Using orientation tests to solve basic problems on polygons

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Universitat Politècnica de Catalunya

Intersection test line - polygon

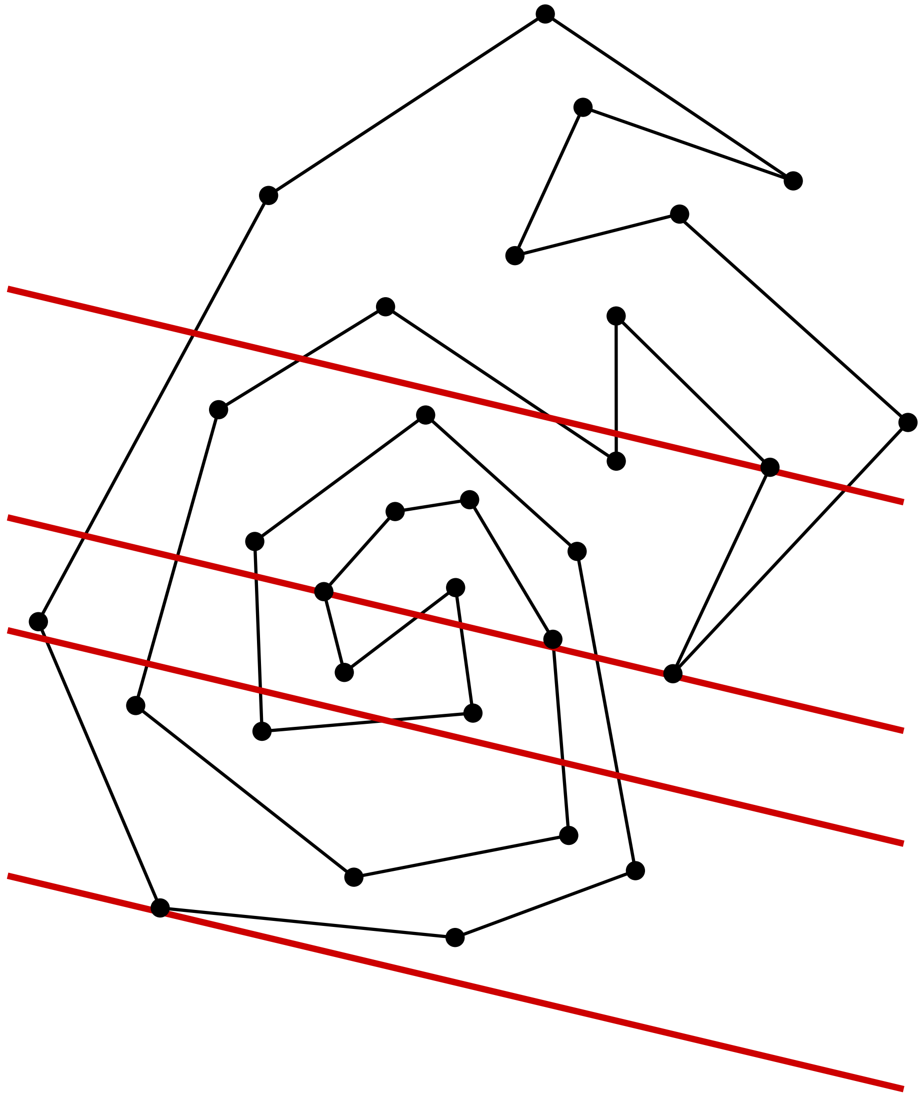
Input:

ℓ: a line (through p and q)

P: a polygon (with vertices $p_1, p_2, \ldots, p_m$)

Yes/No they intersect.

If they do, the edges of P intersecting ℓ



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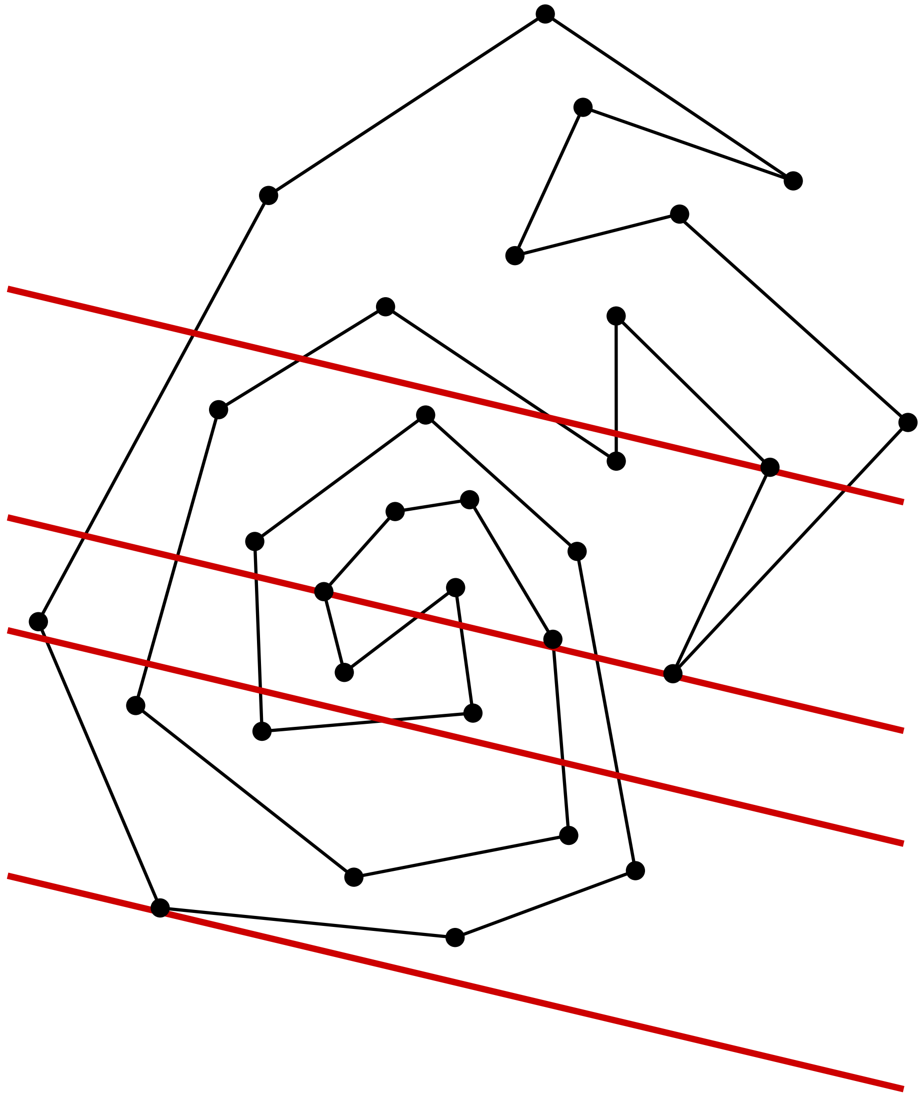
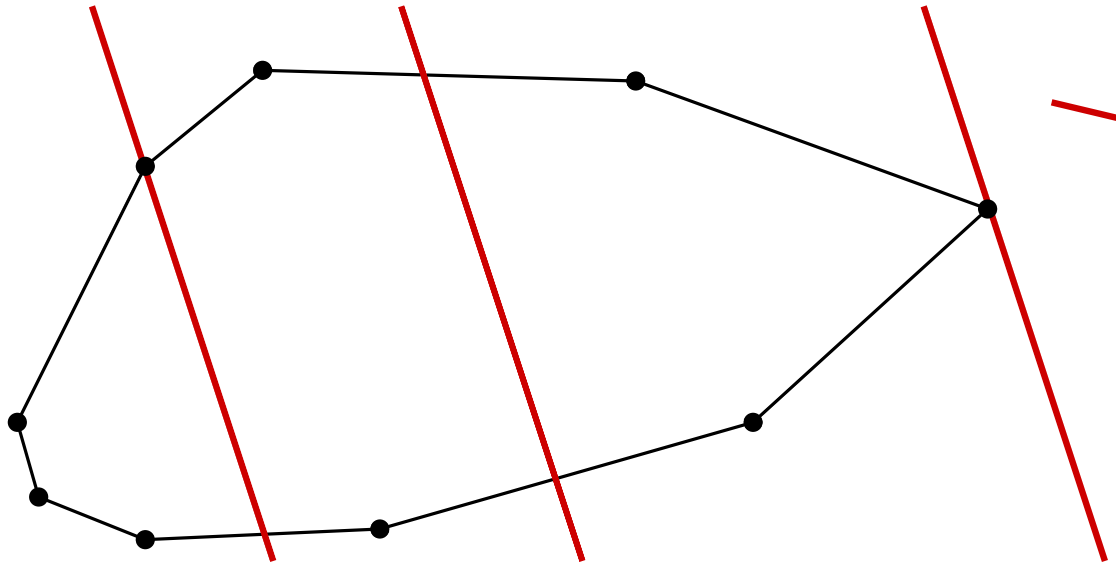


What if the polygon is convex?



Point in polygon test

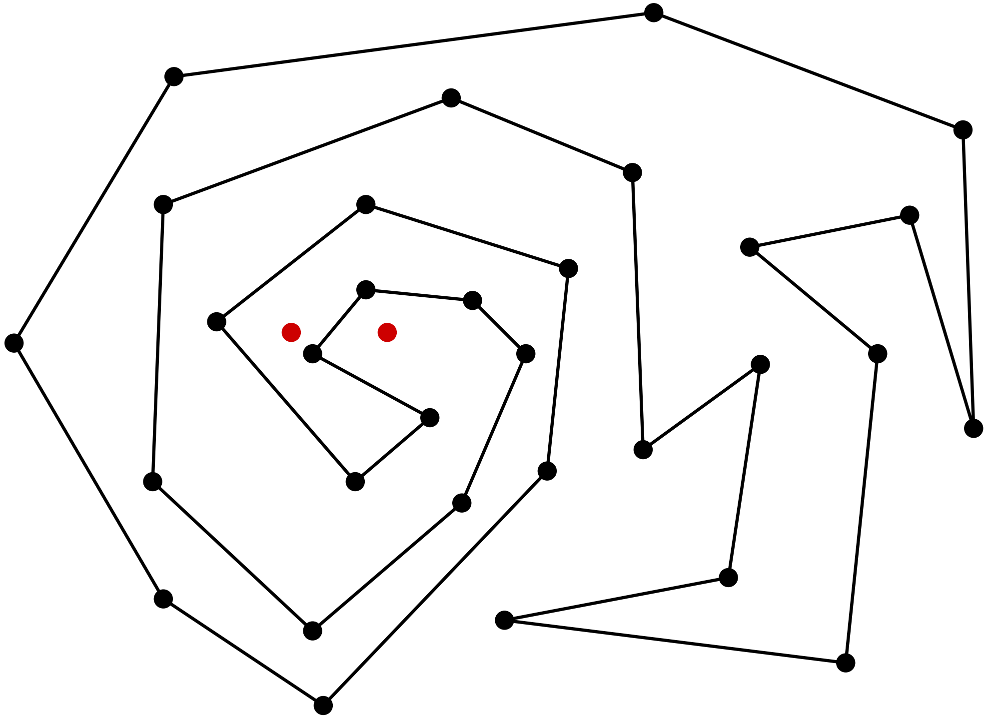
Input:

A polygon $p_1, p_2, \ldots, p_n$

A query point q

Output:

Yes/No $q \in P$



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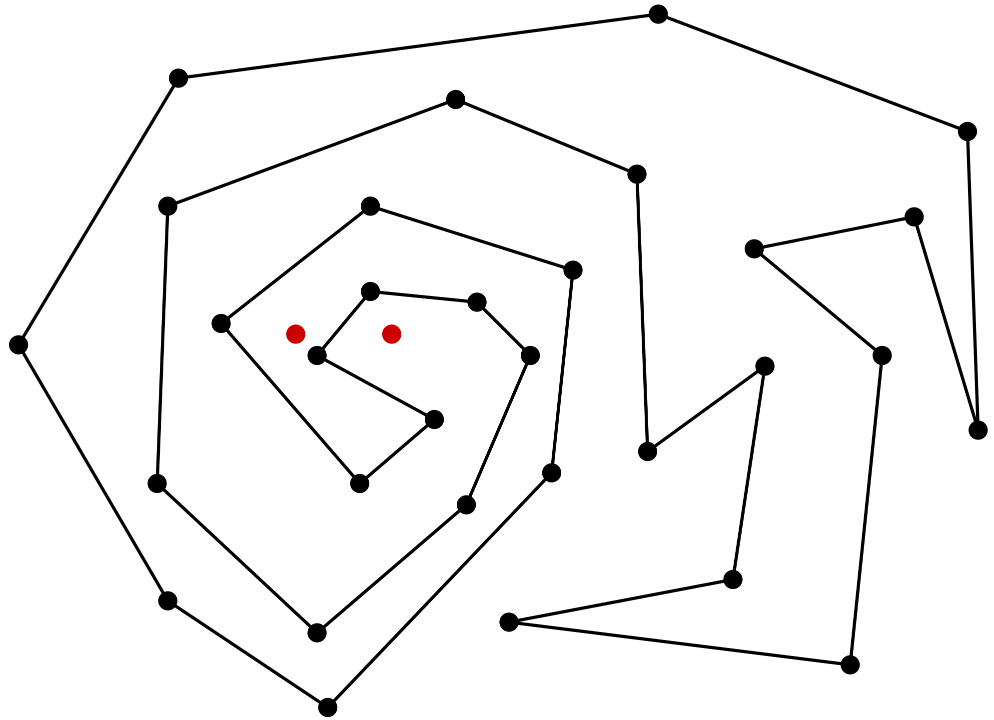

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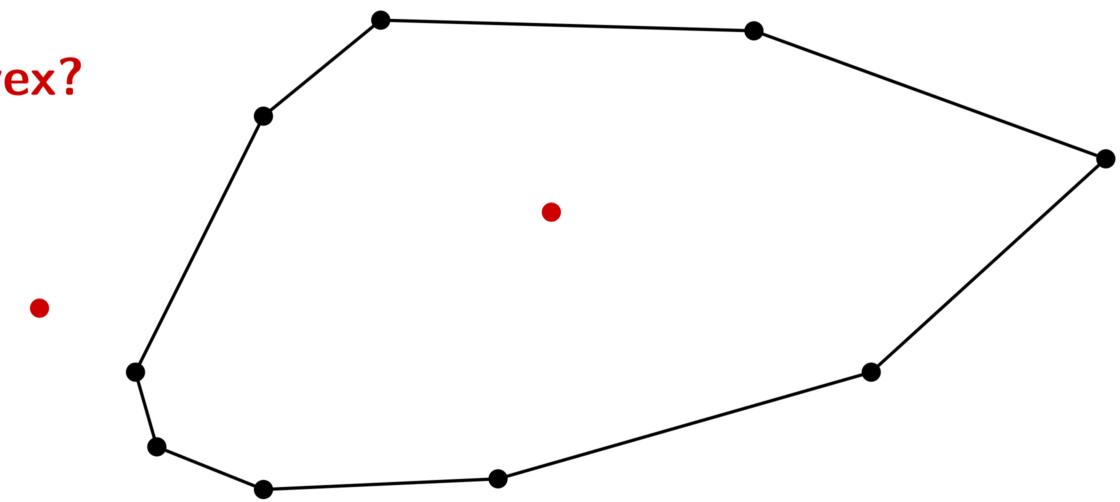


Supporting lines point - polygon

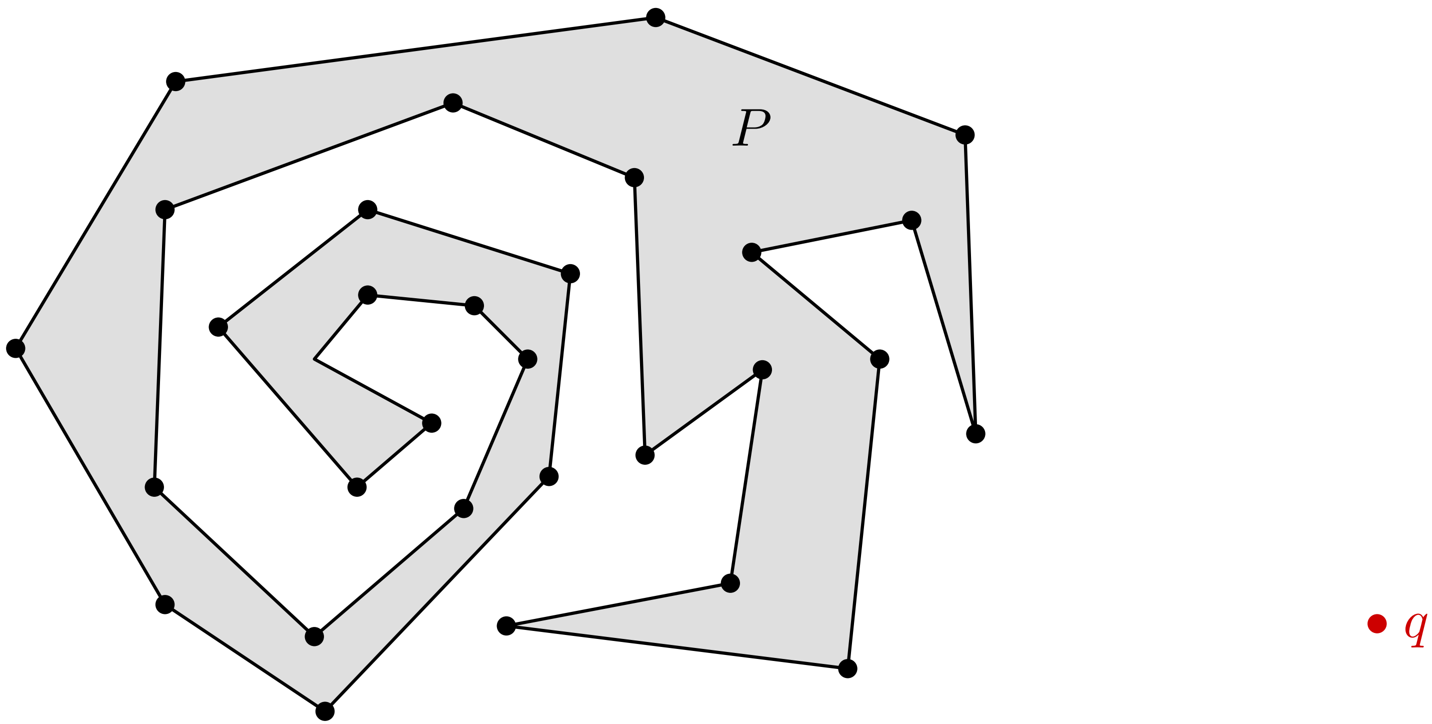
Input:

A polygon P with vertices $p_1, p_2, \ldots, p_n$

A point q not belonging to the convex hull of P

Output:

Lines through q and P that leave all of P to one side



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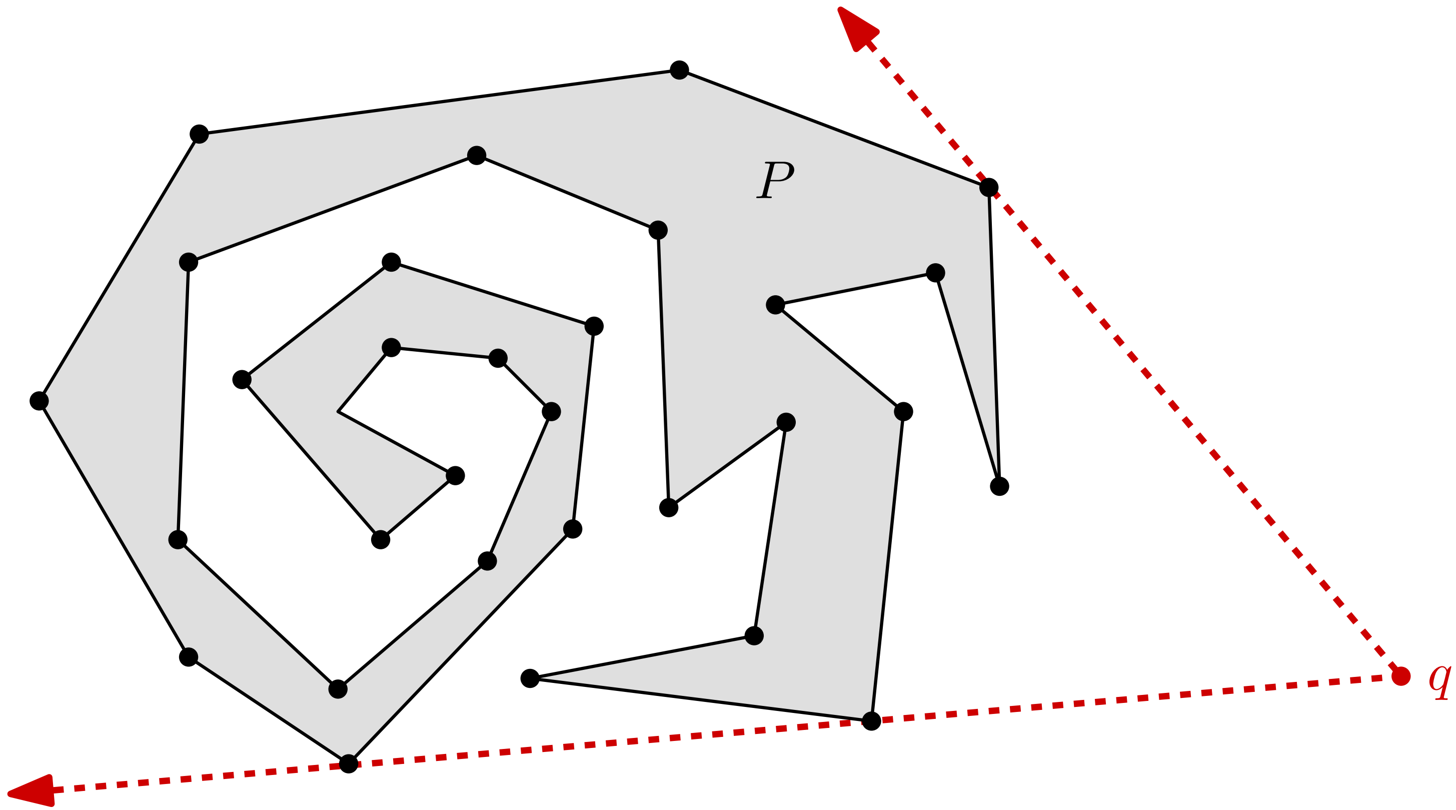

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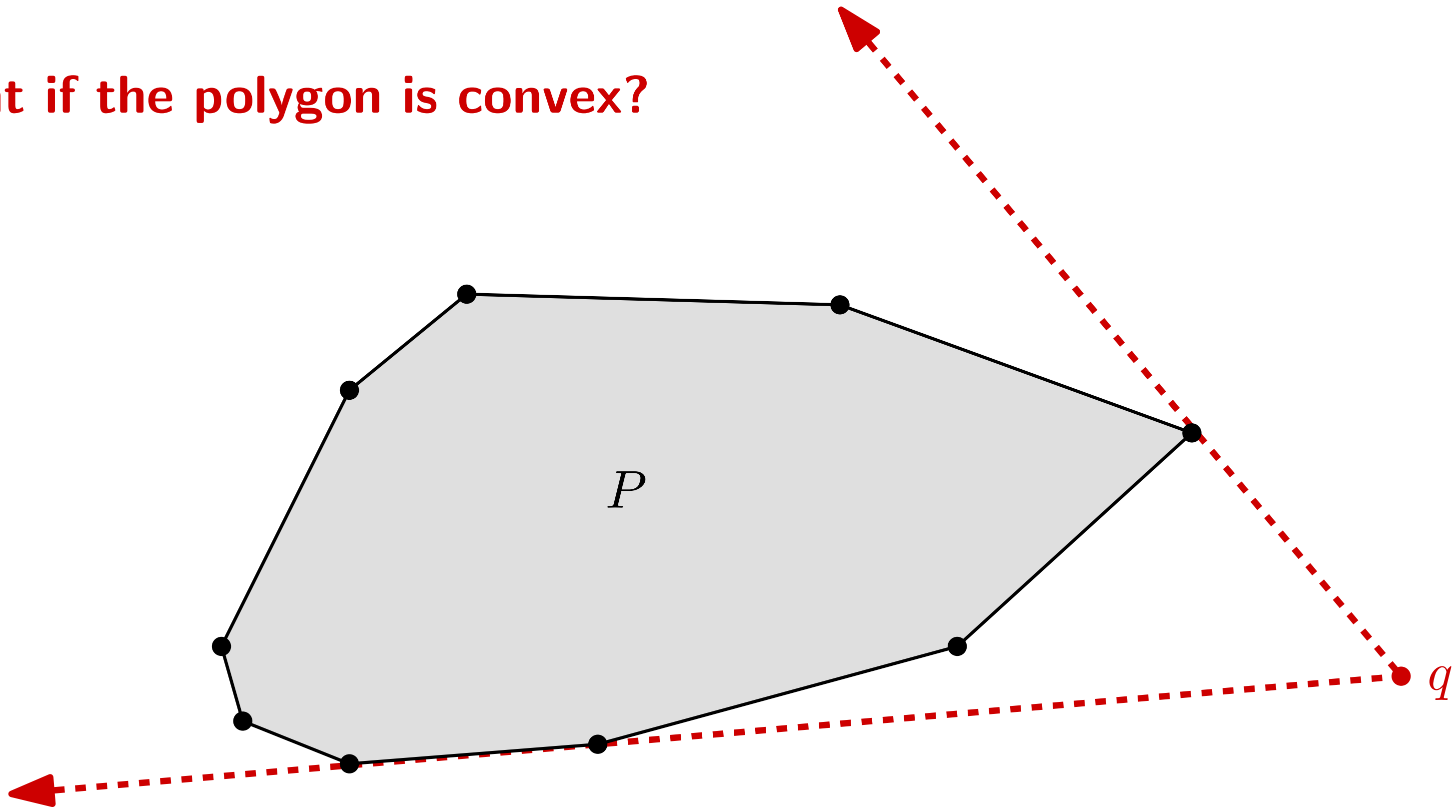

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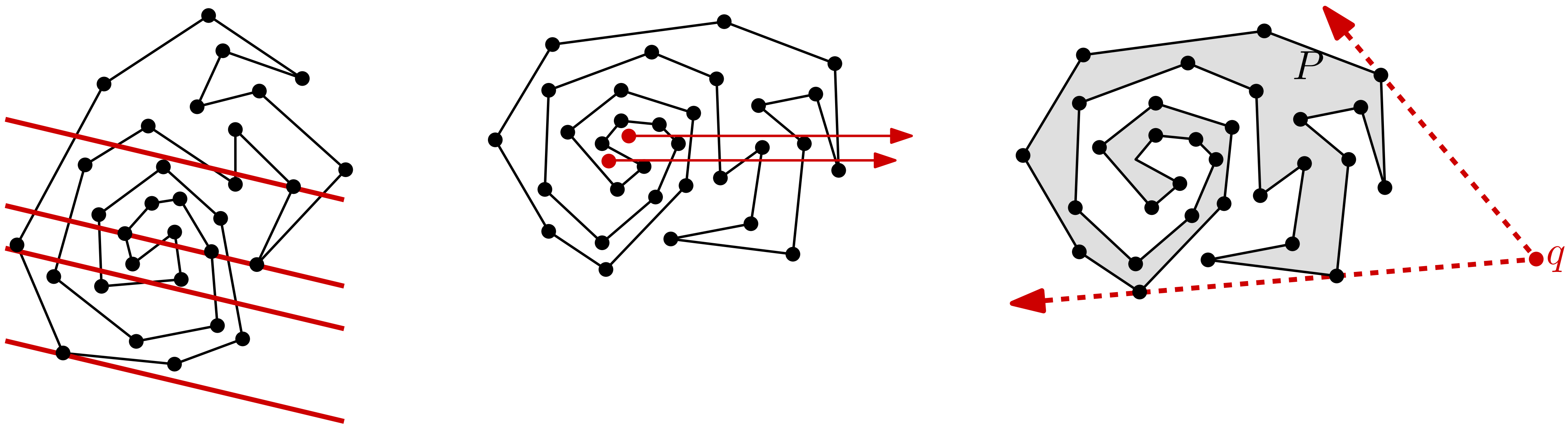

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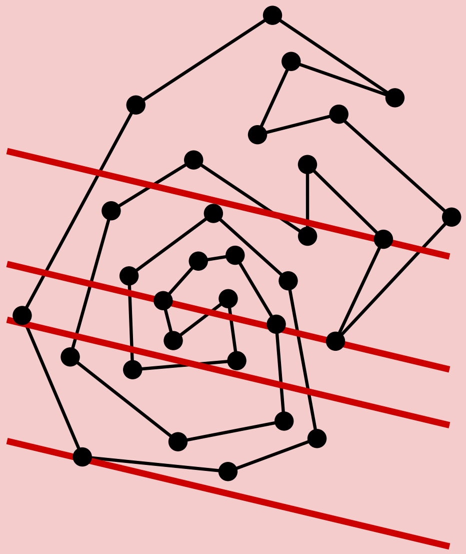

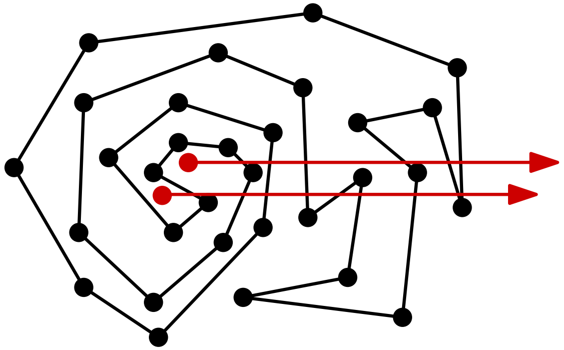

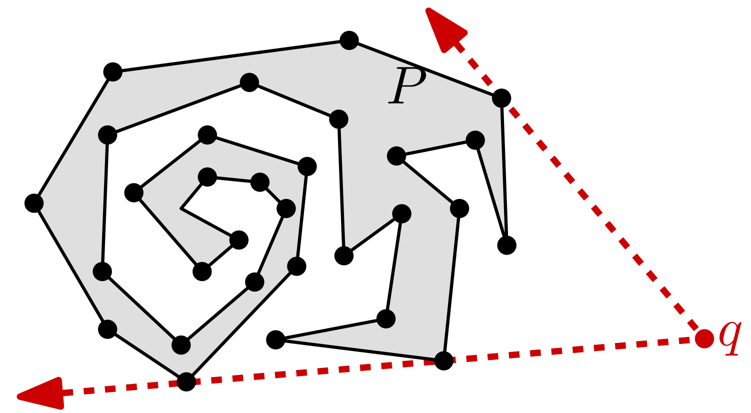

Geometric property:
No particular one.

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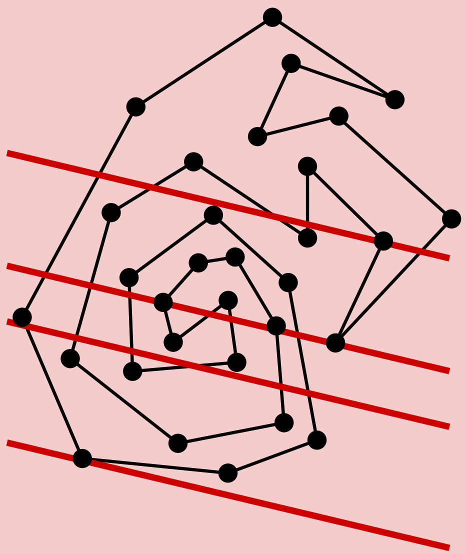

Geometric property:
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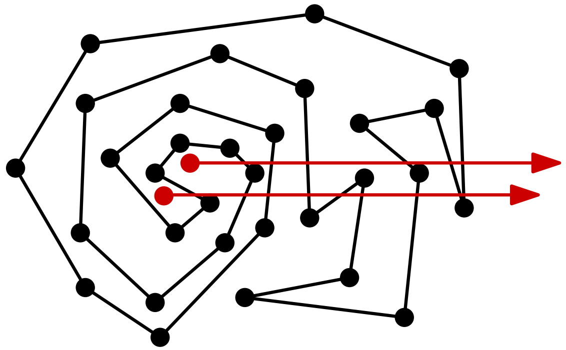
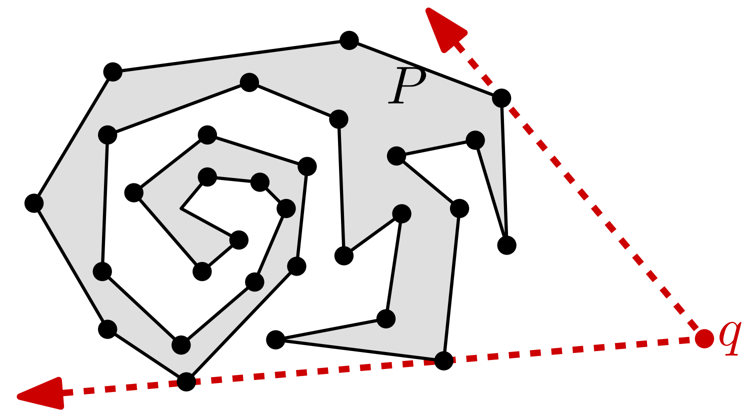
Brute-force solution

$O(n)$ time
$O(n)$ space





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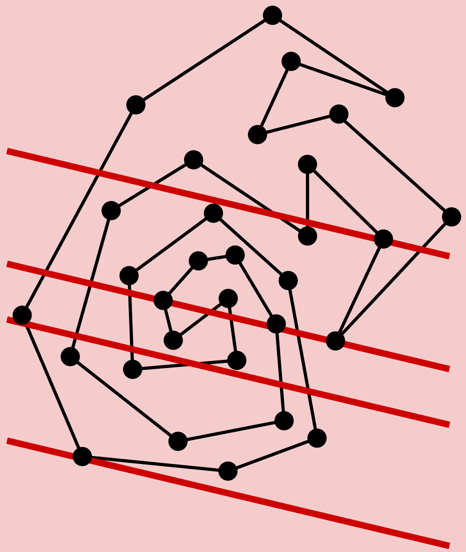

Geometric property:
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⬇

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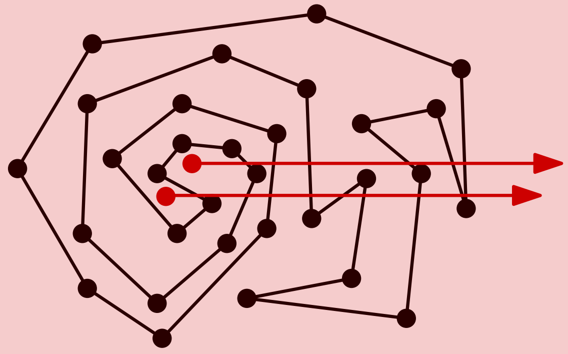
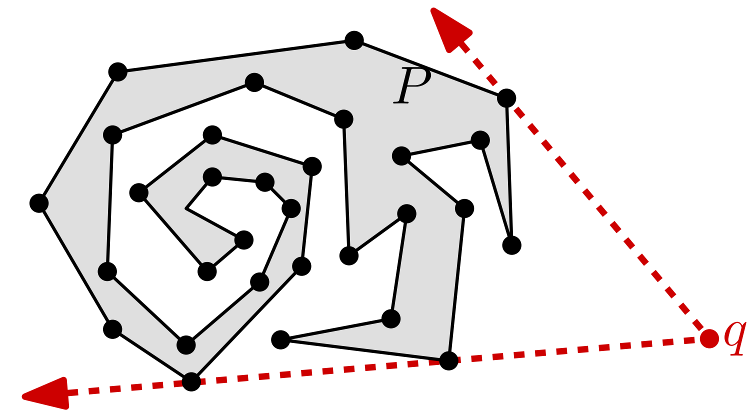


Geometric property:
$p \in P \Leftrightarrow$ The number of intersections of ∂P and any halfline with origin at p is odd.



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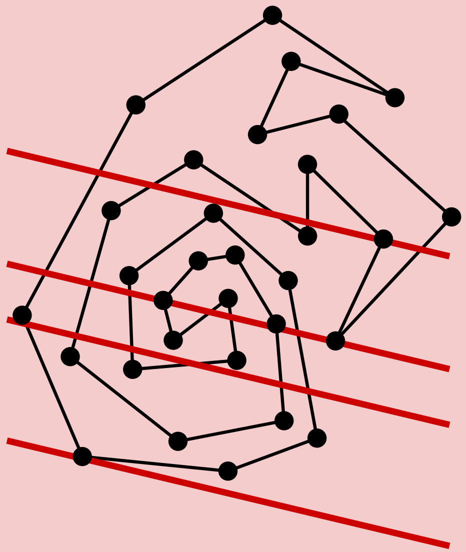

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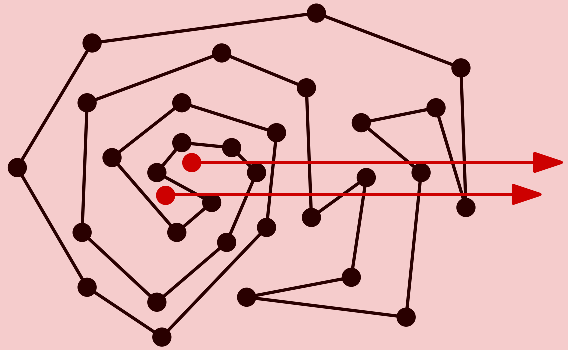

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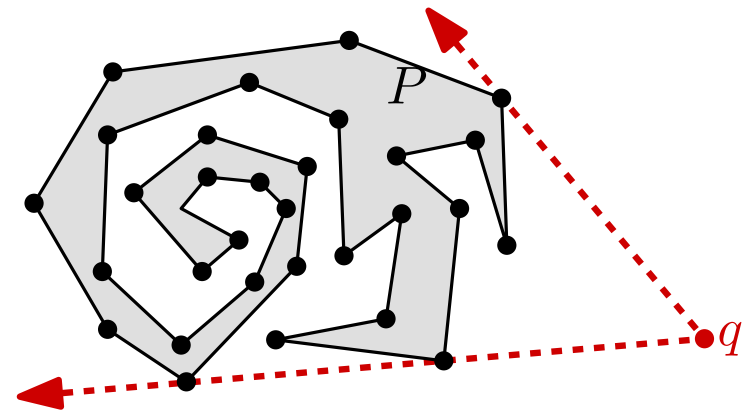

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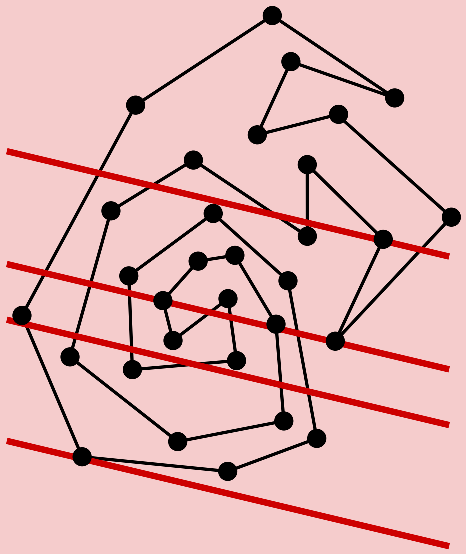

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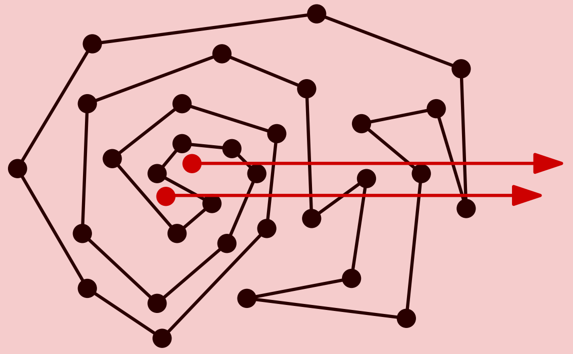

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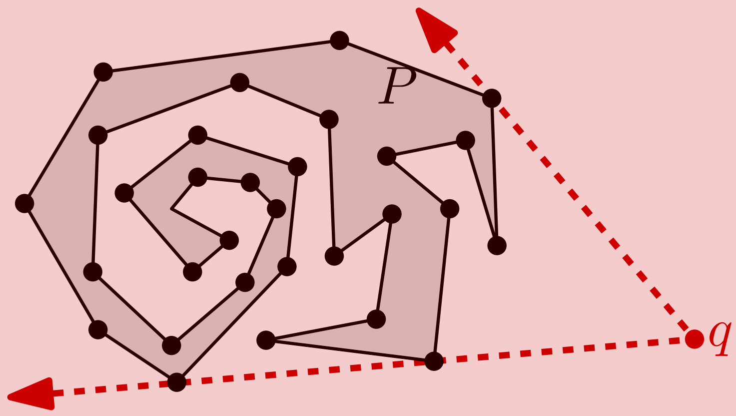

Geometric property:
The solutions are the angularly extreme vertices of P as seen from p.

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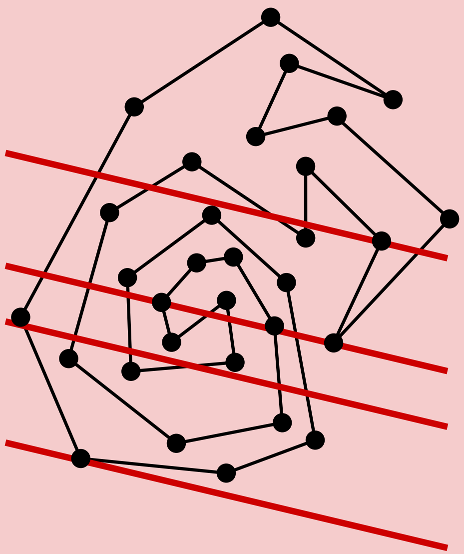

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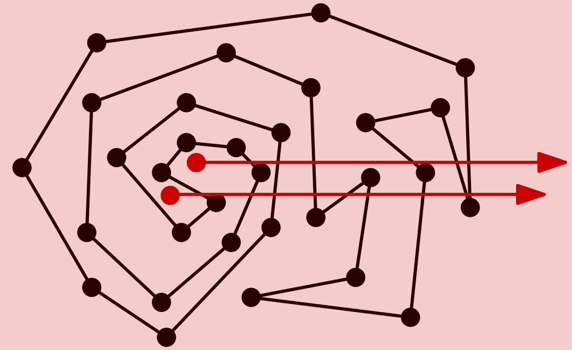

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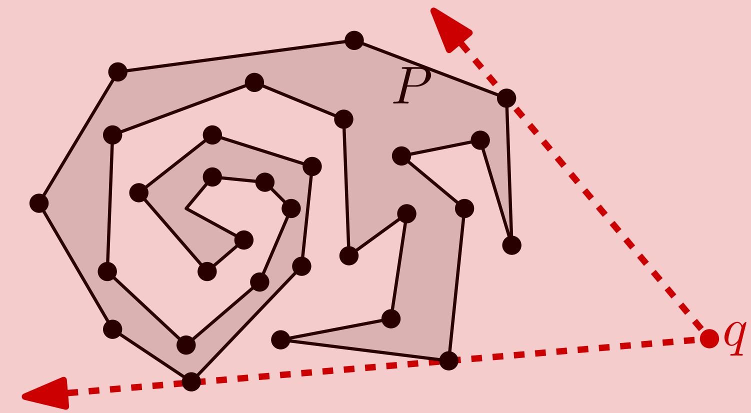

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Use a max/min algorithm

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$O(n)$ space

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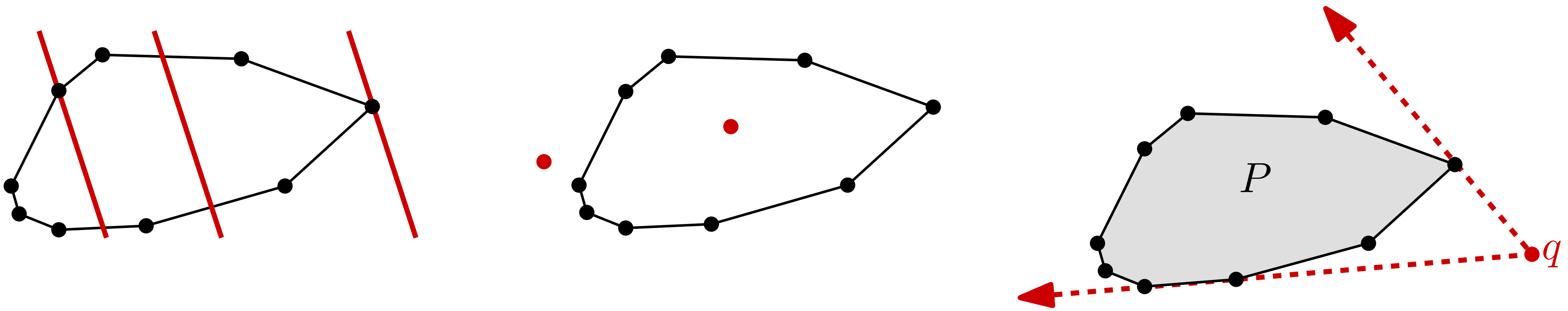

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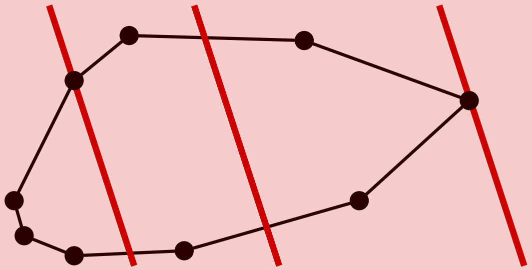
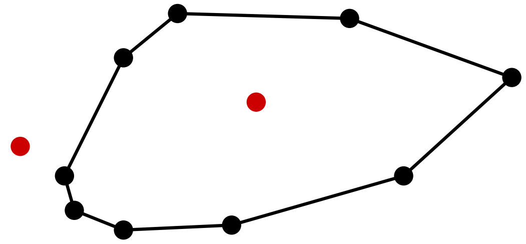
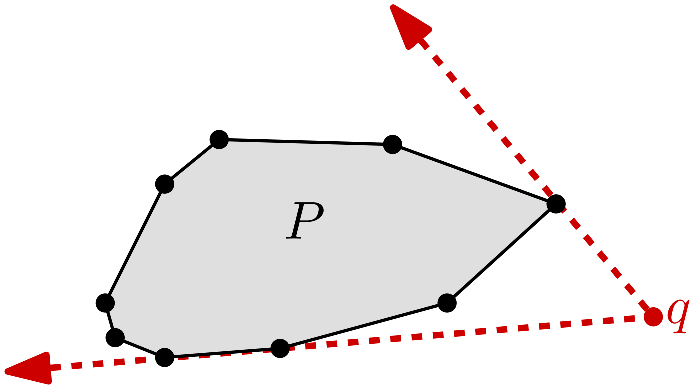


Geometric property: Distance to line is unimodal along each chain of ∂P.





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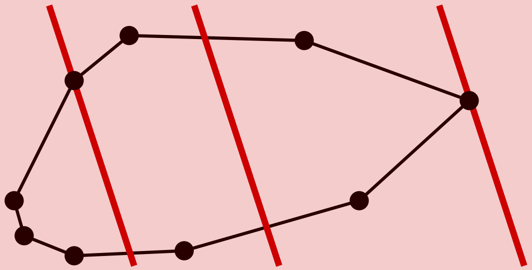

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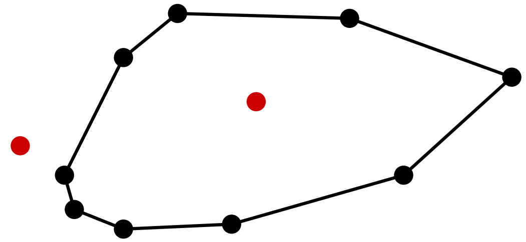
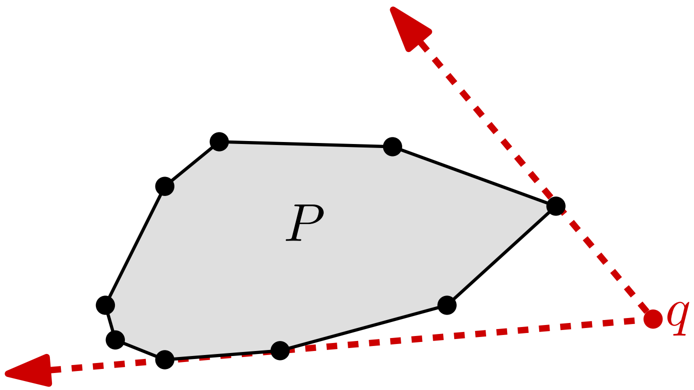
Binary search solution

$O(\log n)$ time
$O(n)$ space
(after preprocess)





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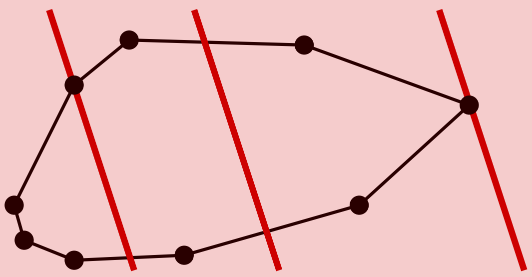

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Binary search solution

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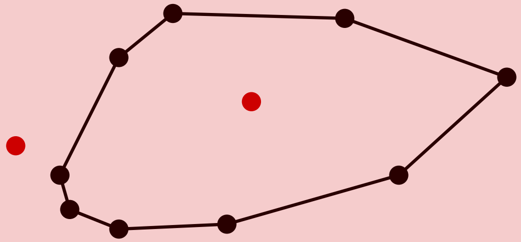
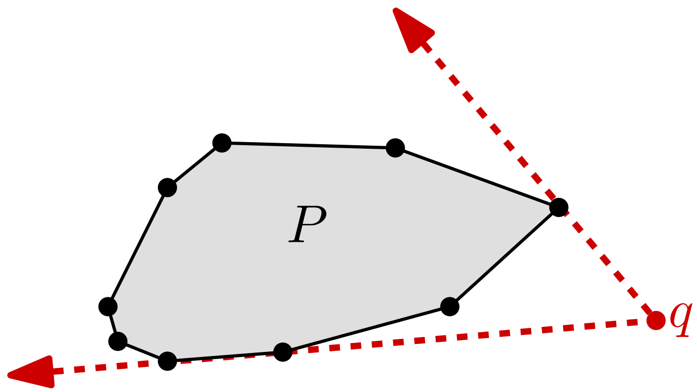


Geometric property:
Segments connecting two vertices decompose P into two convex subpolygons.



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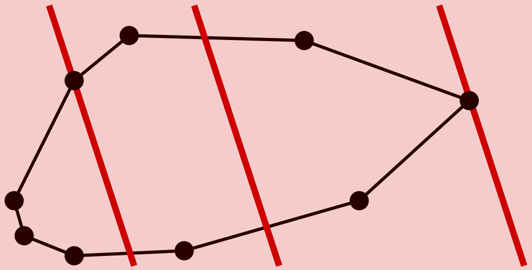

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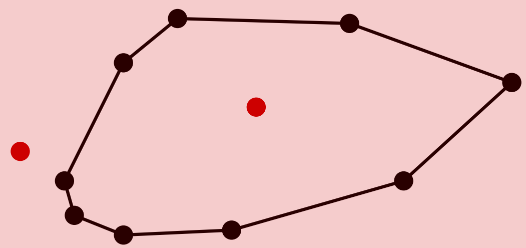

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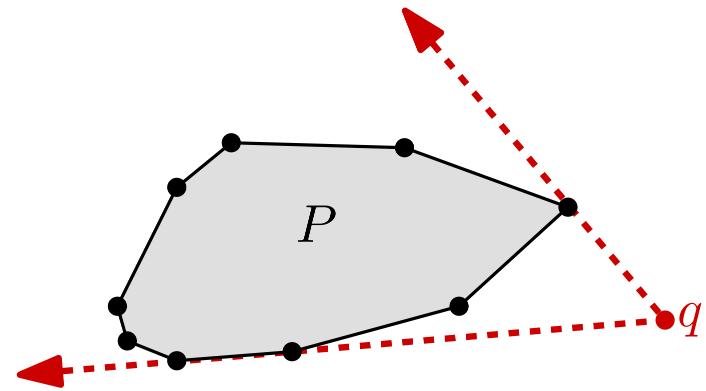

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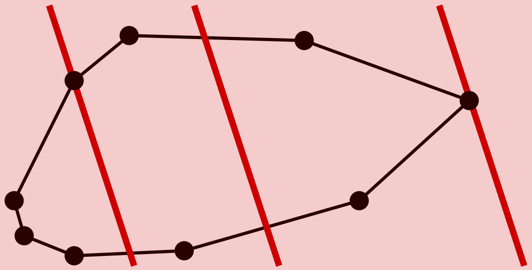

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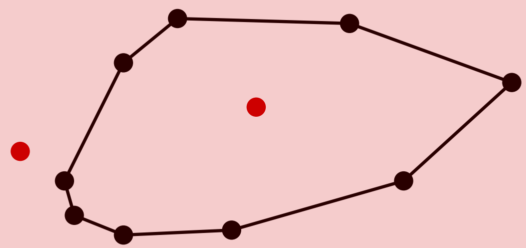

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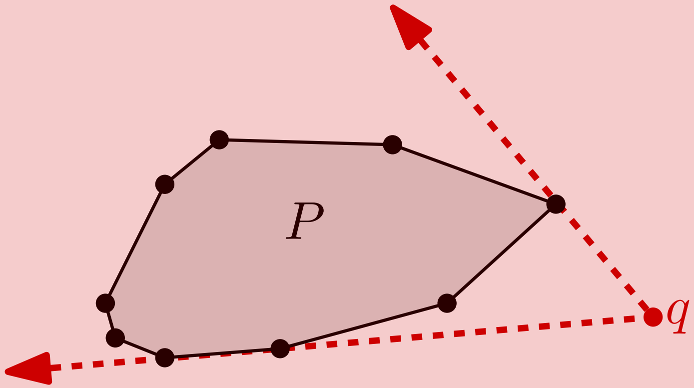

Geometric property: Angle wrt q is unimodal along ∂P.

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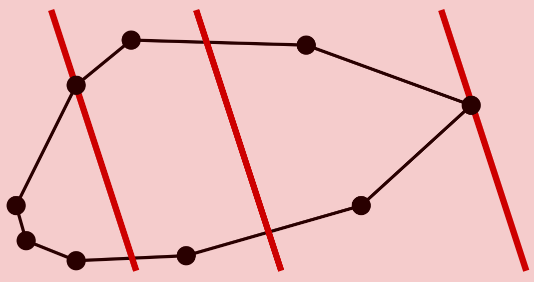

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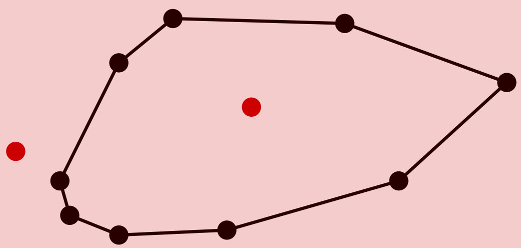

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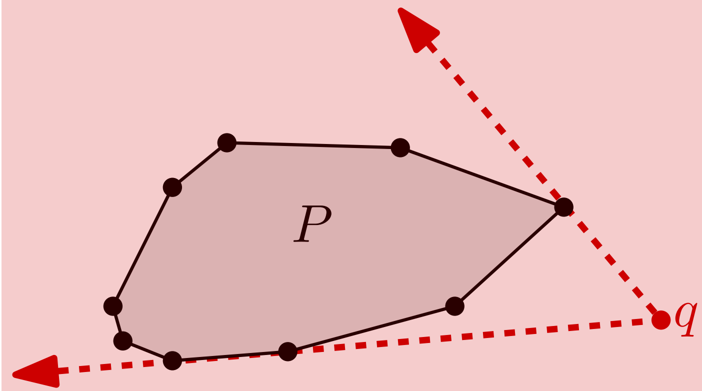

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FURTHER READING

J. O'Rourke
Computational Geometry in C
Cambridge University Press, 1994 (2nd ed. 1998), pp. 17-35.

F. P. Preparata and M. I. Shamos
Computational Geometry: An Introduction
Springer-Verlag, 1985, pp. 36-45.