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CINVESTAV-IPN
Computer Science Departament

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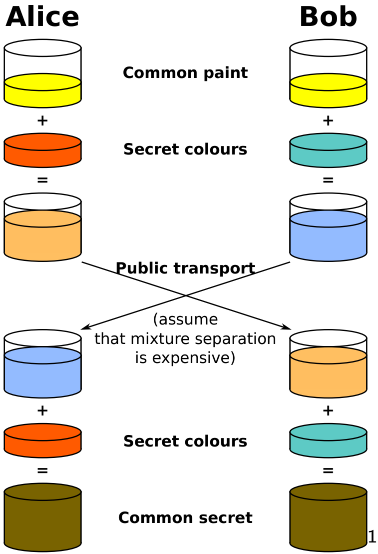
Secret Sharing - Diffie Hellman

Problem:

- Alice and Bob want to paint their houses using the same color.
- They just don't want Eve to know the final color.



[1]

[1] https://commons.wikimedia.org/wiki/File:Diffie-Hellman_Key_Exchange.svg

Discrete Log on finite fields

Public parameters

- Prime p,
- base g (generator).

Alice

Bob

Alice	Bob
Choose a random integer $a \in \{1, \ldots, p-1\}$.	Choose a random integer $b \in \{1, \ldots, p-1\}$.
Compute $g_a := g^a \mod p$.	Compute $g_b := g^b \mod p$.
Send g_a to Bob.	Send g_b to Alice.
Compute $g_{ba} := g_b^a \mod p$.	Compute $g_{ab} := g_a^b \mod p$.

$$g_{ab} = (g^a)^b = (g^b)^a = g_{ba}$$

Discrete log consists in find a knowing g and g_a.

Mathematical Background

An *Elliptic Curve* in Weierstrass short model over a finite field $\mathbb{F}_q$ where $q = p^m$ for some prime $p > 3$, is given by the equation

$$E/\mathbb{F}_q : Y^2 = X^3 + AX + B$$

where $A, B \in \mathbb{F}_q$. The *j-invariant* $j(E)$ of a curve acts like a "fingerprint" of a

curve and it is given by

$$j(E) = \frac{1728 \cdot 4A^2}{4A^2 + 27B^2}.$$

A point P in $E(\mathbb{F}_q)$ is a pair (x, y) such that $x^3 + Ax + B - y^2 = 0$. E is supersingular if

$$\#E(\mathbb{F}_q) = q + 1 + k \cdot p.$$

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Mathematical Background

- (Hasse's Theorem)The number of rational points in an elliptic curve is bounded by

$$\#E(\mathbb{F}_q) = q + 1 - t, \qquad | t | \leq 2\sqrt{q}.$$

- Let E be an elliptic curve and consider the integer t given by Hasse theorem. An elliptic curve is called *supersingular* if $p|t$ otherwise is called *ordinary*.

Mathematical Background

- We can **ADD** points
$$R := P + Q,$$
- **DBL** a point
$$[2]P := P + P$$
- and multiply by an integer
$$[m]P := P + P + \cdots + P, (m-1)(\text{times}).$$
- The minimum integer m shuch that $[m]P = \mathcal{O}$ is called the **order** of P.
- The **subgroup generated** by P is the set $\{P, [2]P, [3]P, \ldots, [m-1]P, \mathcal{O}\}$ and is denoted by $\langle P \rangle$.
- The m-**torsion subgroup** is defined as $E[m] = \{P \in E \mid [m]P = \mathcal{O}\}$.

- An *Isogeny* $\phi : E_0 \to E_1$ is an homomorphism between elliptic curves given by rational functions. Given P and Q in E_0 is fulfilled that
  - $\phi(P+Q) = \phi(P) + \phi(Q)$,
  - $\phi(\mathcal{O}) = \mathcal{O}$.
- The *Kernel* of an Isogeny ϕ is the set

$$K = \{P \in E \mid \phi(P) = \mathcal{O}\}.$$

- The degree of an isogeny is $s := \#K$
- If ϕ has degree s^e then we can "decompose" ϕ as the composition

$$\phi_{e-1} \circ \phi_{e-2} \circ \cdots \phi_1 \circ \phi_0$$

where ϕ_i has degree s.

Mathematical Background

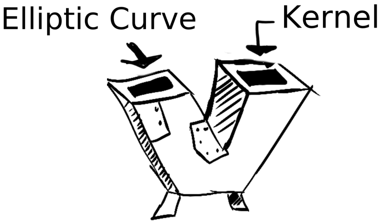
Theorem: Let E and E' be two elliptic curves. If there exists a degree-1 isogeny between E and E' then $j(E) = j(E')$. We say that E and E' are isomorphic. We denote that by $E \cong E'$.

Given an isogeny $\phi : E_0 \to E_1$ of degree d^e then

- We can "decompose" ϕ as the composition $\phi_{e-1} \circ \phi_{e-2} \circ \cdots \phi_1 \circ \phi_0$ where ϕ_i has degree d.
- There exists an isogeny $\hat{\phi} : E_1 \to E_0$ such that $\hat{\phi} \circ \phi = [d^e]$ and $\phi \circ \hat{\phi} = [d^e]$.

- Let E be an elliptic curve and $P \in E$ be an order m point.
- Then there exists an elliptic curve E_P and an isogeny $\phi_P : E \to E_P$ such that the *Kernel* of ϕ_P is $\langle P \rangle$, *i.e.* $\phi_P(p) = \mathcal{O}$ for each $p \in \langle P \rangle$. We write

$$E_P = E/\langle P \rangle$$



Mathematical Background

- Let E be an elliptic curve and $P \in E$ be an order m point.
- Then there exists an elliptic curve E_P and an isogeny $\phi_P : E \to E_P$ such that the *Kernel* of ϕ_P is $\langle P \rangle$, *i.e.* $\phi_P(p) = \mathcal{O}$ for each $p \in \langle P \rangle$. We write

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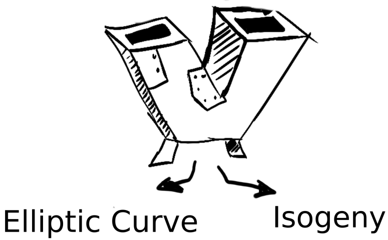

State-of-the-art

SIDH Overview

- Luca De Feo, David Jao and Jérôme Plût proposed in 2014[FJP14] a new instance of Diffie-Hellman protocol using isogenies between supersingular elliptic curves as the core operation and curves as the secret shared (actually their j-invariants).
- They use a special kind of primes $p := \ell_a^{e_a} \ell_b^{e_b} f - 1$ which satisfies:
  - ℓ_a and ℓ_b are small primes,
  - $\log_2(\ell_a^{e_a}) \approx \log_2(\ell_b^{e_b})$,
  - f is a small integer which makes p to be a prime number.
- Public parameters are: prime p, an elliptic curve E_0, and points $P_a, Q_a, P_b, Q_b \in E_0$ such that $\langle P_a, Q_a \rangle = E[\ell_a^{e_a}]$ and $\langle P_b, Q_b \rangle = E[\ell_b^{e_b}]$.

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SIDH public parameters

$$\mathsf{p} := \quad 2^{e_2} \quad 3^{e_3} \quad f - 1$$

Such that $3^{e_3} \approx 2^{e_2}$

SIDH public parameters

Choose P_2 and Q_2
such that $\langle P_2, Q_2 \rangle = E[2^{e_2}]$

Choose P_3 and Q_3
such that $\langle P_3, Q_3 \rangle = E[3^{e_3}]$

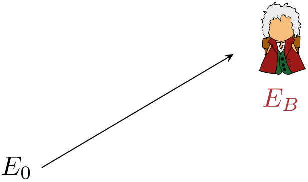
p := $2^{e_2} \quad 3^{e_3} \quad f - 1$

Such that $3^{e_3} \approx 2^{e_2}$

SIDH protocol

$K_B := P_2 + [n_2]Q_2$

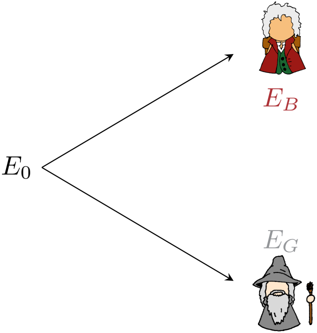
Get ϕ_B and $E_B = E_0/\langle K_B \rangle$



SIDH protocol

$$K_G := P_3 + [n_3]Q_3$$

Get ϕ_G and $E_G = E_0/K_G$



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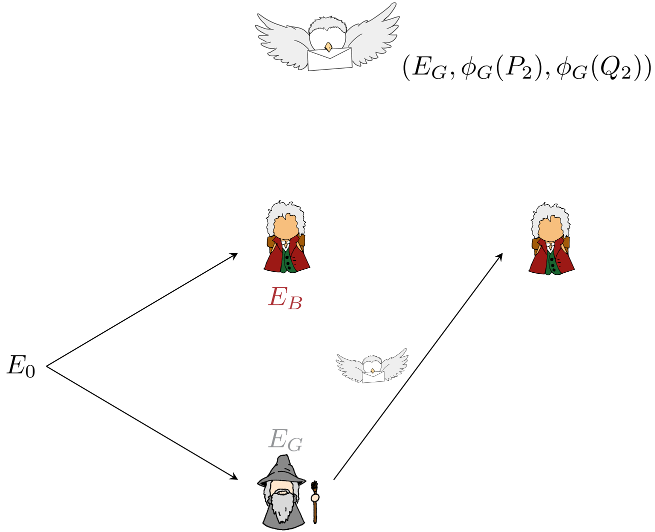
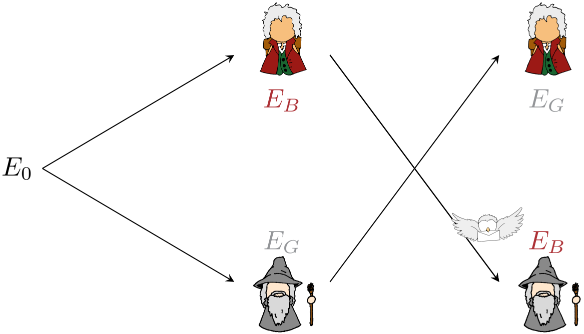
SIDH protocol



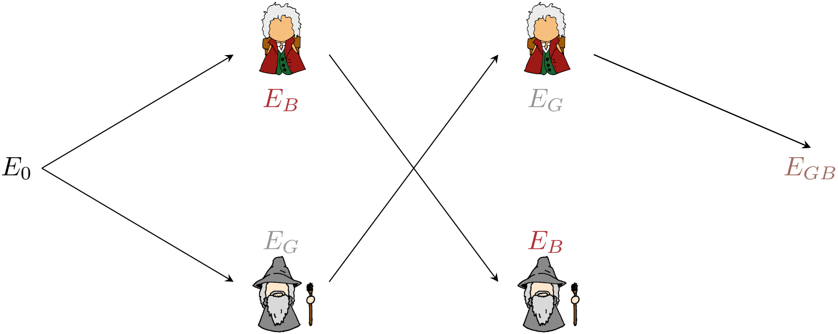
SIDH protocol

$(E_B, \phi_B(P_3), \phi_B(Q_3))$



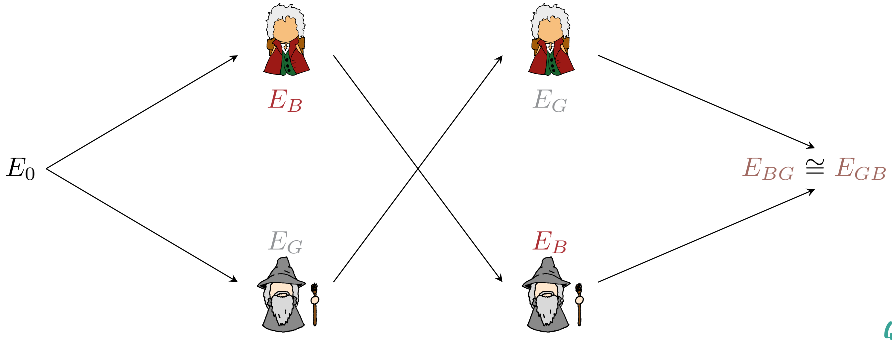
SIDH protocol

$$K'_B := \phi_G(P_2) + [n_2]\phi_G(Q_2)$$
Get $E_{GB} = E_G/\langle K'_B \rangle$



SIDH protocol

$$K'_G := \phi_B(P_3) + [n_3]\phi_B(Q_3))$$
Get $E_{BG} = E_B / \langle K'_G \rangle$



The Models

- There are different Models (equations) for elliptic curves.

Twisted Edwards Model

Twisted Edwards Curves:

$$E_{(a,d)}/\mathbb{F}_q : ax^2 + y^2 = 1 + dx^2y^2.$$

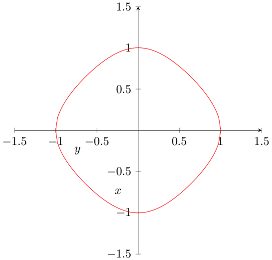
Advantages:

- Faster enough to be considered in some standards.
- Allows a y-only arithmetic.

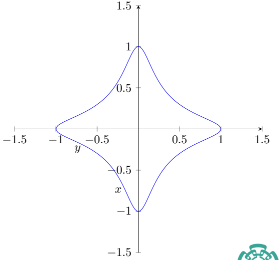
$$P = \left(\frac{x}{z}, \frac{y}{z}\right)$$

$$\mathcal{Y}(P) = (y_P : z_P)$$

- Complete addition formulas.
- Dustin Moody and Daniel Shumow[MS11] proposed formulas for computing isogenies between Twisted Edwards curves[2].



$x^2 + y^2 = 1 - x^2y^2$



$x^2 + y^2 = 1 - 30x^2y^2$

[2]Also for non-twisted Edwards curves and Huff curves

Montgomery Model

Projective Constant Montgomery Curves:

$$E_{(A:C)}/\mathbb{F}_q : Cy^2 = x(Cx^2 + Ax + C).$$

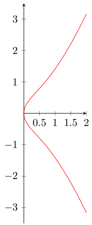
Advantages:

- Faster enough to be considered in some standards.
- Allows an x-only arithmetic.

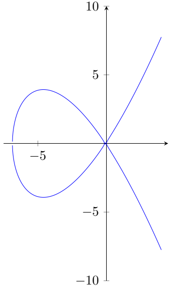
$$P = \left(\frac{x}{z}, \frac{y}{z}\right)$$

$$\mathcal{X}(P) = (x_P : z_P)$$

- Costello and Hisil [CH17] proposed formulas for computing isogenies between Montgomery curves.



$y^2 = x(x^2 + 1)$



$3y^2 = x(x^2 + 7x + 1)$

There and back again

We can transfer back and forth from Montgomery to Edwards curves "almost for free":

$$
\begin{aligned}
E_{(a,d)} &\to E_{(A:C)} \\
(y : z) &\mapsto (z + y : z - y), \\
(a, d) &\mapsto \left(\frac{a+d}{2} : \frac{a-d}{4}\right)
\end{aligned}
$$

$$
\begin{aligned}
E_{(A:C)} &\to E_{(a,d)} \\
(x : z) &\mapsto (x - z : x + z), \\
(A : C) &\mapsto (A + 2C, A - 2C)
\end{aligned}
$$

"How to get" an s-isogeny for $s = 2\ell + 1$.

Edwards	Montgomery
Order s point $K_e \in E_{a,d}$.	Order s point $K_m \in E_{(A:C)}$.
$E_{a,d} \xrightarrow{\phi} E_{a',d'}$	$E_A \xrightarrow{\phi} E_{A'}$
$a' := B_z a^s, \quad d' = B_y^8 d^s,$	$A' = (6\sigma + A) \cdot \pi^2$
$B_y = \prod_{i=1}^{\ell} y_{[i]K_e}.$	$\sigma_x = \sum_{i=1}^{\ell} \frac{z_{[i]K}^2 - x_{[i]K}^2}{x_{[i]K} z_{[i]K}}$
$B_z = \prod_{i=1}^{\ell} z_{[i]K_e}.$	$\pi_x = \prod_{i=1}^{\ell} x_{[i]K_m}, \pi_z = \prod_{i=1}^{\ell} z_{[i]K_m}.$

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Eval s-isogeny

Eval an s-isogeny for $s = 2\ell + 1$.

Edwards...

Eval s-isogeny

Eval an s-isogeny for $s = 2\ell + 1$.

It does not works in the sense that there is not an evaluation using only YZ-coordinates

Eval an s-isogeny for $s = 2\ell + 1$.

MontgomeryOrder s point $K_m \in E_{(A:C)}$.Point $Q \in E_{(A:C)}$ not in

$\langle K_m \rangle$.$\phi_{K_m}(\mathcal{X}(Q)) = (x_{Q'} : z_{Q'})$.$x_{Q'} =$

$$x_Q \cdot \left(\prod_{i=1}^{\ell} \left[(x_Q - z_Q)(x_{[i]K_m} + z_{[i]K_m}) + (x_Q + z_Q)(x_{[i]K_m} - z_{[i]K_m}) \right] \right)^2$$

$z_{Q'} =$

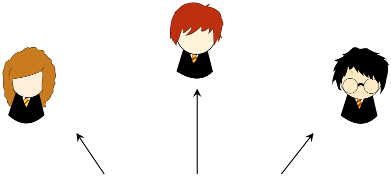
$$z_Q \cdot \left(\prod_{i=1}^{\ell} \left[(x_Q - z_Q)(x_{[i]K_m} + z_{[i]K_m}) - (x_Q + z_Q)(x_{[i]K_m} - z_{[i]K_m}) \right] \right)^2$$

Cost per iteration: 2**M** + 2**S**

Contribution

3. Extended SIDH
   - eSIDH
   - CRT + eSIDH
   - Parallelism in s^e-degree isogenies
4. Edwards-Montgomery Hybridization
   - There and back again
5. Results
   - Proposals
   - Results

Parameters



$$p := 2^{e_2}\ 3^{e_3}\ 5^{e_5}\ f - 1$$

Such that $3^{e_3}5^{e_5} \approx 2^{e_2}$
and $3^{e_3} \approx 5^{e_5}$

Parameters

Choose P_3 and Q_3
such that $\langle P_3, Q_3 \rangle = E[3^{e_3}]$

Choose P_2 and Q_2
such that $\langle P_2, Q_2 \rangle = E[2^{e_2}]$

Choose P_5 and Q_5
such that $\langle P_5, Q_5 \rangle = E[5^{e_5}]$

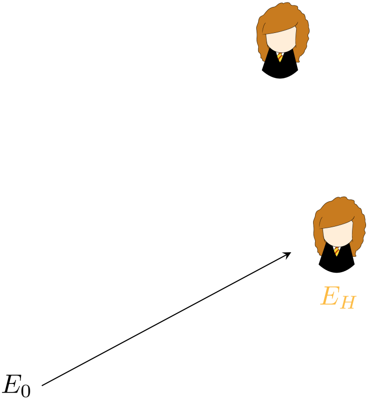
$$p := 2^{e_2} \quad 3^{e_3} \quad 5^{e_5} \quad f - 1$$

Such that $3^{e_3} 5^{e_5} \approx 2^{e_2}$
and $3^{e_3} \approx 5^{e_5}$

Define $S := P_3 + P_5$ and $T := Q_3 + Q_5$
to be the public parameters of Ron and Harry

eSIDH

$K_2 := P_2 + [n_2]Q_2$
Get ϕ_H and E_H

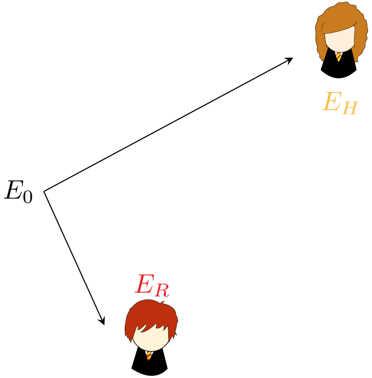


eSIDH

Parallel

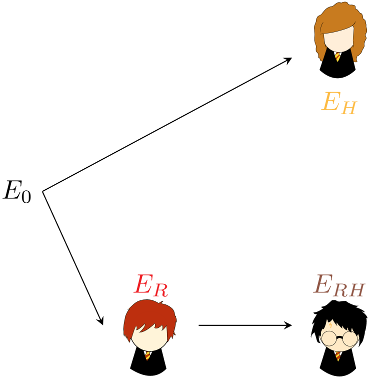
$K_3 := P_3 + [n_3]Q_3$ $\qquad$ $K_5 := P_5 + [n_5]Q_5$

Get ϕ_R and E_R. Send $\phi_R(K_5)$ to Harry.

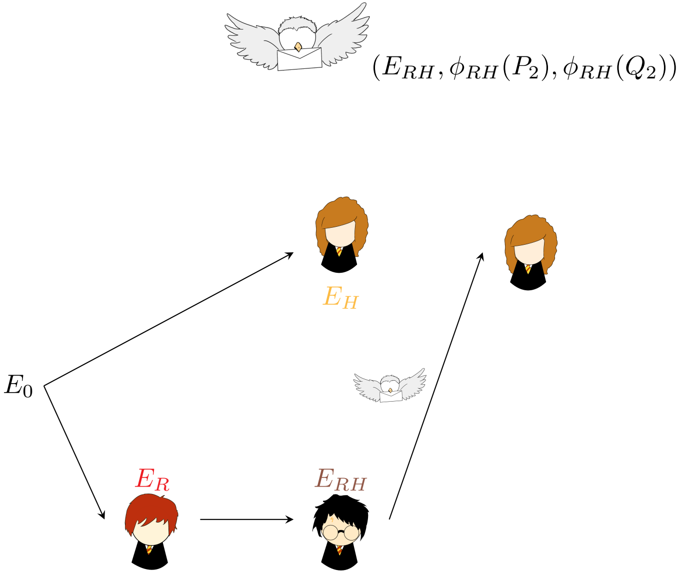


eSIDH

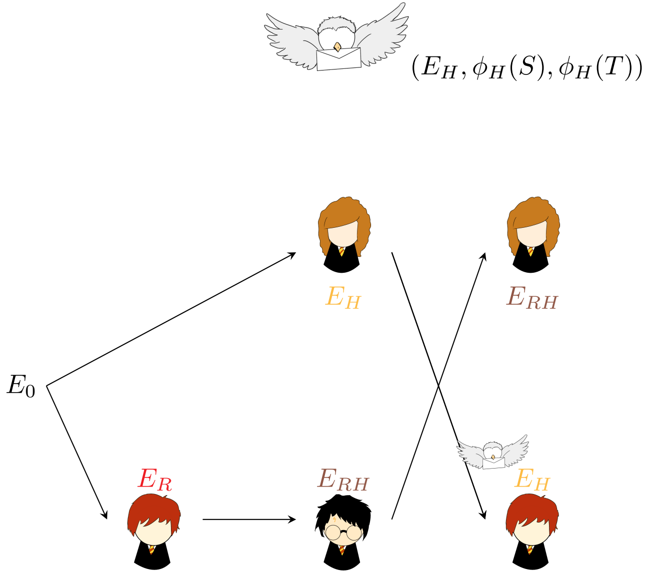
Use $\phi_R(K_5)$ to get E_{RH} and ϕ_{RH}



eSIDH

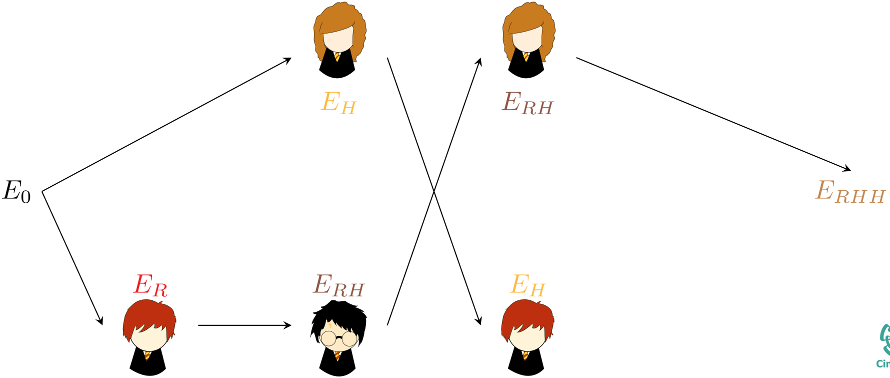


eSIDH



$$K_2' := \phi_{RH}(P_2) + [n_2]\phi_{RH}(Q_2)$$

Get E_{RHH}

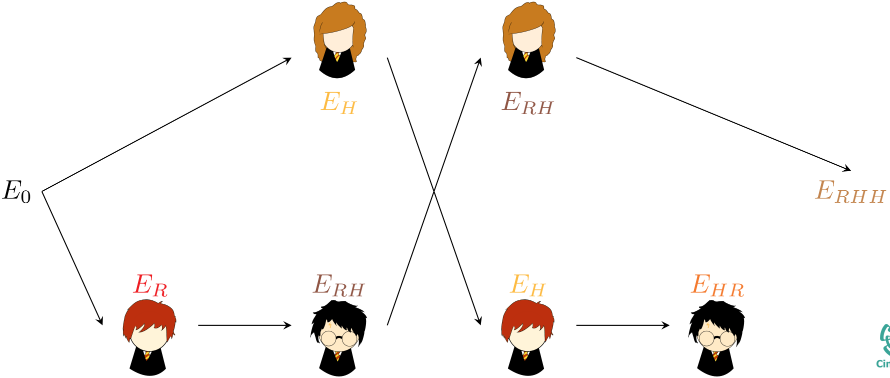


eSIDH

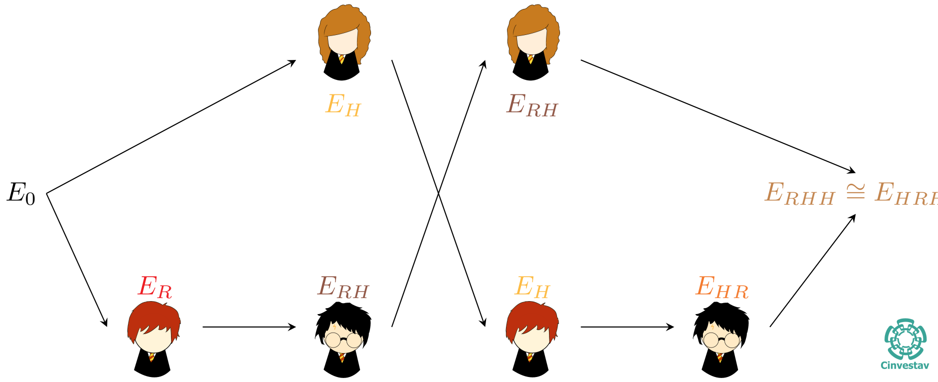
Parallel

$K'_3 := [5^{e_5}](\phi_H(S) + [n_3]\phi_H(T))$ $\quad$ $K'_5 := [3^{e_3}](\phi_H(S) + [n_5]\phi_H(T))$

Get ϕ'_R and E_{HR}. Send $\phi'_R(K'_5)$ to Harry.



Use $\phi'_R(K'_5)$ to get E_{HRH}



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CRT + eSIDH

- Choose $n_3 \in [1, 3^{e_3}]$ and $n_5 \in [1, 5^{e_5}]$ such that $(n_3, 5^{e_5}) = (n_5, 3^{e_3}) = 1$.
- Compute $\hat{n}_3 := n_3^{-1} \mod 5^{e_5}, \quad \hat{n}_5 := n_5^{-1} \mod 3^{e_3}$.
- Finally compute the integer private keys
  - $(\bar{n}_3 := n_3 \cdot \hat{n}_3 \mod 3^{e_3}, \ \bar{n}_5 := n_5 \cdot \hat{n}_5 \mod 5^{e_5})$.
  - $n_{35} := n_3 \cdot \hat{n}_3 \cdot n_5 \cdot \hat{n}_5 \mod (3^{e_3} 5^{e_5})$.

CRT + eSIDH

- Choose $n_3 \in [1, 3^{e_3}]$ and $n_5 \in [1, 5^{e_5}]$ such that $(n_3, 5^{e_5}) = (n_5, 3^{e_3}) = 1$.
- Compute $\hat{n}_3 := n_3^{-1} \mod 5^{e_5}, \quad \hat{n}_5 := n_5^{-1} \mod 3^{e_3}$.
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  - $n_{35} := n_3 \cdot \hat{n}_3 \cdot n_5 \cdot \hat{n}_5 \mod (3^{e_3} 5^{e_5})$.

Key Generation is the same as in previous approach

1. Ron computes: $K_3 := P_3 + [\bar{n}_3]Q_3$,
2. Harry computes: $K_5 := P_5 + [\bar{n}_5]Q_5$.
3. Ron computes: E_R, ϕ_R using K_3.
4. Harry computes: E_{RH}, ϕ_{RH} using $\phi_R(K_5)$.

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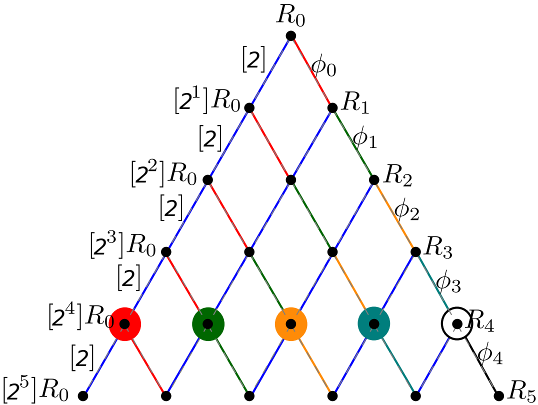
CRT + eSIDH

- Choose $n_3 \in [1, 3^{e_3}]$ and $n_5 \in [1, 5^{e_5}]$ such that $(n_3, 5^{e_5}) = (n_5, 3^{e_3}) = 1$.
- Compute $\hat{n}_3 := n_3^{-1} \mod 5^{e_5}, \quad \hat{n}_5 := n_5^{-1} \mod 3^{e_3}$.
- Finally compute the integer private keys
  - $(\bar{n}_3 := n_3 \cdot \hat{n}_3 \mod 3^{e_3},\ \bar{n}_5 := n_5 \cdot \hat{n}_5 \mod 5^{e_5})$.
  - $n_{35} := n_3 \cdot \hat{n}_3 \cdot n_5 \cdot \hat{n}_5 \mod (3^{e_3} 5^{e_5})$.

Key agreement phase:

1. Ron computes: $K' := \phi_H(P_3) + [n_{35}]\phi_H(Q_3)$,
2. Ron computes: $K'_3 := [5^{e_5}]K'$.
3. Ron computes: E_{HR}, ϕ'_R using K_3.
4. Harry computes: E_{HRH}, ϕ'_{RH} using $\phi'_R(K')$.

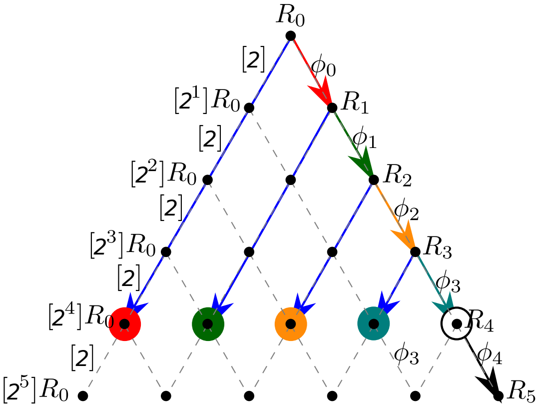


Example for a 2^5-isogeny.

Rules:

- Once you go down, you can't go back.
- The only way to go down along a non-blue line is reaching first the dot rounded by the same color of the line. Example: if you want to go down by a red line, first you need to reach the dot rounded by a red circle.

Parallelism in s^e-degree isogenies



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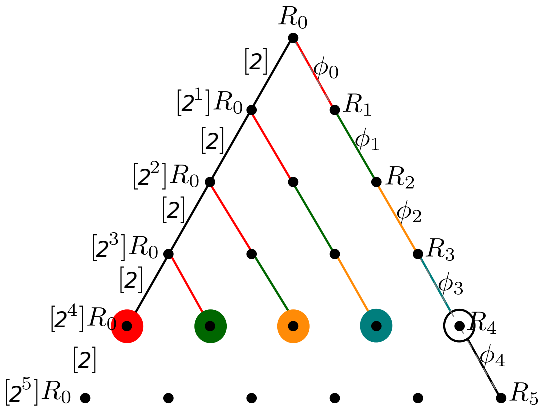
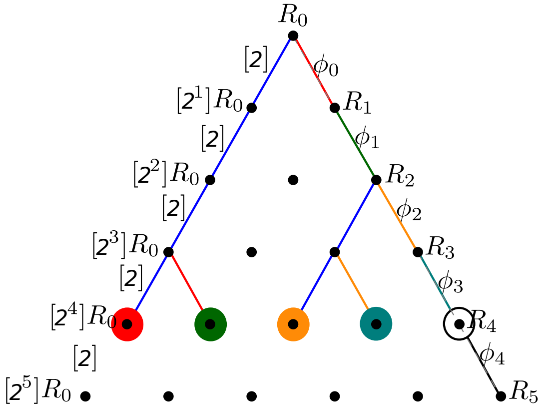


Unbalanced path: Evaluation oriented

Costs:

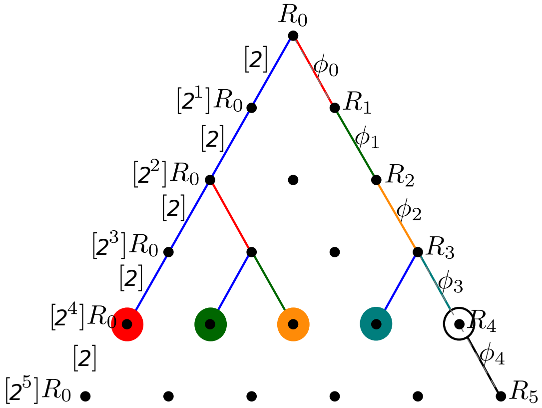
- $[2]$: 4
- Evaluations : 10

Fully parallelizable. (Need more than 250 cores in real life)



Balanced path
Costs:

- $[2]$: 6
- Evaluations : 6



Balanced path
Costs:

- [2] : 6
- Evaluations : 6

There and back again

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There and back again

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- Montgomery evaluation use Kernel points in YZ-Coordinates.

$$
\begin{aligned}
x' =& \mathbf{x_Q} \cdot \left(\prod_{i=1}^{\ell} \left[z_Q y_{[i]P} + y_Q z_{[i]P} \right] \right)^2 \\
z' =& \mathbf{z_Q} \cdot \left(\prod_{i=1}^{\ell} \left[z_Q y_{[i]P} - y_Q z_{[i]P} \right] \right)^2 .
\end{aligned}
$$

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There and back again

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- Once that the Kernel points are in YZ-coordinates it is not necessary to go back to Montgomery anymore.

There and back again

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- Montgomery evaluation use Kernel points in YZ-Coordinates.
- Once that the Kernel points are in YZ-coordinates it is not necessary to go back to Montgomery anymore.
- Translate `xDBL` and `xADD` into `yDBL` and `yADD` respectively to compute $[i]K$.

Proposals

Our proposals	[JAC+17] proposals
$P_{508} = 2^{258}3^{74}5^{57} - 1$	$P_{503} = 2^{250}3^{159} - 1$
$P_{764} = 2^{391}3^{121}5^{78} - 1$	$P_{751} = 2^{372}3^{239} - 1$
$P_{1013} = 2^{512}3^{157}5^{108} - 1$	$P_{964} = 2^{486}3^{301} - 1$
	[ACVCD+18] proposals
$P_{443} = 2^{222}3^{73}5^{45} - 1$	$P_{434} = 2^{216}3^{137} - 1$
$P_{557} = 2^{280}3^{86}5^{61} - 1$	$P_{546} = 2^{273}3^{172} - 1$

Table 1: Our proposals for eSIDH primes in comparison with the current state-of the art

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Table 1: Our proposals for eSIDH primes in comparison with the current state-of the art

- Our primes are Montgomery Friendly so we can achieve a faster modular reduction.
- There are more eSIDH primes than SIDH primes.
- It is possible to improve the security (few bits).

Implementation considerations

- We compare against the recent library version of Costello-Longa-Naehrig instead of the reported one [CLN16].
- We do not compare with results of Faz-López-Ochoa-Rodríguez article [FHLOJRH18] because the specifications submitted to NIST[JAC$^+$17] does not allow the use of all improvements reported by them.
- All the timings were measured using an Intel core i7-6700K processor with micro-architecture Skylake at 4.0 GHz. Using the Clang-3.9 compiler and the flags `-Ofast -fwrapv -fomit-frame-pointer -march=native -madx -mbmi2`.

Arithmetic Results

Operation	[JAC+17] p_{503}	**Ours** p_{509}	[JAC+17] p_{751}	**Ours** p_{765}	**Ours** p_{1013}
Mult $\mathbb{F}_{p^2}$	557	500	1,054	972	1,610
Sqr $\mathbb{F}_{p^2}$	411	370	769	711	1,217
Inv $\mathbb{F}_{p^2}$	110,927	102,530	314,354	250,131	675,623

Operation	[ACVCD+18] p_{434}	**Ours** p_{443}	[ACVCD+18] p_{546}	**Ours** p_{557}
Mult $\mathbb{F}_{p^2}$	509	467	774	680
Sqr $\mathbb{F}_{p^2}$	345	340	519	515
Inv $\mathbb{F}_{p^2}$	79,018	80,253	207,854	154,931

Table 2: Arithmetic cost comparison. Timings are reported in clock cycles measured over a Skylake processor at 4.0GHz.

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Protocol Results

	Alice KeyGen			Bob KeyGen			Alice KeyAgr			Bob KeyAgr		
	NP	**P**	**AF**	**NP**	**P**	**AF**	**NP**	**P**	**AF**	**NP**	**P**	**AF**
P503 [JAC+17]		8.24			9.13			6.70			7.71	
$2^{258} \cdot 3^{74} \cdot 5^{57} \cdot 1 - 1$	7.50	5.92	1.39	8.04	5.46	1.67	6.11	5.38	1.43	7.58	5.55	1.38
P751 [JAC+17]		23.72			26.70			19.38			22.81	
$2^{391} \cdot 3^{121} \cdot 5^{78} \cdot 1 - 1$	22.27	16.72	1.42	24.10	15.43	1.73	18.35	15.32	1.26	22.77	15.78	1.44
$2^{512} \cdot 3^{157} \cdot 5^{108} \cdot 1 - 1$	49.27	36.44		54.79	34.57		40.84	33.26		51.78	35.40	
P434 [ACVCD+18]		5.3			5.9			5.0			5.8	
$2^{222} \cdot 3^{73} \cdot 5^{45} \cdot 1 - 1$	5.93	4.68	1.13	6.60	4.61	1.28	4.79	4.27	1.17	6.17	4.69	1.23
P546 [ACVCD+18]		10.6			11.6			9.9			11.3	
$2^{280} \cdot 3^{86} \cdot 5^{61} \cdot 1 - 1$	11.17	8.63	1.23	12.45	8.29	1.40	9.09	7.83	1.26	11.65	8.48	1.33

Table 3: Performance comparison of the eSIDH against the proposed in [JAC+17] and [ACVCD+18]. The running time is reported in 10^6 clock cycles measured in an Intel Skylake proccessor at 4.0 GHz.Parallel version performance using 3 cores.

Results

Publications

Accepted:

- Gora Adj, Daniel Cervantes-Vázquez, Jesús-Javier Chi-Domínguez, Alfred Menezes and Francisco Rodríguez-Henríquez. *On the cost of computing isogenies between supersingular elliptic curves*. Selected Areas in Cryptology 2018(Conference).

Work in progress:

- Daniel Cervantes-Vázquez, Eduardo Ochoa-Jiménez and Francisco Rodríguez-Henríquez. *A parallel approach for SIDH*.
- Daniel Cervantes-Vázquez, Mathilde Chenu-de-La Morinerie, Luca de Feo, Jesús Chi-Domínguez, Francisco Rodríguez-Henríquez and Ben Smith. *Stronger and Faster Side-Channel Protections for CSIDH*. Submitted.

Future Work

- To implement different parallel strategies and analyze those strategies.
- To study other models to improve performance (Huff, Split/Twisted Normal Form).

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