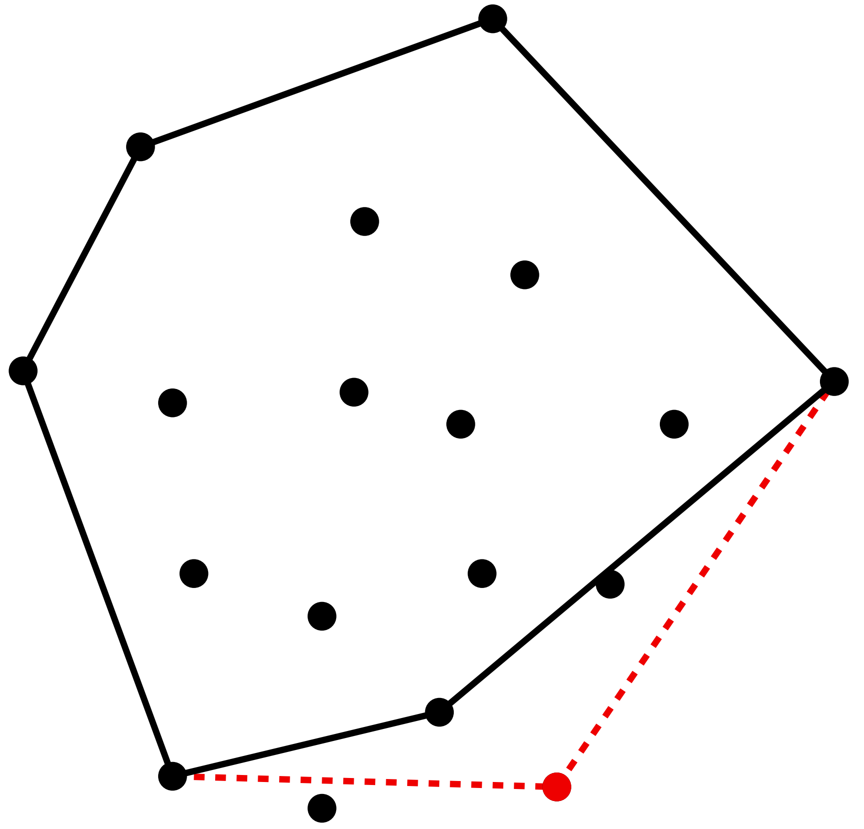

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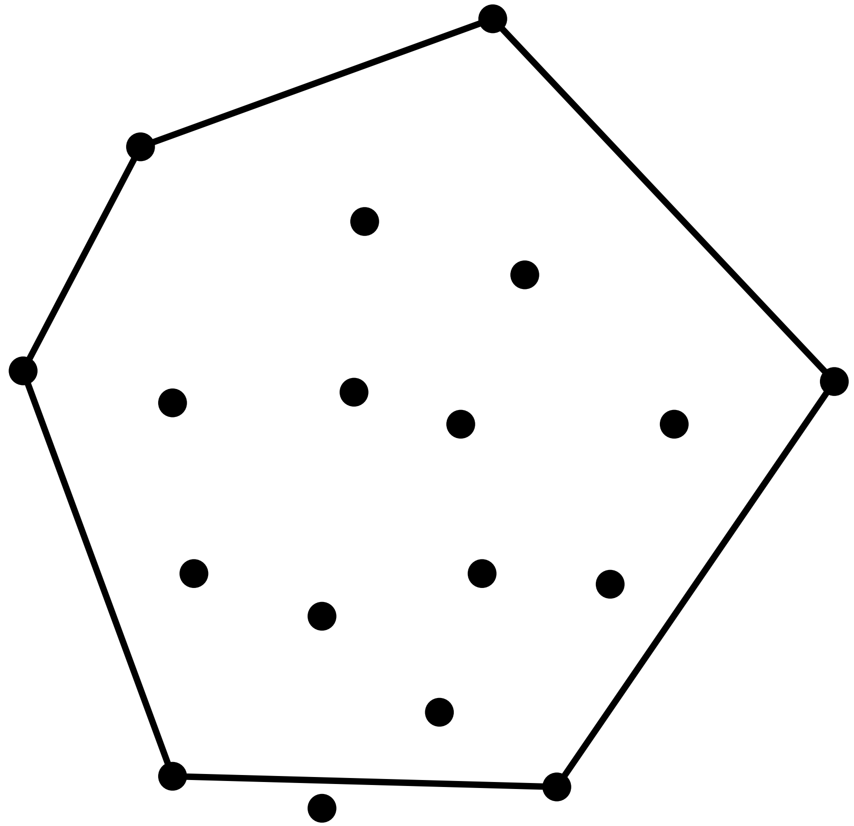

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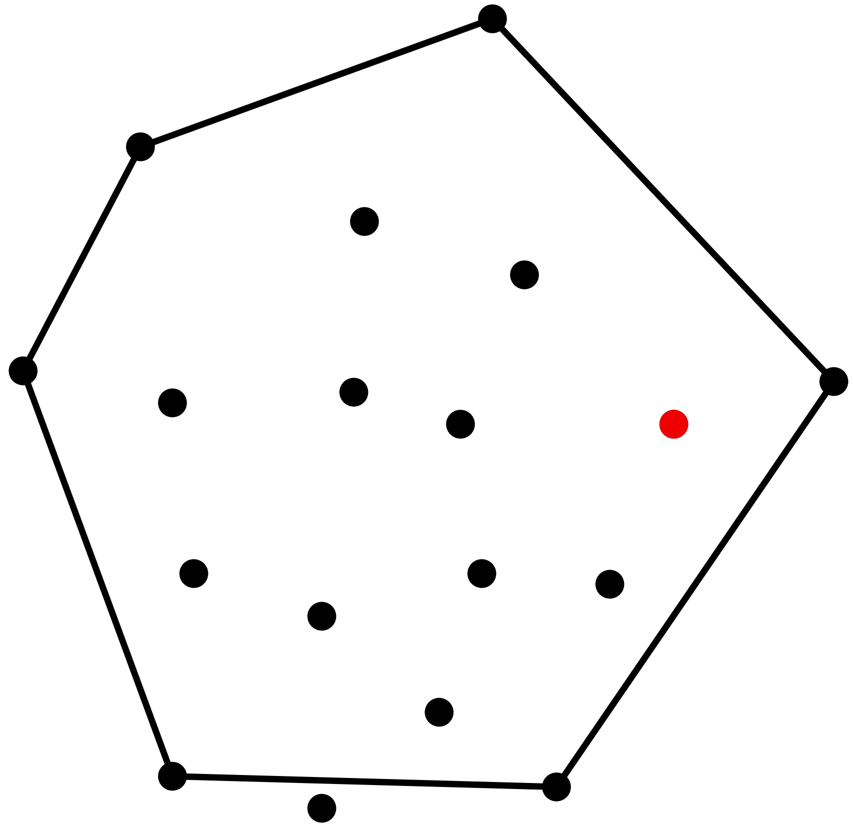

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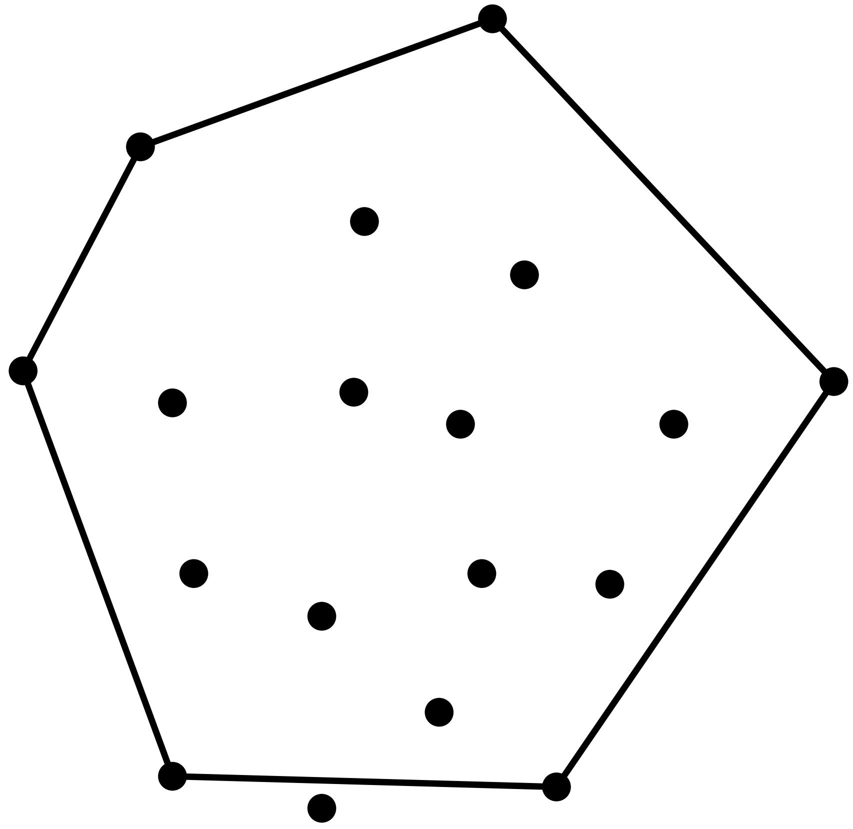

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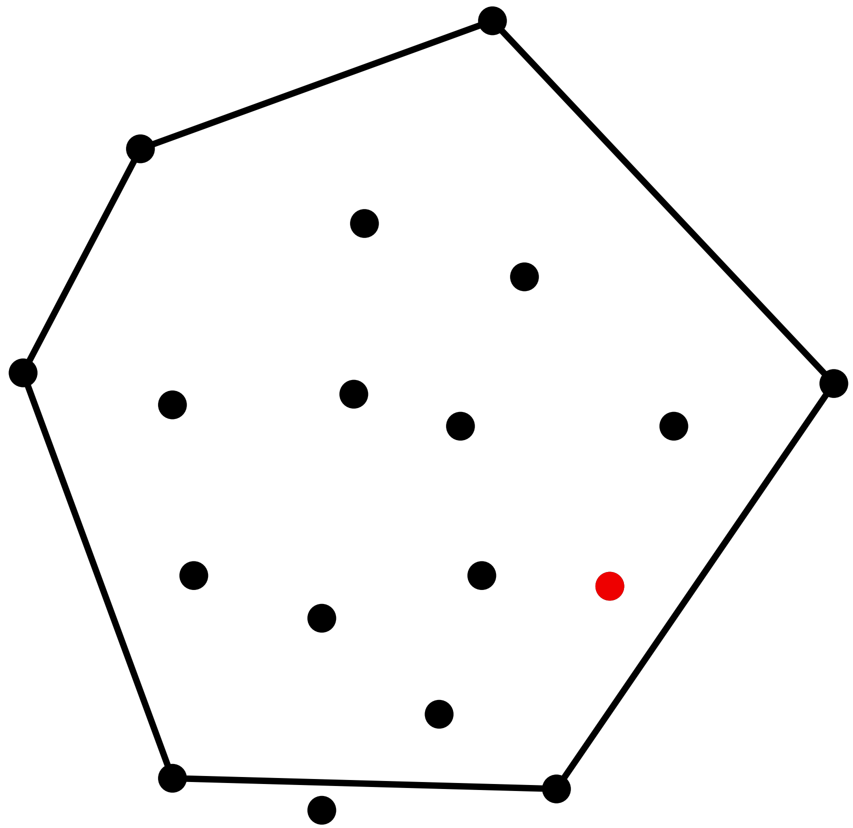

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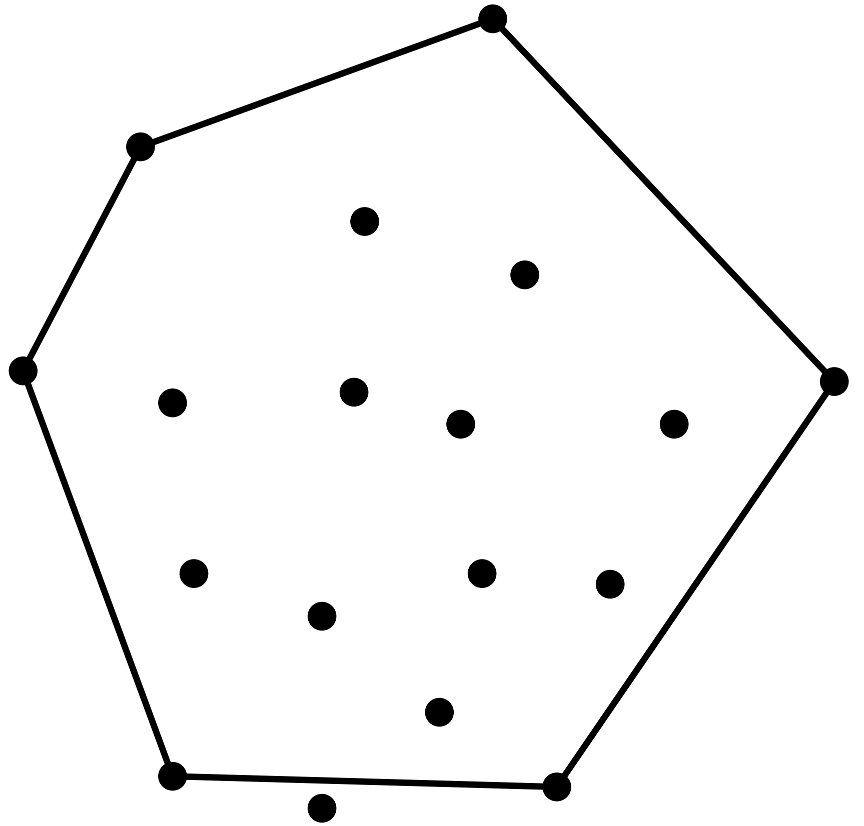

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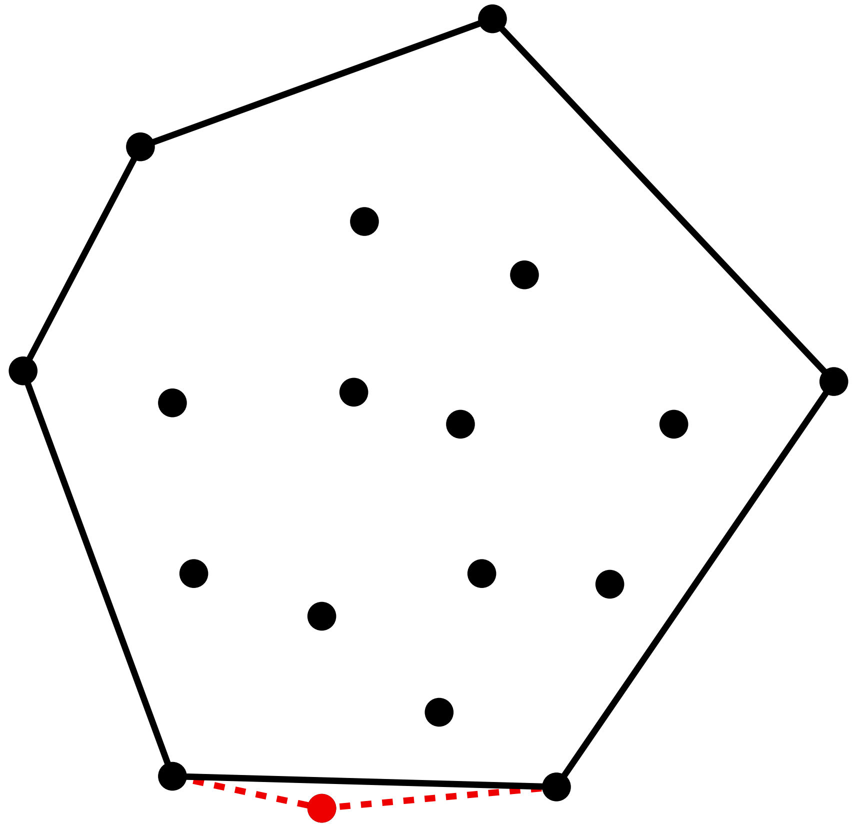

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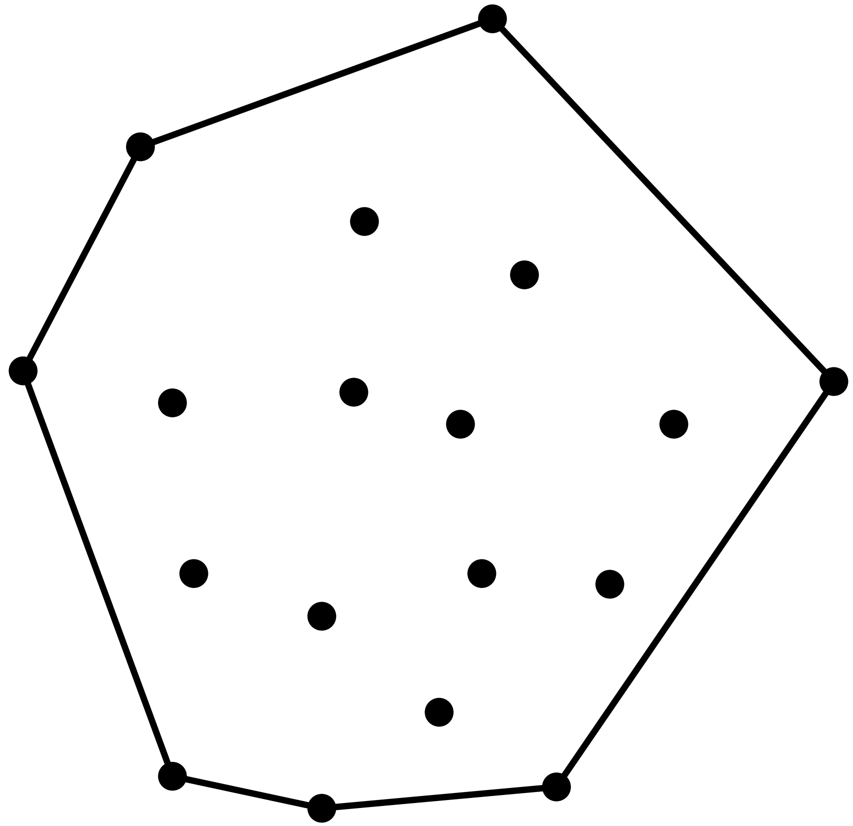

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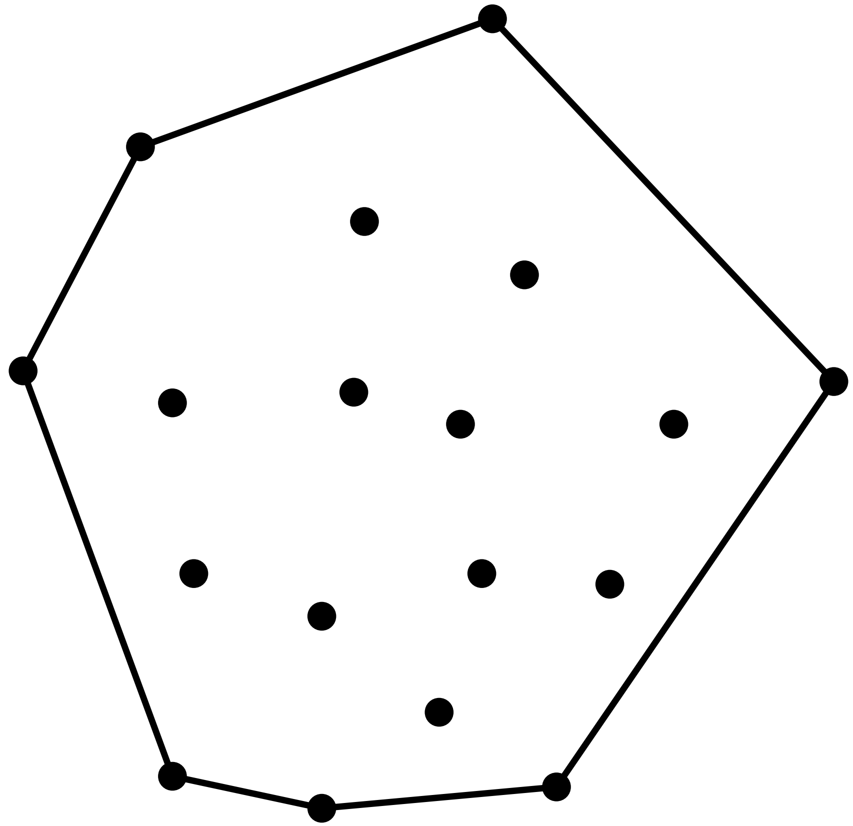

Running time: $O(n \log n)$

Incremental algorithm

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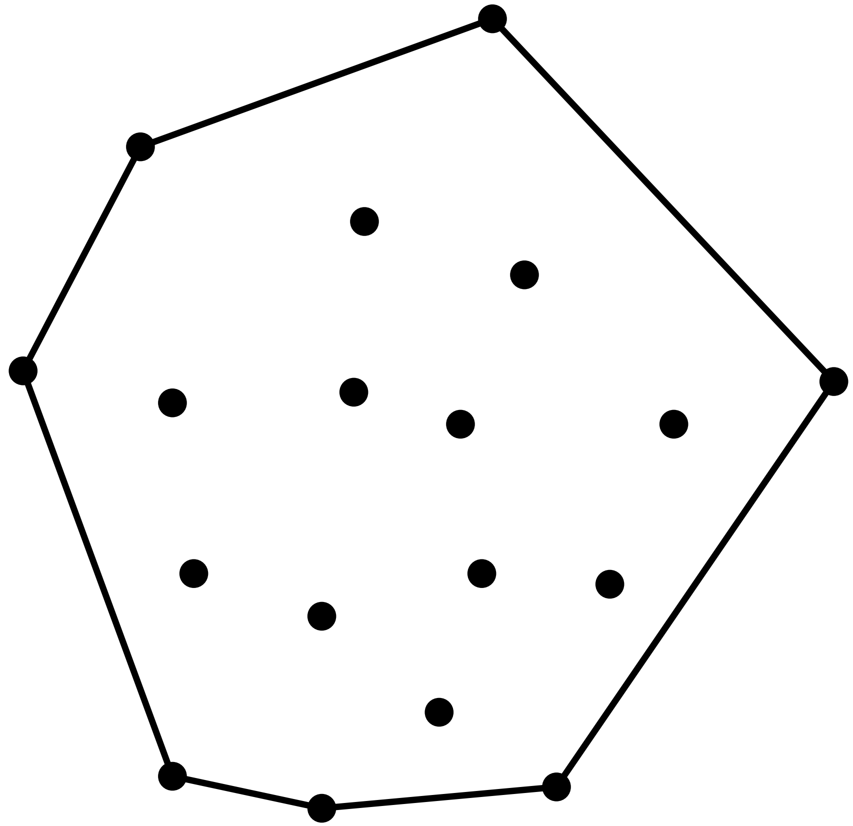
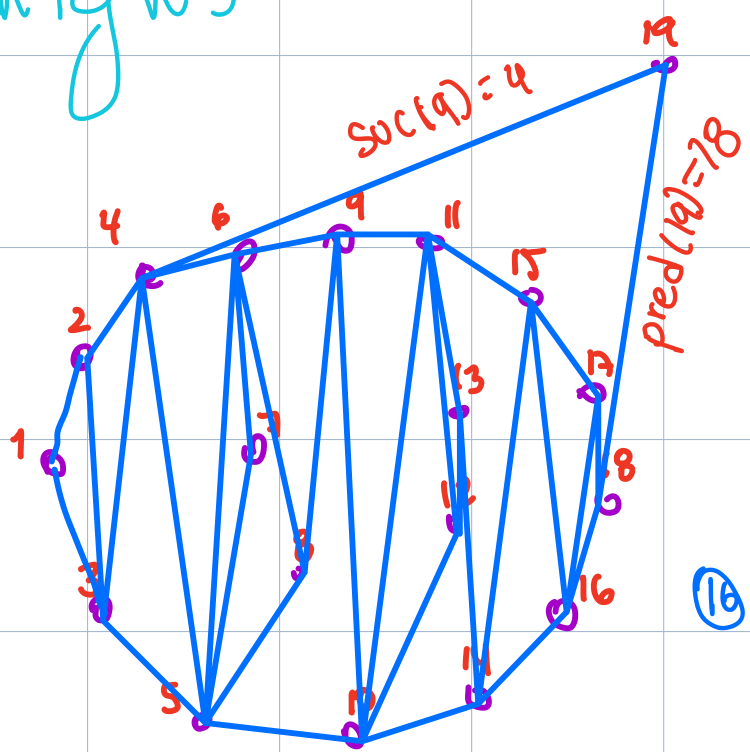


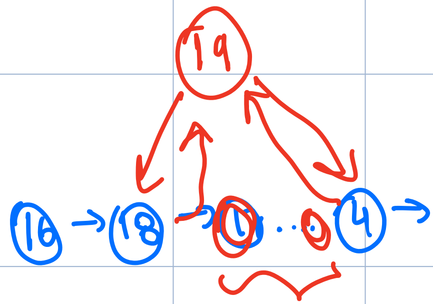
Running time: $O(n \log n)$

By storing l in a structure allowing binary search and updatings (insertions and deletions) in $O(\log n)$ time.

Algoritmo incremental modificado.

$O(n \lg n)$





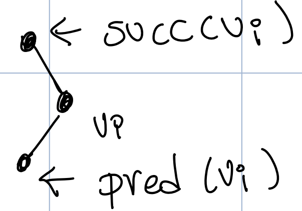
Condiciones:

① $p_{i-1} \in ch(S)$

② Segmento $p_{i-1} p_i$ no intersecta el interior de ch(S).

③ p_i está en el exterior de ch(S).

④ Los vértices de ch(S) están dados en sentido anti-horario.



$p_1, p_2, p_3, \dots, p_n \leftarrow S.$

$p_1', p_2', \dots, p_n'$ $\quad p_n' = p_{i-1}$

= Timothy Chan =

Algoritmo

1. Encontrar el vértice v_t, como sigue:

    $v = p_{i-1}$
    while p_i a la derecha de la recta $L(v, suc(v))$
        $v = suc(v)$

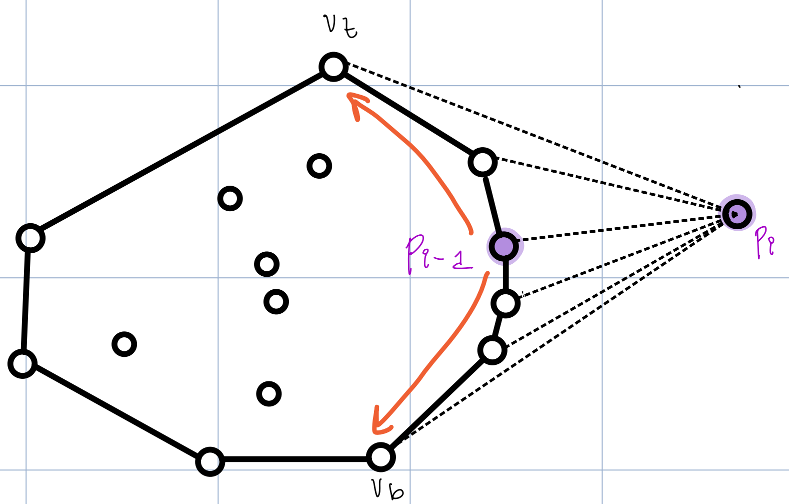
    $v_t = v$

2. Encontrar el vértice v_b, como sigue:

    $v = p_{i-1}$
    while p_i a la izquierda de la recta $L(v, pred(v))$
        $v = pred(v)$

    $v_b = v$

3. Eliminar la cadena $v_b, suc(v_b), \ldots, v_t$ en la estructura de datos.

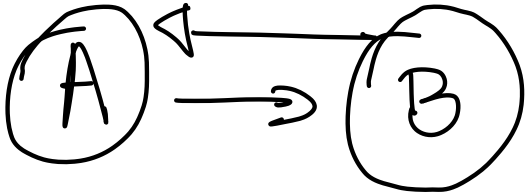


Godfried Toussaint, 1986.

Científico de la Comp. Canadiense.

Divide-and-conquer algorithm

Reducción.



Técnica: Recursión.

- Reducimos (A) a una instancia más pequeña de sí mismo.
- Construir la solución de (A) a partir de las instancias más pequeñas debe ser trivial.

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Divide-and-conquer algorithm

Initialization

1. Sort the points by abscissae

Division

1. Divide the points (x_i, y_i) into two subsets, wrt the median value of the abscissae

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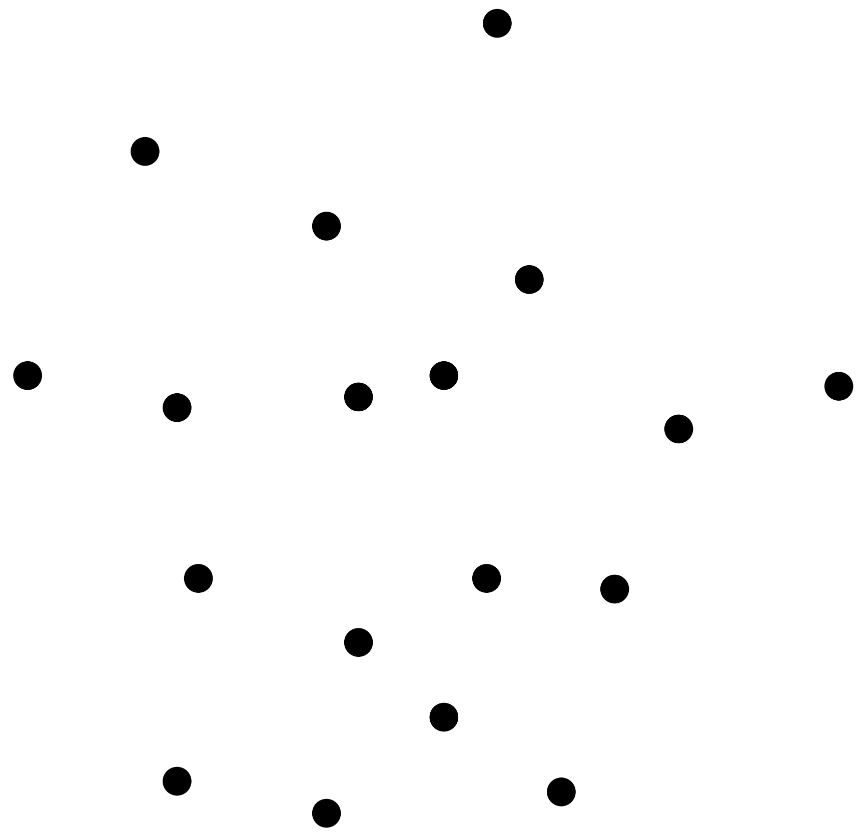

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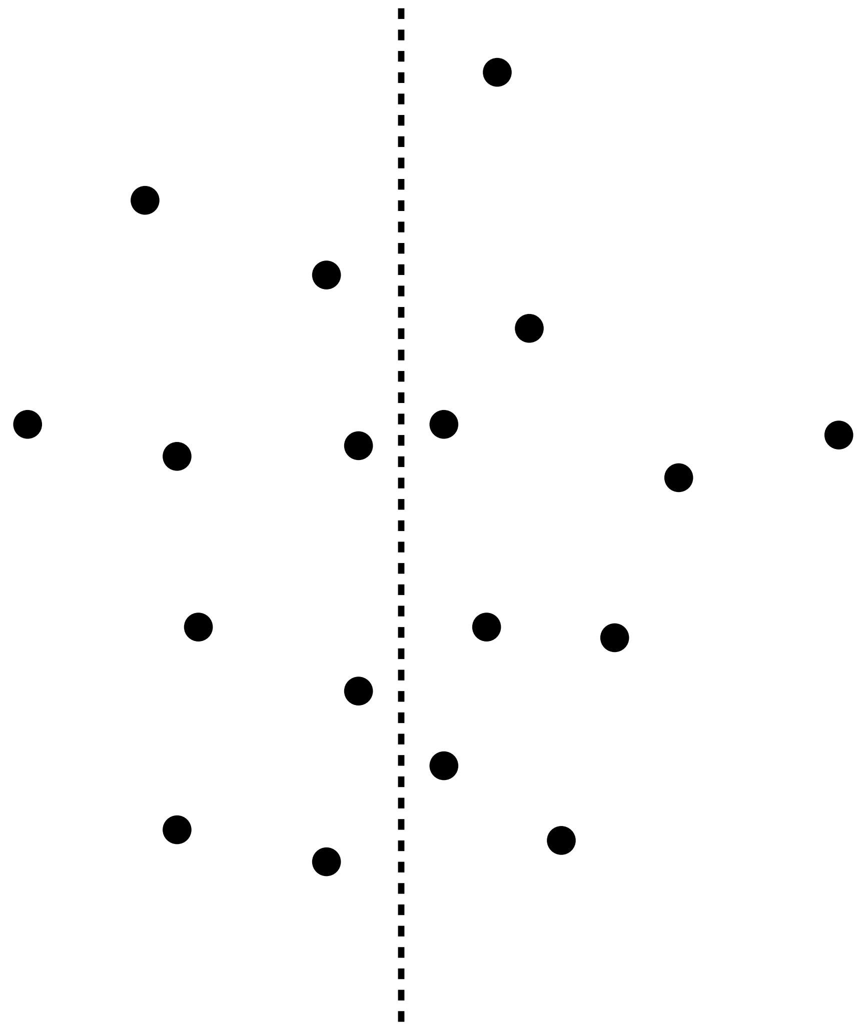

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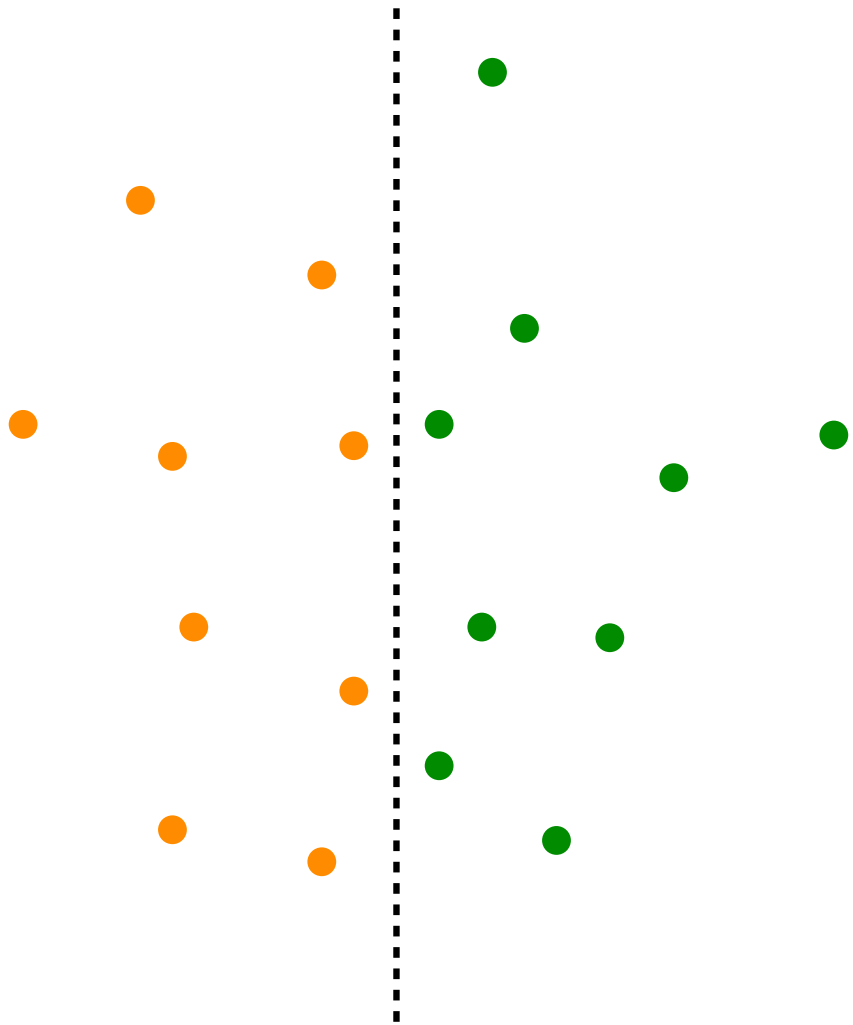

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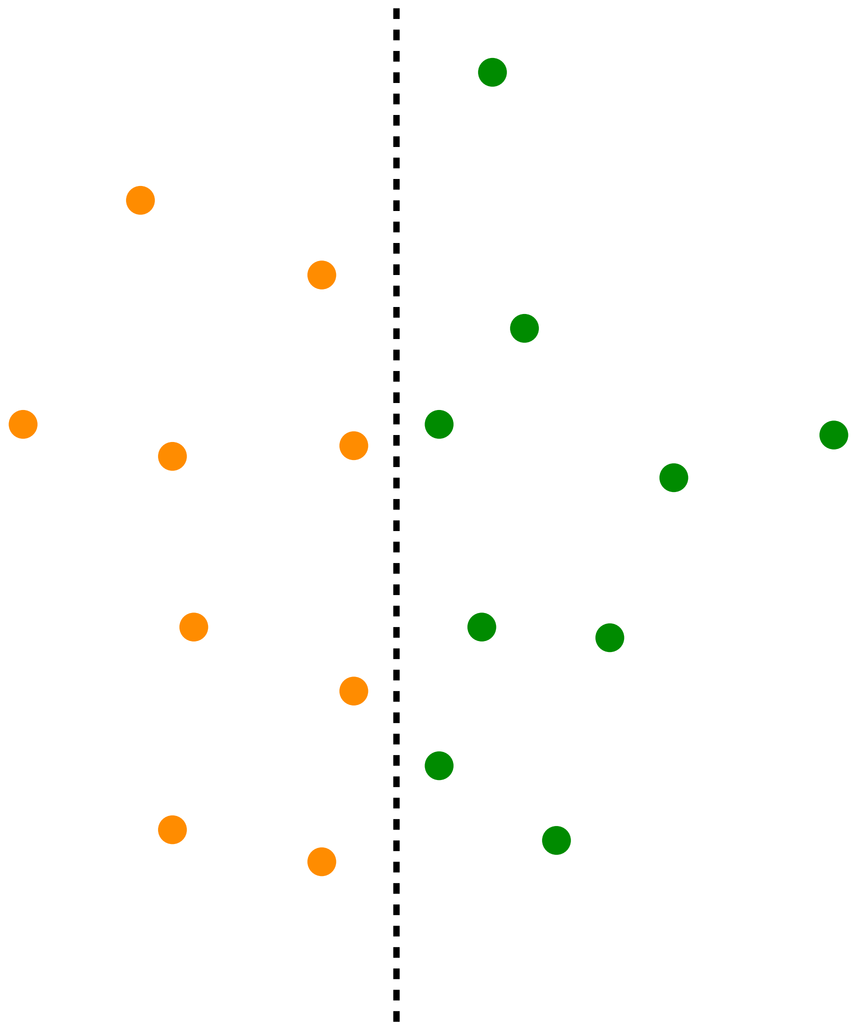
Recursion

1. Recursively compute the convex hull of the two subsets



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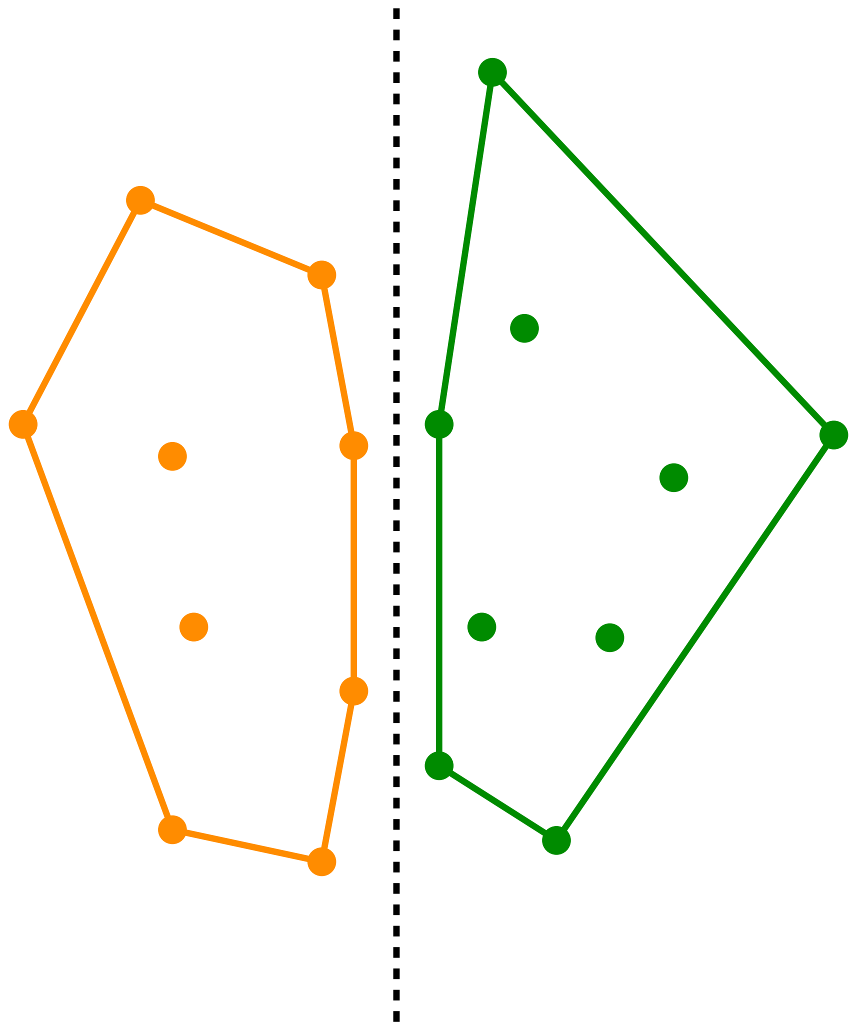

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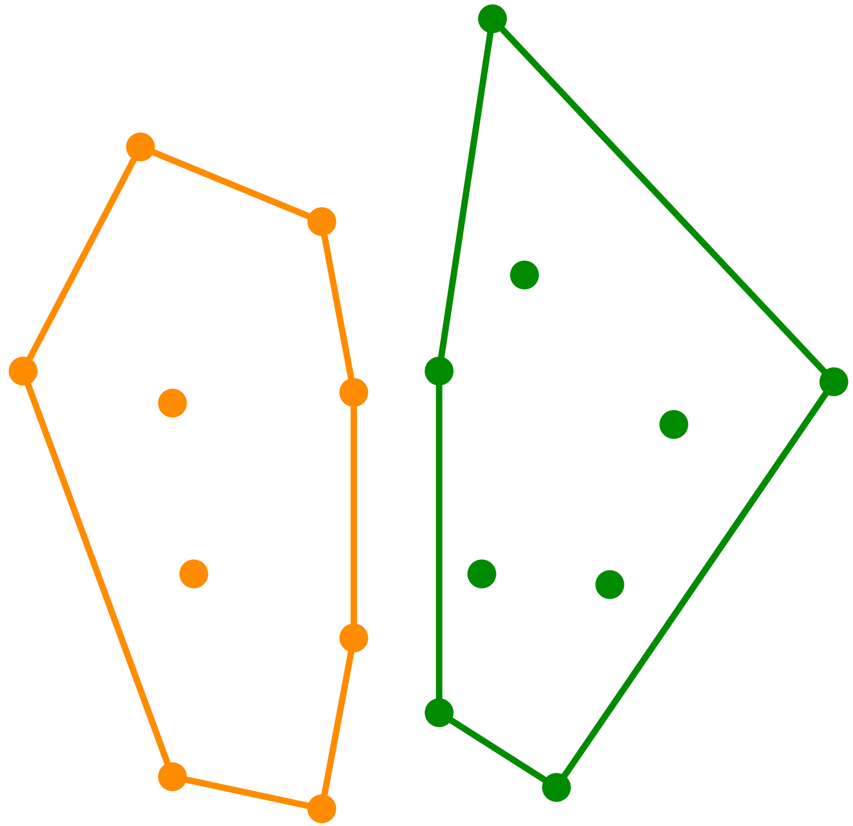

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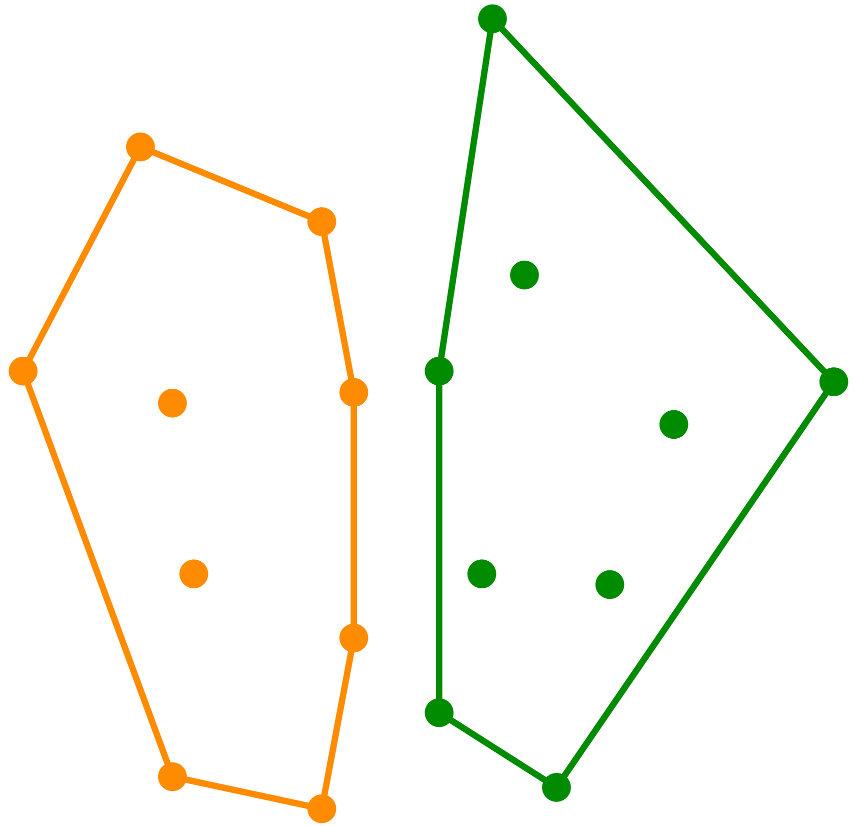
Merge

1. Compute the external common tangents of the two convex polygons

2. Delete the interior chains of the two polygons and join the external chains through the supporting segments



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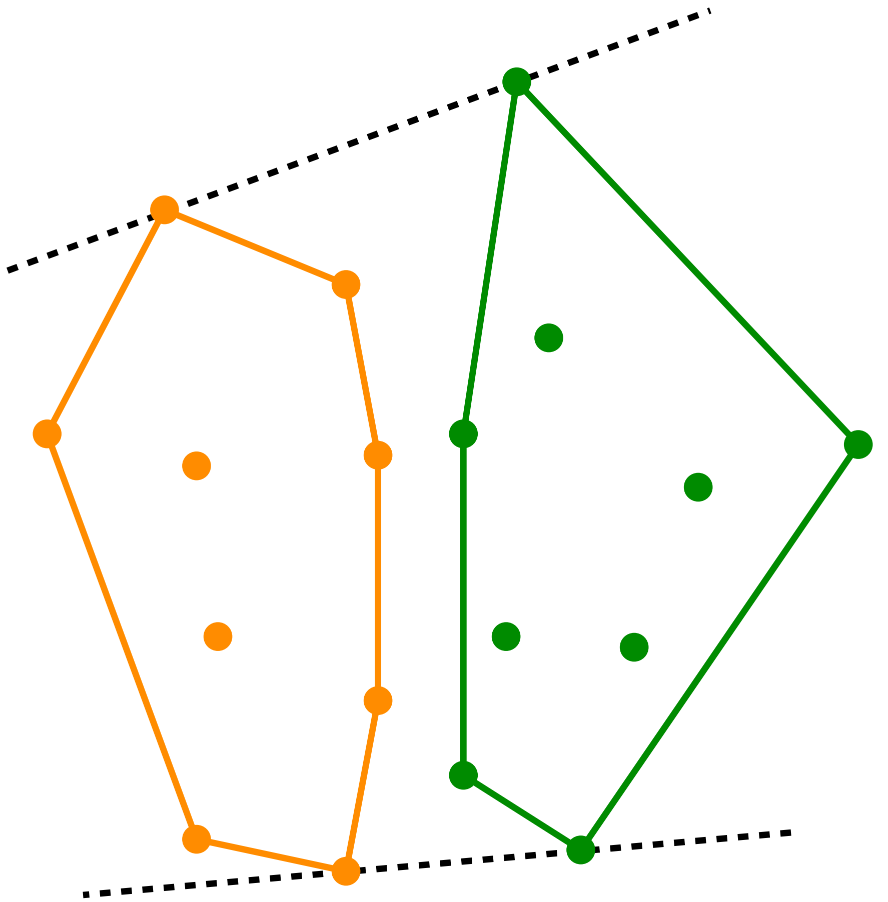

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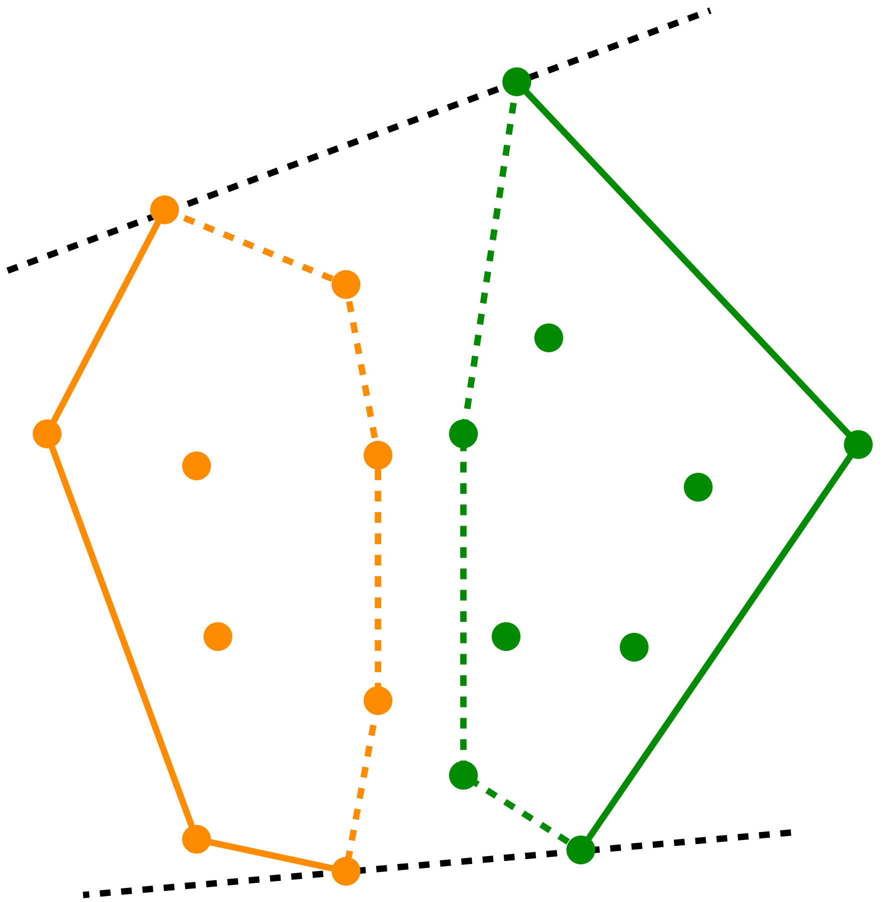

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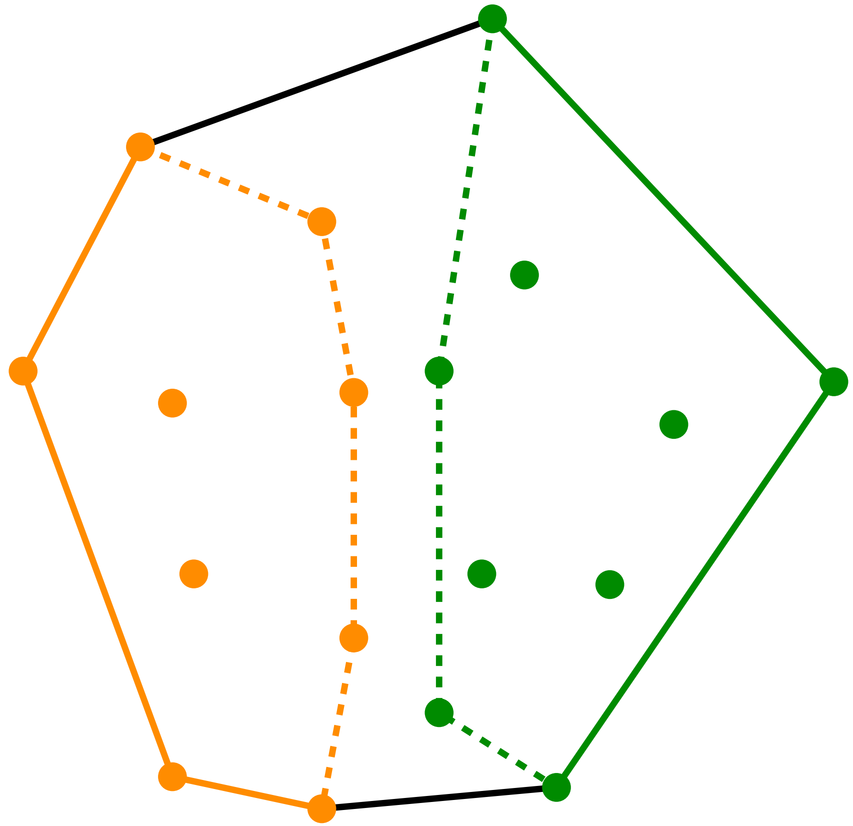

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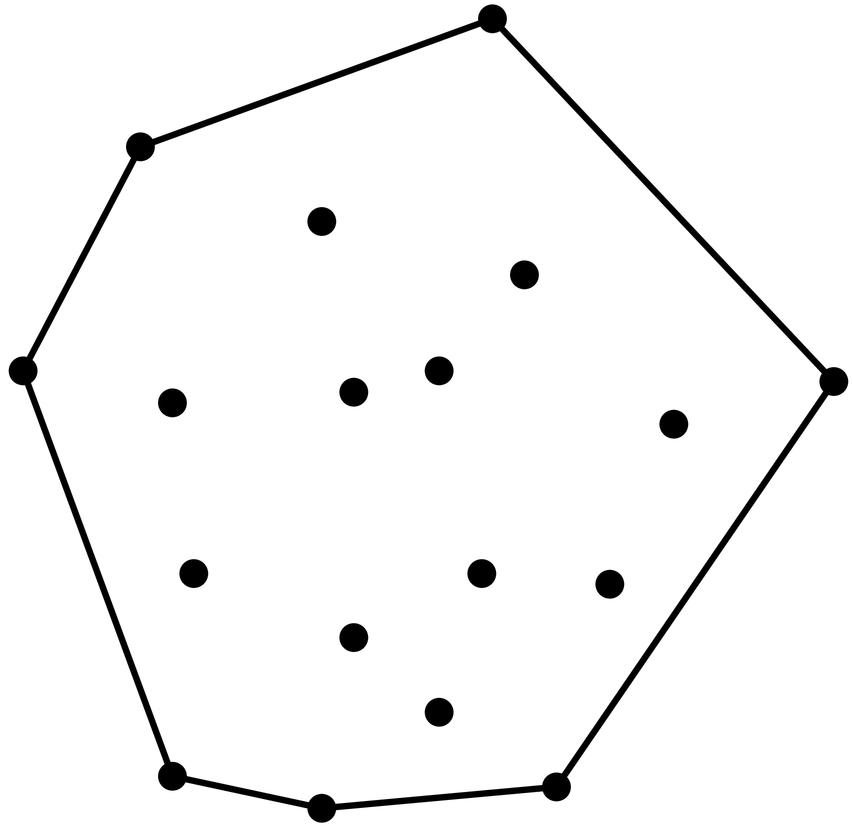
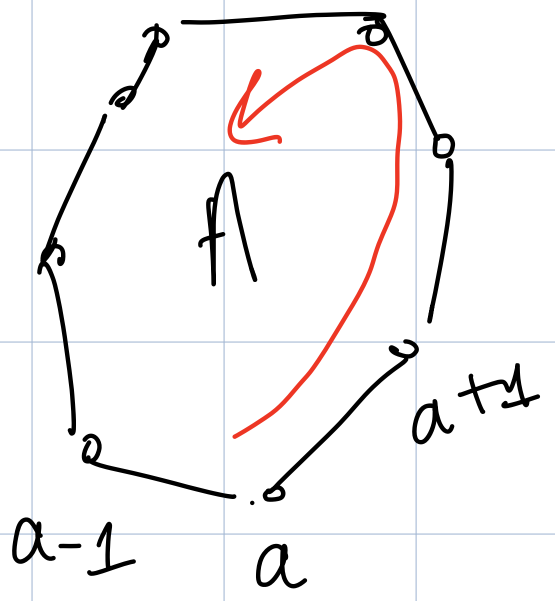
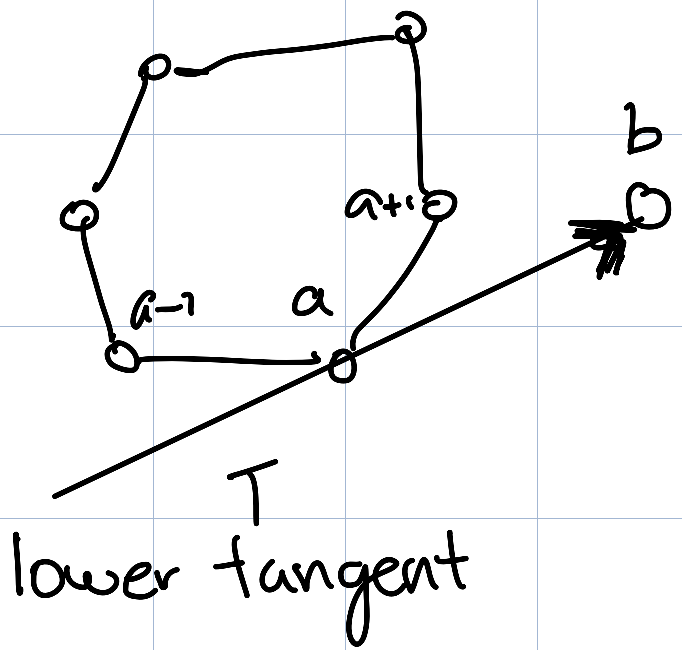
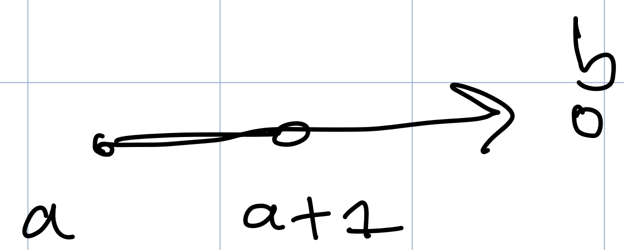


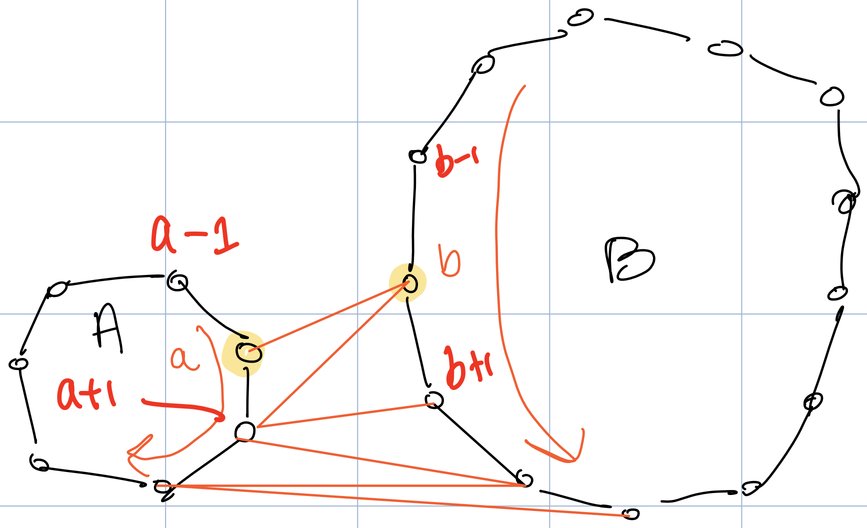
¿Cómo encontramos las tangentes?

Definición: Una recta T=ab es una tangente inferior en a si ambos a-1 y a+1 están a la izquierda (o sobre) T.









① Es correcto?

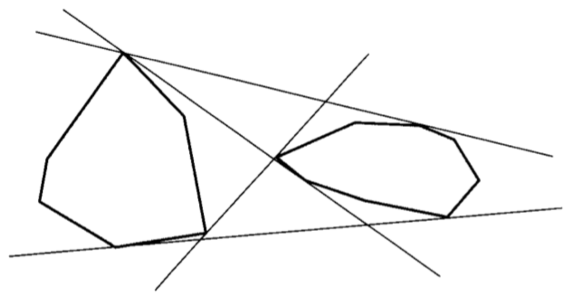
② Cuál su complejidad?

Algoritmo: Tangente Inferior

a ← rightmost point A
b ← left most point B
while T=ab not lower tangent of A & B.
  while T=ab not lower tangent A
    a ← a − 1
  while T=ab not lower tangent B
    b ← b + 1

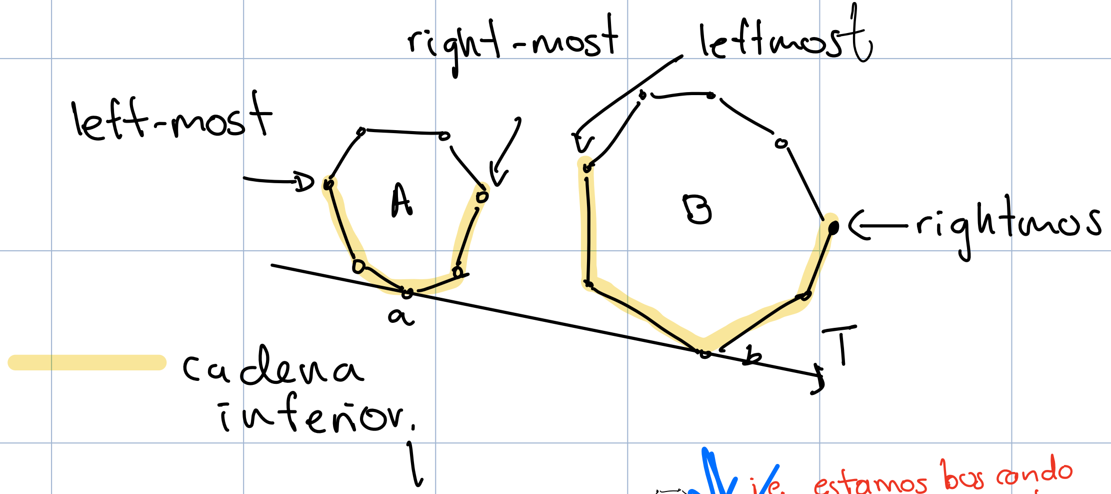
② Pregontas

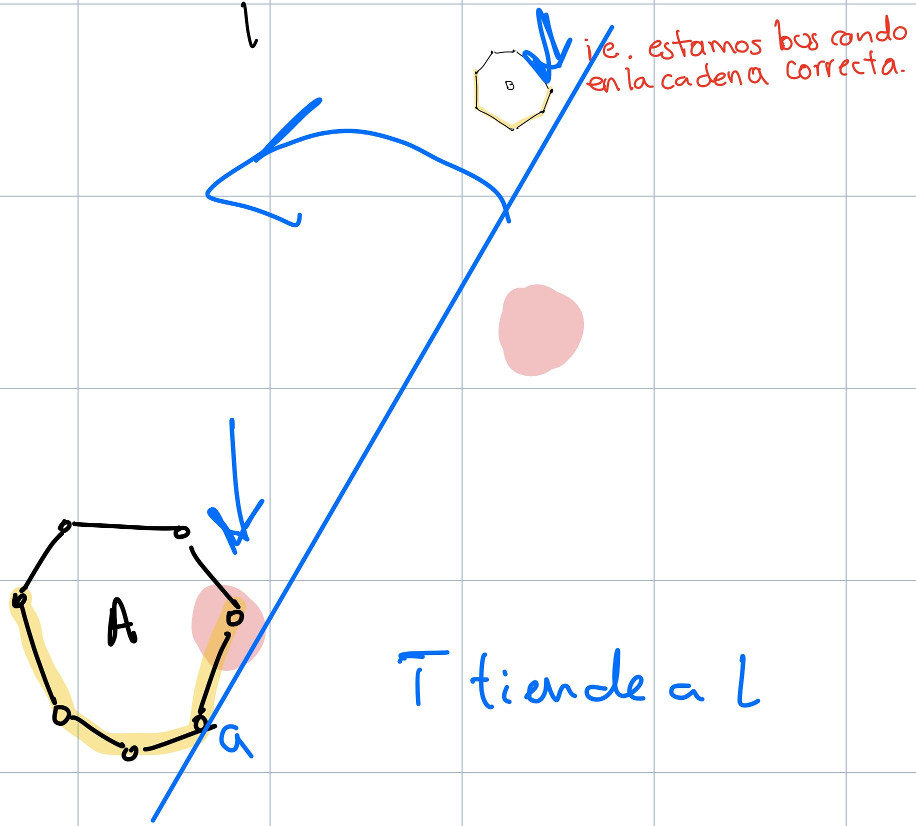
1. ¿Termina el algoritmo?
2. Hay 4 tangentes ¿cómo podemos estar segurxs de que el algoritmo encuentra la tangente inferior?

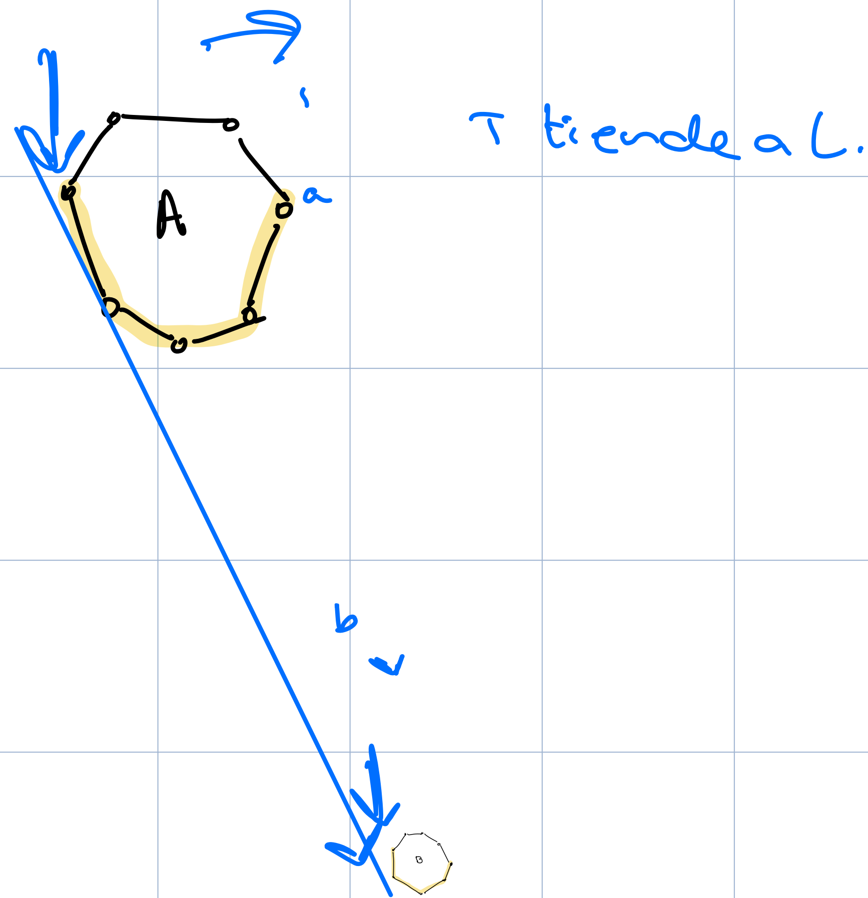


⇓ el algoritmo es correcto.

Lema: La tangente inferior T=ab toca tanto a A como a B en sus cadenas inferiores.



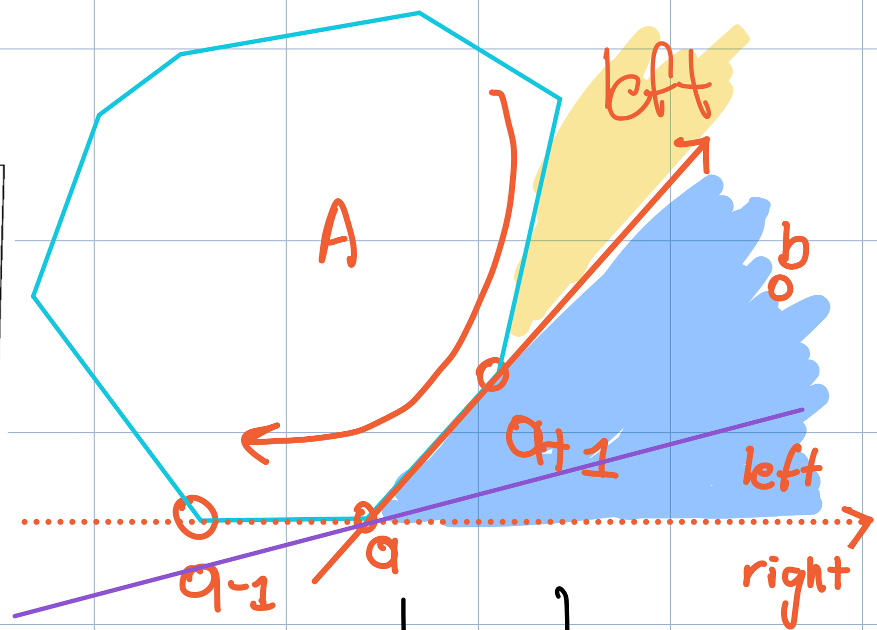




Demostraremos ahora que T nunca toca el interior de A, y por lo tanto el ciclo que itera sobre A termina eventualmente.

Lema: Durante la ejecución del algoritmo, ab nunca intersecta el interior ni de A ni de B.

Algorithm: Lower Tangent
$a \leftarrow$ rightmost point of A.
$b \leftarrow$ leftmost point of B.
while $T = ab$ not lower tangent to both A and B do
  while T not lower tangent to A do
    $a \leftarrow a - 1$
  while T not lower tangent to B do
    $b \leftarrow b + 1$



Dem:
Por inducción sobre a.

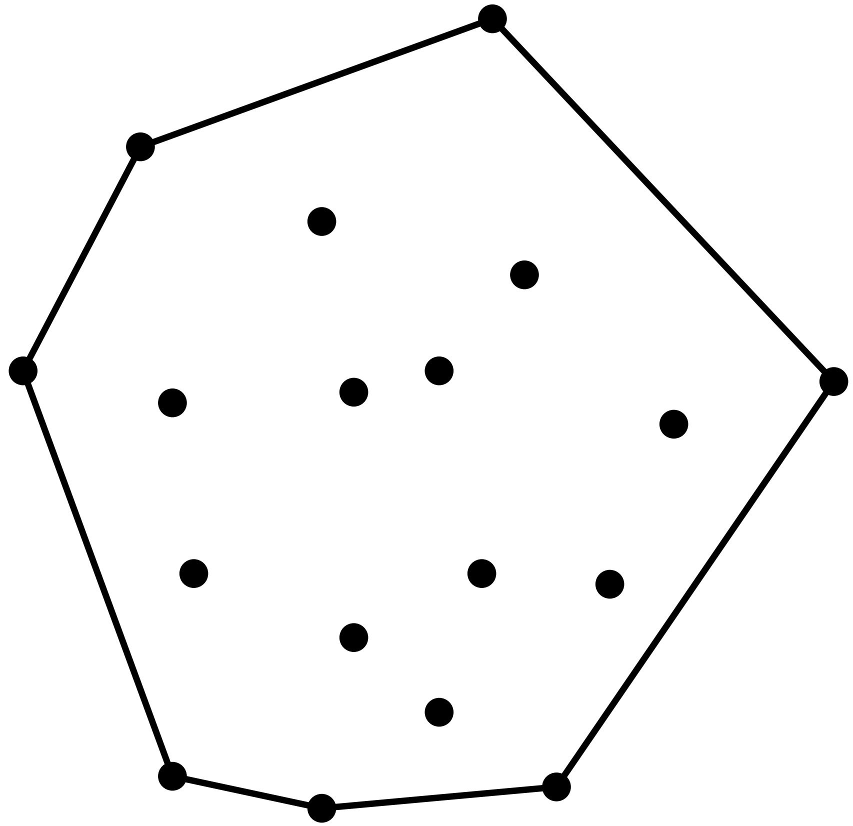
Hipótesis: Al revisar el segmento ab, b no intersecta el interior de A, es decir que b no está a la izquierda de la recta (a, a+1)

Supongamos que a fue decrementada, y que el segmento (a−1, b) pasa por el interior de A. Es decir que b está a la izq. de la recta (a−1, a), pero entonces la recta (a, b) es tangente inferior de A y el alg. nunca hubiera decrementado a.

Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

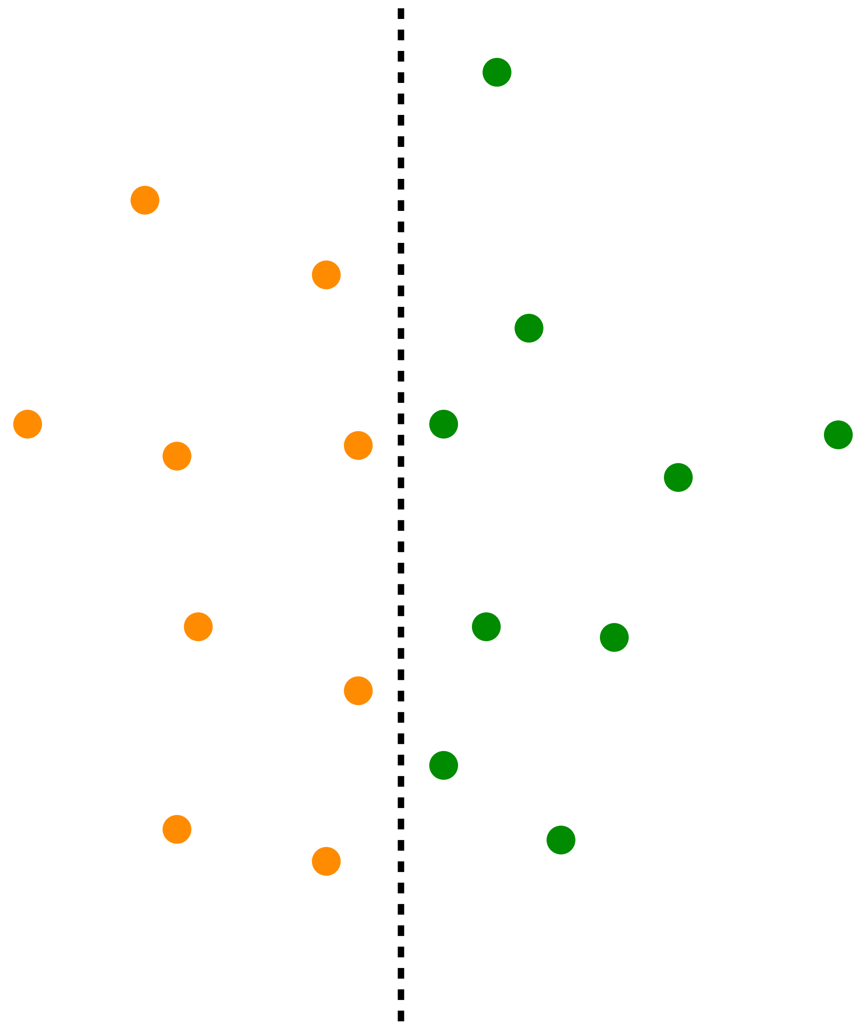


Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$



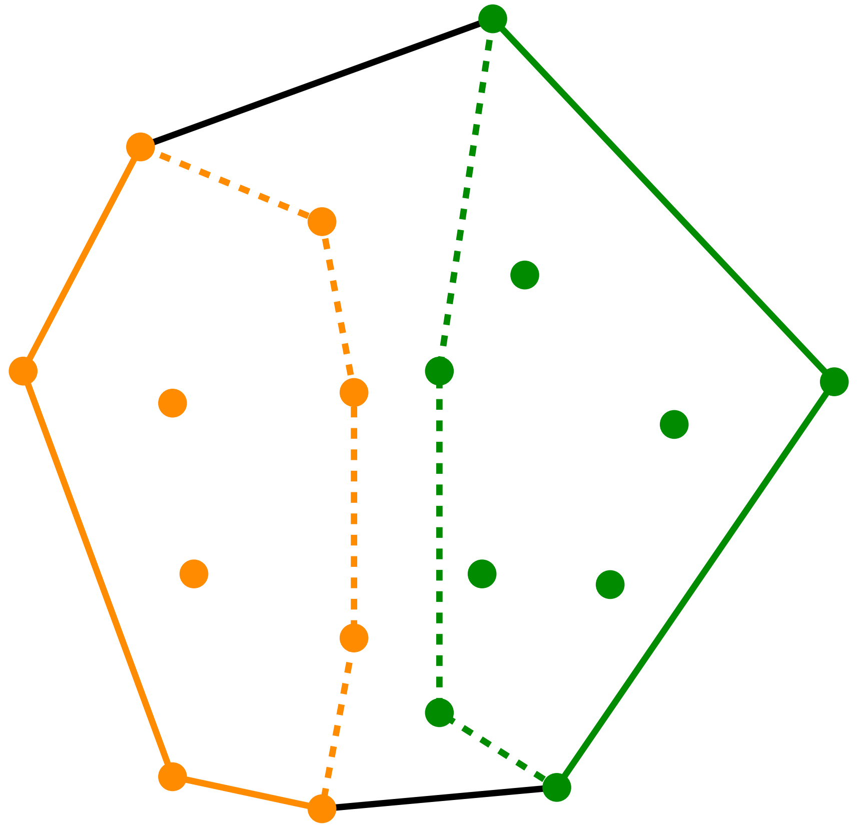
Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

Division: $O(n)$

Merge: $O(n)$



Divide-and-conquer algorithm

Running time

Initialization: $O(n \log n)$ (only once)

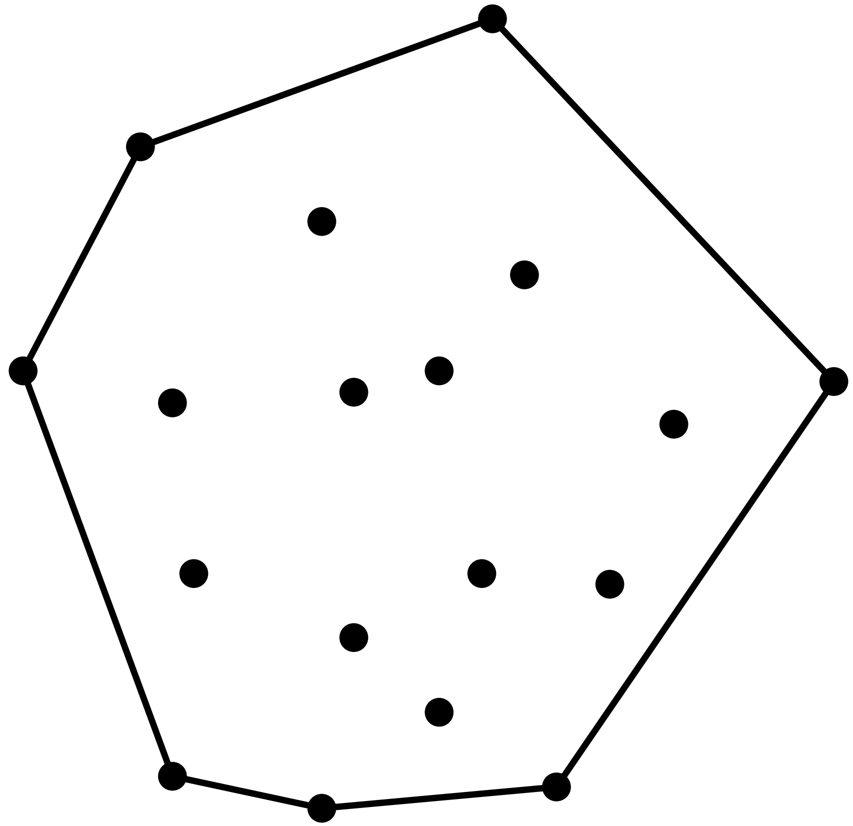
Division: $O(n)$

Merge: $O(n)$

Advance:

$$T(n) = 2\,T\left(\frac{n}{2}\right) + O(n) = O(n \log n)$$

Overall: $O(n \log n)$



Lower bound

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Input: n real numbers

$x_1, \ldots, x_n$ real numbers

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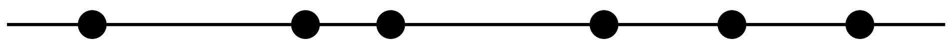


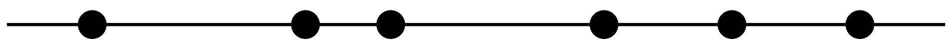
Lower bound

Input: n real numbers

$x_1, \ldots, x_n$ real numbers

Input: n points

$p_1, \ldots, p_n$, with $p_i = (x_i, x_i^2)$



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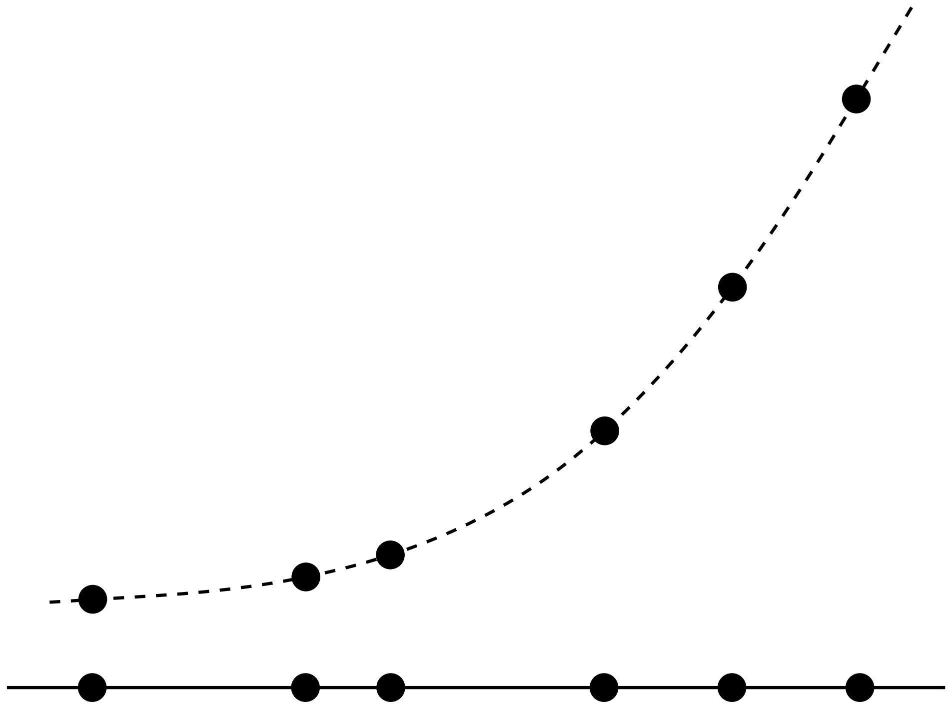

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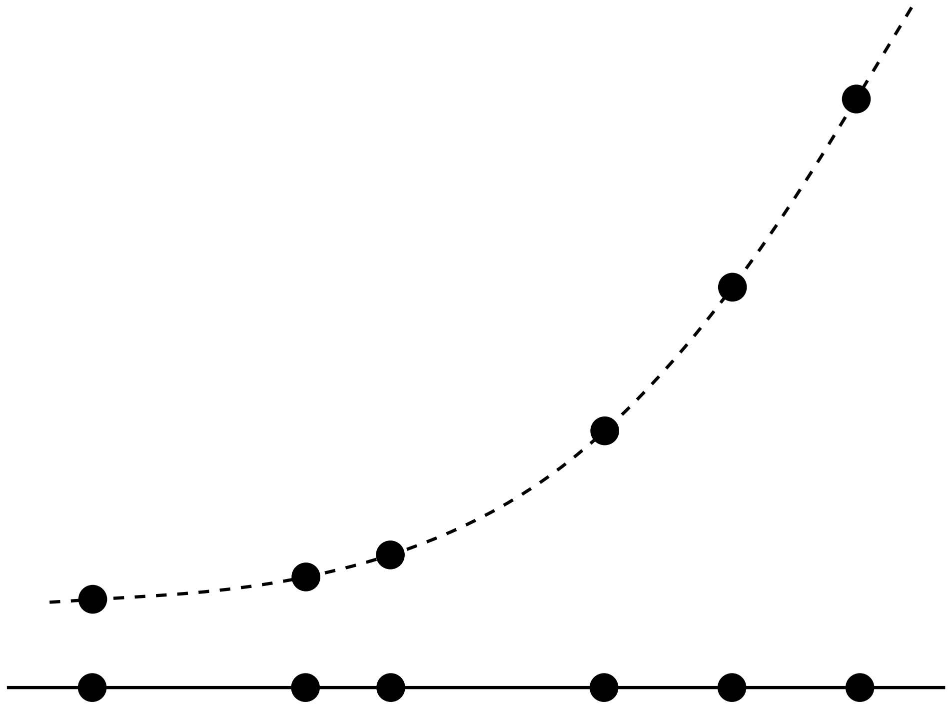
Output: convex hull of the points

Sorted list of the vertices of the convex hull



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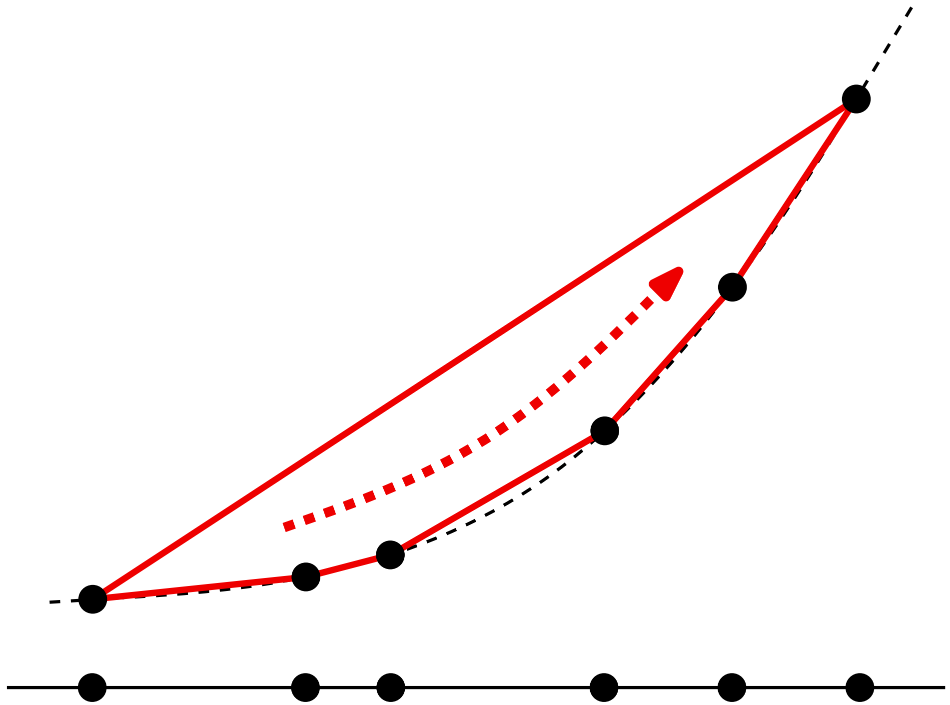

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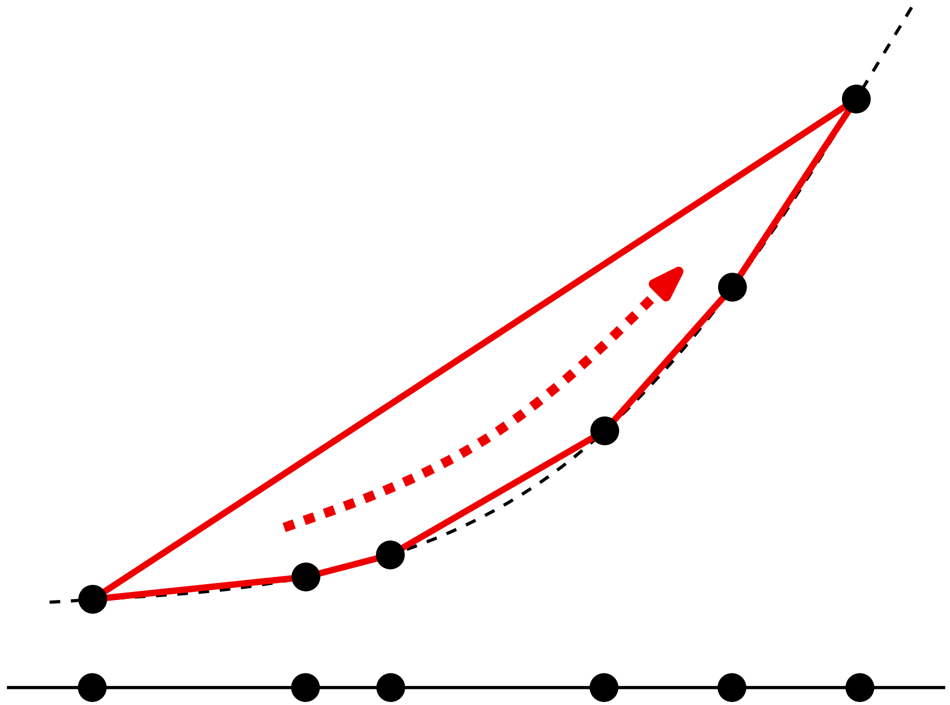

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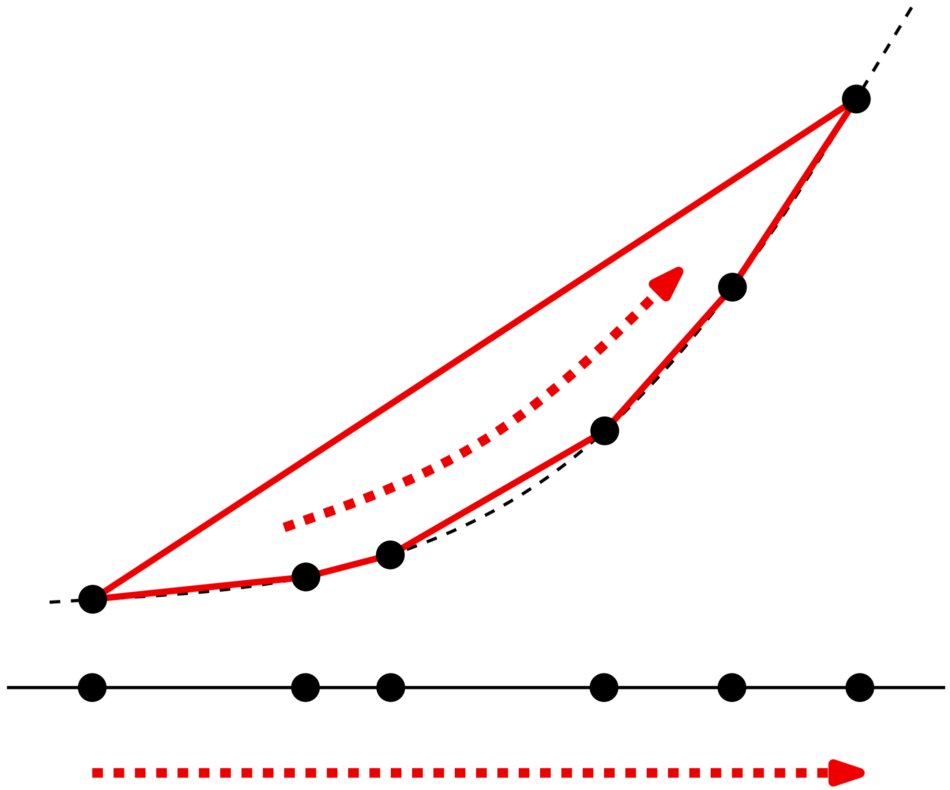

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Input: n real numbers
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$\downarrow$ $O(n)$

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$\downarrow$

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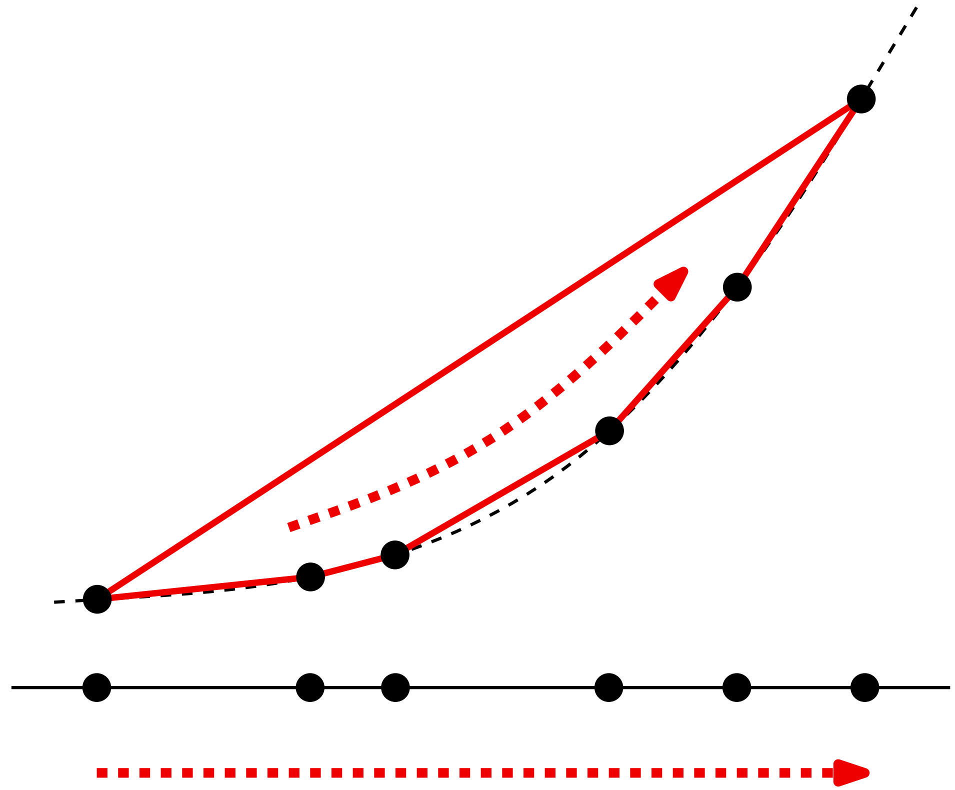

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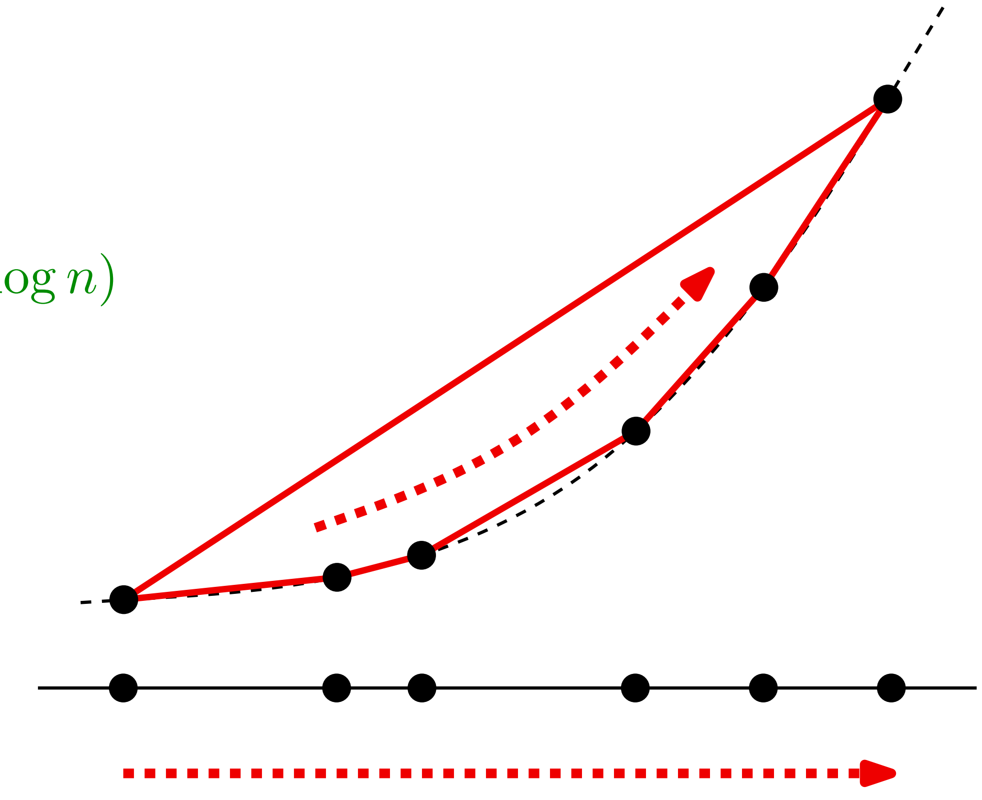
$\Omega(n \log n)$



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$\downarrow$ $\Omega(n \log n)$

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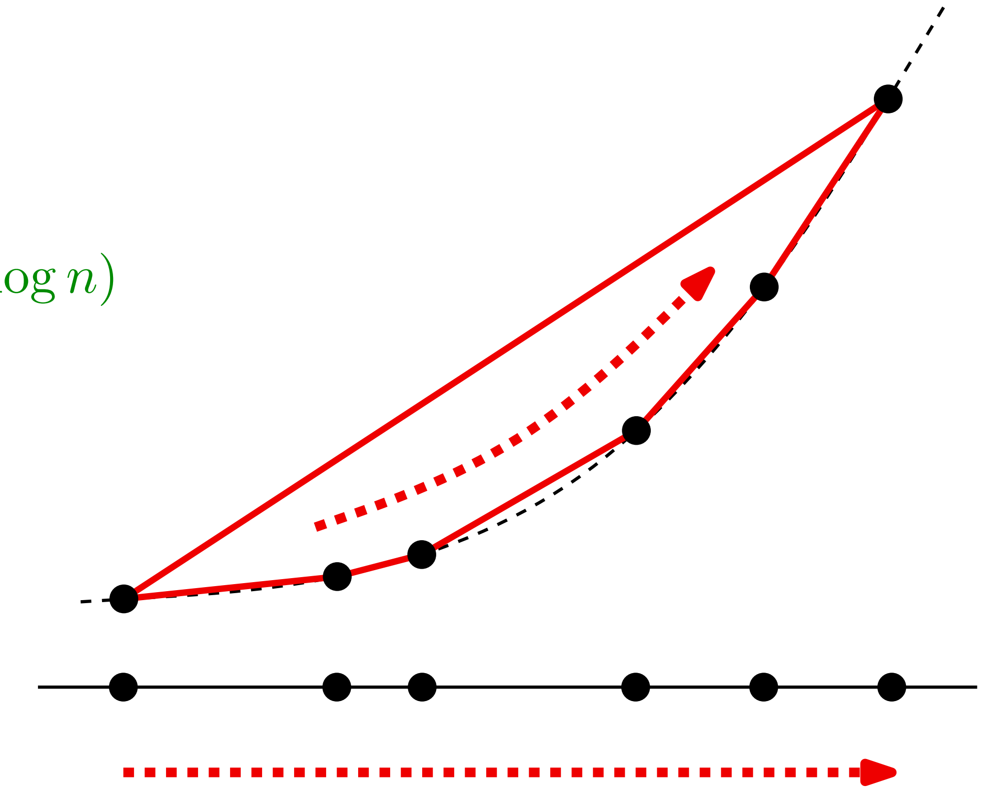
(Overall: $\Omega(n \log n)$)



Extensions

Extensions

- Convex hull of a set of n points in 3D

  (proposed for a theory presentation)

  - Gift wrapping
  - Divide-and-conquer
  - Incremental

- Convex hull of a simple polygon

  (proposed for a theory presentation)

  - Is it possible to design an $o(n \log n)$ time algorithm by exploiting the order of the vertices of the polygon?
  - Is it possible, for example, to apply Graham's algorithm using the order of the vertices of the polygon?

SOME LINKS TO PLAY WITH THE CONSTRUCTION OF CONVEX HULLS

In 2D:

http://www.dma.fi.upm.es/recursos/aplicaciones/geometria_computacional_y_grafos/

In 3D:

http://www.cse.unsw.edu.au/~lambert/java/3d

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TO LEARN MORE

- J. O'Rourke, **Computational Geometry in C (2nd ed.)**, Cambridge University Press, 1998.
- F. Preparata, M. Shamos, **Computational Geometry: An introduction (revised ed.)**, Springer, 1993.