Implementing pairing-based protocols

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Joint work with:

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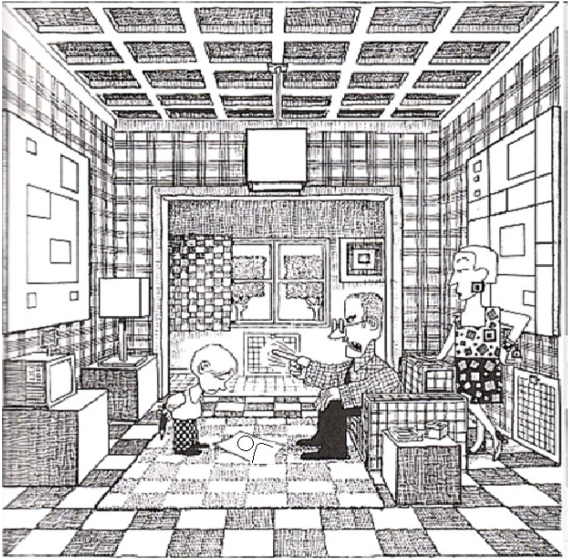
Elliptic curves



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- Let r be a large prime with $r \mid \#E(\mathbb{F}_p)$ and $\gcd(r, p) = 1$. The embedding degree k is the smallest positive integer such that $r \mid (p^k - 1)$

Discrete logarithm cryptography

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- We assume that the discrete logarithm problem (DLP) in $\mathbb{G}_1$ is hard

Bilinear pairings

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Bilinear pairing, basic definitions and properties (1/2)

Bilinear pairings were introduced by the French mathematician André Weil in 1940 under the name of couplages. Here, we define a bilinear pairing, or pairing for short, as a non-degenerate bilinear mapping,

$$\hat{e} : \mathbb{G}_2 \times \mathbb{G}_1 \rightarrow \ \mathbb{G}_T,$$

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Note: There exist also Type 2 and Type 4 pairings. Type 2 pairings can be safely ignored [Chatterjee-Menezes D. Appl. Math 2011]

Bilinear pairing, basic definitions and properties (2/2)

A pairing is non-degenerate iff $\hat{e}(Q, P) \neq 1_{\mathbb{G}_T}$. The most important property of a pairing is its bilinearity, denoted as:

$$\begin{aligned} \hat{e}(Q_1 + Q_2, P) &= \hat{e}(Q_1, P) \cdot \hat{e}(Q_2, P); \\ \hat{e}(Q, P_1 + P_2) &= \hat{e}(Q, P_1) \cdot \hat{e}(Q, P_2). \end{aligned}$$

where $P_1, P_2 \in \mathbb{G}_1$; $Q_1, Q_2 \in \mathbb{G}_2$, and the result is in $\mathbb{G}_T$. From the above property, it follows that for any two integers n_1 and n_2,

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- Intuitively, scalar multiplication in $\mathbb{G}_2$ is much more expensive than in $\mathbb{G}_1$, hence, it is wise to place such operation in the latter group
- It is also believed that an exponentiation in $\mathbb{G}_T$ is cheaper than a pairing computation. Thus, some protocol designers may try to exploit this too

The rise of pairing-based cryptography: 20 years of history

- September 1993: The Menezes-Okamoto-Vanstone (MOV) attack polynomially reduces elliptic curve discrete problems into a discrete problem over a finite extension field using the Weil pairing
- April 1994: Frey and Rück introduced the Tate pairing in cryptography to carry out an attack similar to the MOV one
- January 2000: Sakai-Ohgishi-Kasahara discovered constructive properties of pairings (identity-based key exchange)
- July 2000: Joux presented a one round protocol for tripartite Diffie-Hellman
- August 2001: Boneh and Franklin proposed identity-based encryption using the Weil pairing [4800+ citations]
- December 2001: Boneh, Lynn and Shacham presented short signatures using the Weil pairing[1900+ citations]

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- Excellent timing: Intense debates about the usage of RSA Vs ECC were mostly over after 2000
- Many crypto conferences emerged during the last decade, including Pairing (back in 2007)

Pairing-based cryptography from the protocol design perspective

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- Most protocols use symmetric pairings
- There is no good notion of the individual costs of the main cryptographic blocks within a protocol. As a consequence some protocol designers have a bad intuition of the costs associated to their schemes

A recent example of a practical pairing-based protocol

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In their paper the authors give the following concluding remark,

> *"In this paper we gave the first aggregate signature scheme which is provably secure without random oracles; the first multisignature scheme which is provably secure without random oracles; and the first verifiably encrypted signature scheme which is provably secure without random oracles [...] All our constructions are quite practical."*

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Sequential Aggregate Signatures, Multisignatures, and Verifiably Encrypted Signatures 369

Table 1. Comparison of aggregate signature schemes. Signatures are by l signers; k is the output length of a collision resistant hash function; "R.O." denotes if the security proof uses random oracles. Neven's scheme supports message recovery, which can reduce the effective signature overhead.

Scheme	R.O.	Sequential	Key model	Sig. size	Key size	Verification	Signing
BGLS	YES	NO	Chosen	160 bits	1920 bits	$l+1$ pair.	1 exp.
LMRS-1	YES	YES	Chosen	1024 bits	2048 bits	$2l$ exp.	verify + 1 exp.
LMRS-2	YES	YES	Registered	1024 bits	1024 bits	$4l$ mult.	verify + 1 exp.
Neven	YES	YES	Chosen	1184 bits	1024 bits	$2l$ mult.	verify + 1 exp.
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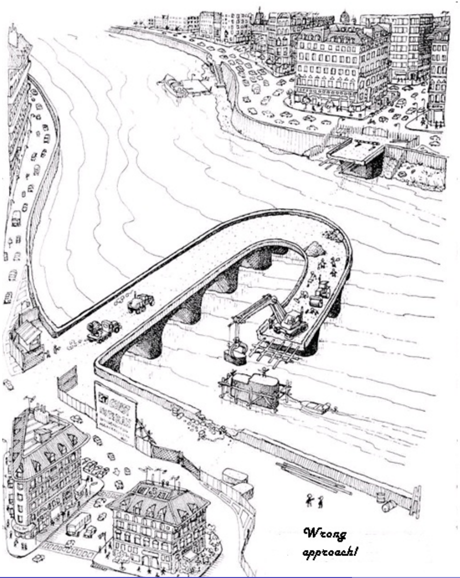
- The authors also pointed out that the cost of hashing to the groups $\mathbb{G}_1$ and $\mathbb{G}_2$ is "an expensive operation". This claim is inexact at best

Pairing-based cryptography from the cryptographic implementation perspective



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- 2010: optimal pairings [Vercauteren IEEE TIT]

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- Several crucial building blocks [notably the map-to-point hash function] have not been subject of a careful C level or assembly implementation
- In spite of some efforts, as of today there are no good benchmarks of the relative computational costs associated to the most important building blocks of pairing-based protocols

Previous work

1. [Pairing 2013]: Chuengsatiansup et al. “PandA: Pairings and Arithmetic”
2. [ACNS 2013]: A. Guillevic. “Comparing the pairing efficiency over composite-Order and prime-order elliptic curves”
3. [ACNS 2013]: Sánchez-Ramírez and RH. “NEON implementation of an attribute-based encryption scheme”
4. [eprint 2012]: M. Scott. “Replacing username/password with software-only two-factor authentication”
5. [IMA 2011]: M. Scott. “On the efficient implementation of pairing-based protocols”
6. [Comp.Comm 2011]: Oliveira et al. “TinyPBC: Pairings for authenticated identity-based non-interactive key distribution in sensor networks”
7. [DCC 2010]: Chatterjee et al. “Comparing two pairing-based aggregate signature schemes”

Cryptographic libraries available

- Charm, a framework for rapidly prototyping cryptosystems
- RELIC is an Efficient Library for Cryptography
- MIRACL Cryptographic SDK
- PBC, The Pairing-Based Cryptography Library

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  Once again, none of these three papers had a section dealing with implementation considerations

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- A plausible conclusion is that everybody is having fun in his/her own way, which is great... but

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- Wouldn't it be also nice to have soon in Pairing a session named something like:
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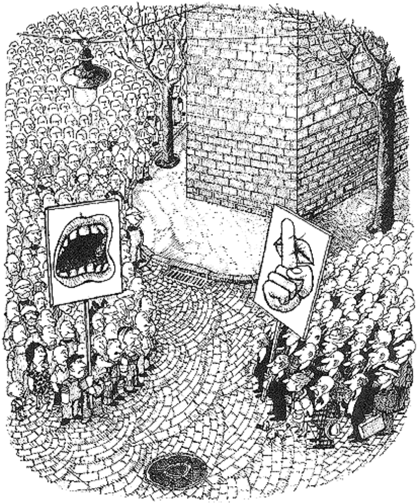
- The purpose of this talk is to present some steps forward in that direction

Implementing popular cryptographic building blocks used in pairing-based protocols



borrowed from Quino.

Barreto-Naehrig curves

Barreto-Naehrig (BN) elliptic curves are a family of elliptic curves with embedding degree $k = 12$ defined by the equation

$$E/\mathbb{F}_p : y^2 = x^3 + b, b \neq 0,$$

where the prime p, the group order $r = \#E(\mathbb{F}_p)$, and the trace of Frobenius t are parametrized as,

$$\begin{aligned} p(u) &= 36u^4 + 36u^3 + 24u^2 + 6u + 1; \\ r(u) &= 36u^4 + 36u^3 + 18u^2 + 6u + 1; \\ t(u) &= 6u^2 + 1, \end{aligned}$$

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BN curves admit a sextic twist curve, defined as $\tilde{E}(\mathbb{F}_{p^2}) : Y^2 = X^3 + b/\xi$, where $\xi \in \mathbb{F}_{p^2}$ is neither a square nor a cube in $\mathbb{F}_{p^2}$.

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3. Compute the scalar multiplication cQ, where the cofactor c is given as, $c = \#\tilde{E}(\mathbb{F}_{p^2})/r$. This scalar multiplication can be greatly accelerated using the Frobenius endomorphism plus lattice reduction techniques [Fuentes-Castañeda, Knapp and RH, SAC 2011]

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- If the point Q is known in advance, the pairing can be computed $\approx 15\%$ faster using precomputation

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- Otherwise, the pairing can still be accelerated by using strategies analogous to multi-exponentiation computations
  [all the point doubling computations in the Miller loop can be shared using a single accumulator]

Popular cryptographic building blocks for pairing-based protocols

Operation	Operation count	10^3 Clock cycles	op/$\mathbb{G}_1$_UK_mul
single pairing (unknown point)	$10,312 m_E + 4,954 r_E$	1,162	5.95
single pairing (known point)	$8,738 m_E + 3,792 r_E$	980	5.03
1 more pairing (unknown point)	$4,604 m_E + 2,301 r_E$	483	2.48
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Timings measured on an Intel Core i7-4770 with the micro-architecture Haswell running at 3.4GHz

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known sc. mult. in $\mathbb{G}_2$, $w = 8$	636	0.77
unknown sc. mult. in $\mathbb{G}_2$, $w = 3$	$1,571$	1.91
known exp. in $\mathbb{G}_T$, $w = 8$	$1,121$	1.36
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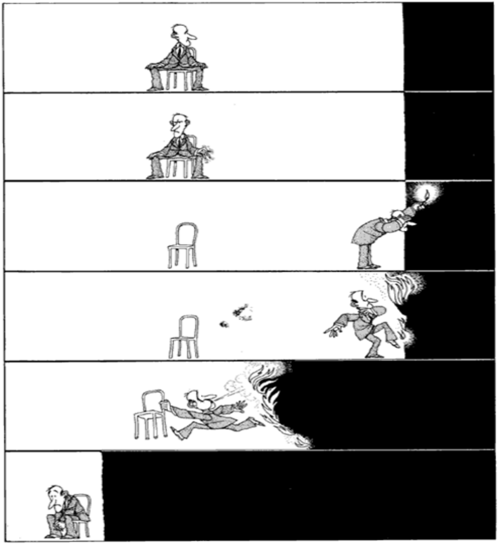
Timings measured on an Exynos 5 Cortex-A15 running at 1.7GHz using NEON as reported in [Sánchez-Ramírez and RH ACNS 2013]

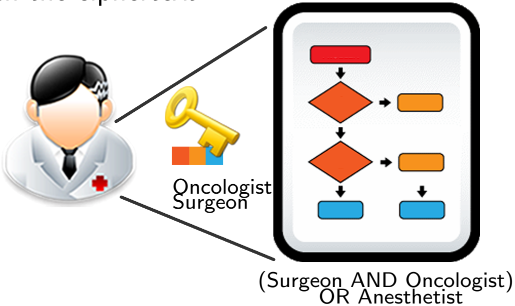
Case Study: Attribute-based encryption



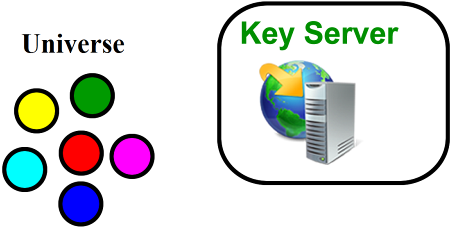
borrowed from Quino.

Attribute-based encryption overview

- In 2004, Sahai and Waters introduced attribute-based encryption (ABE) as a new method for encrypted access control
- In this scheme both, the user's private key and the ciphertext are associated with a set of attributes or with an access policy defined by a set of attributes
- A user can decrypt the ciphertext if her private key satisfies the access policy associated with the ciphertext or covers the set of attributes related with the ciphertext.



Attribute-based encryption overview



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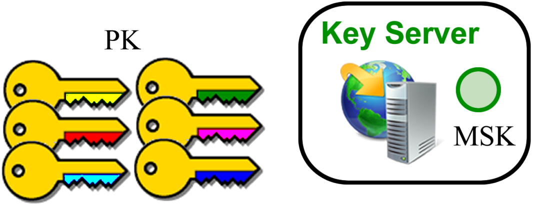

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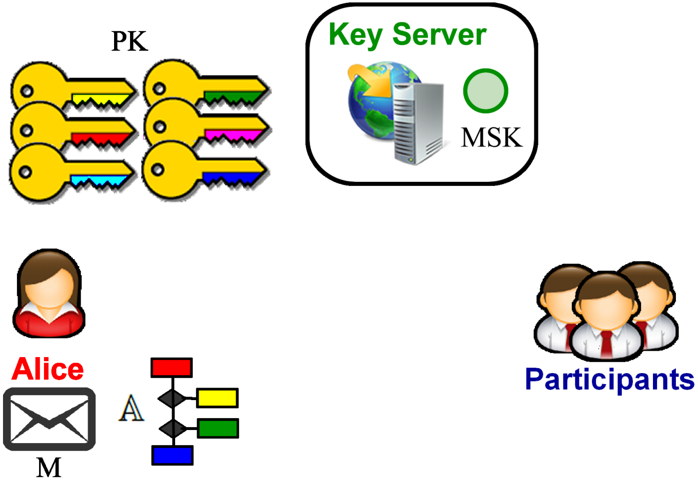

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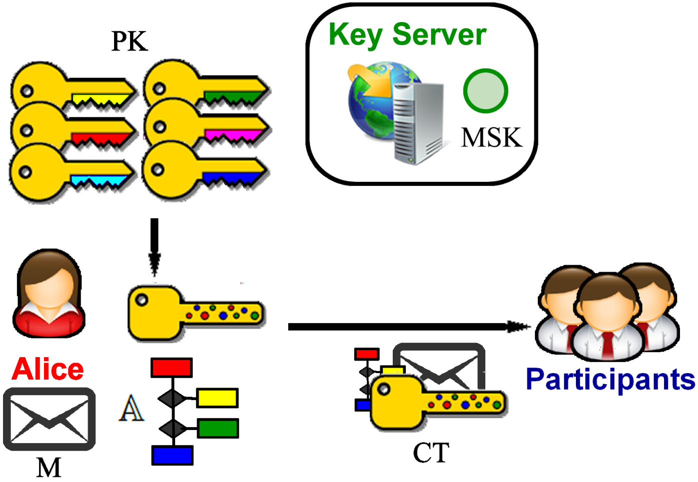

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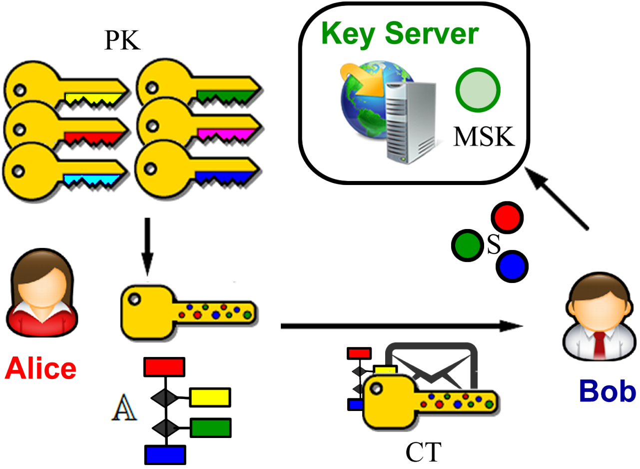

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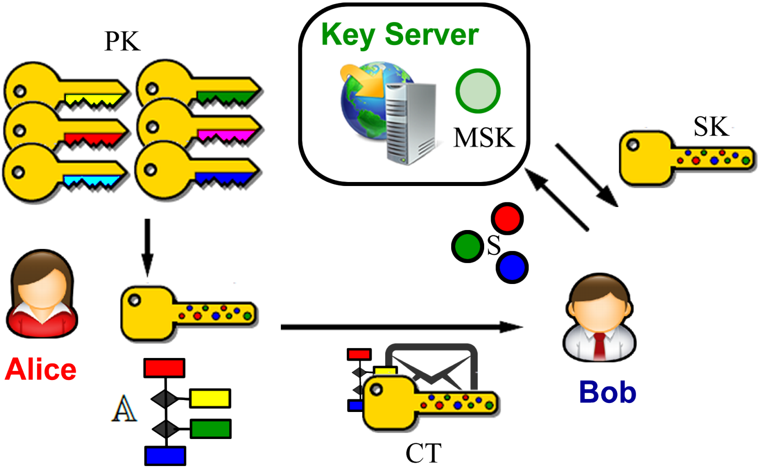

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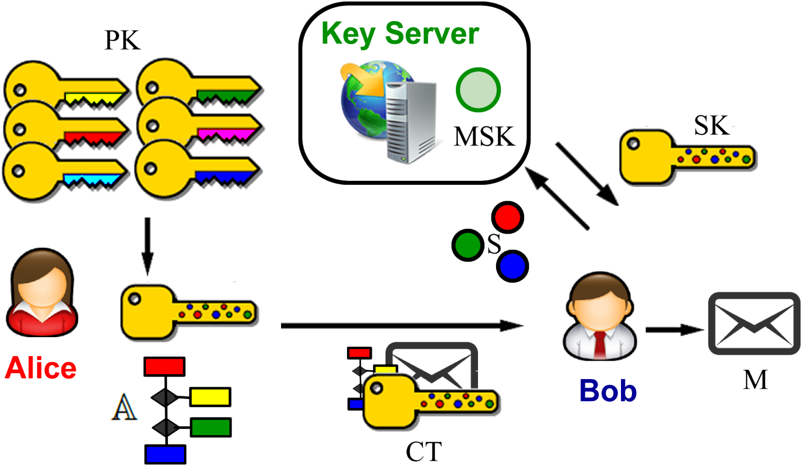

Access Policy

- The access policy $\mathbb{A}$ is initially specified as a boolean formula over a subset of attributes. Let us assume that the number of distinct attributes in that boolean formula is u
- The boolean formula describing the access policy is converted into a Linear Secret-Sharing Scheme (LSSS) matrix $\mathcal{S} \in \mathbb{F}_r$ of size $u \times t$, along with a function ρ that associates rows of $\mathcal{S}$ to attributes in $\mathcal{H}$. Here t is the number of shares to be produced.

$$\mathcal{S} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1t} \\ a_{21} & a_{22} & \dots & a_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ a_{u1} & a_{u2} & \dots & a_{ut} \end{bmatrix} \xrightarrow[\rho(i)]{} \begin{bmatrix} \text{Attribute}_1 \\ \text{Attribute}_2 \\ \vdots \\ \text{Attribute}_u \end{bmatrix}$$

Sharing/recovering a secret in LSSS

- Sharing a secret. Let us consider the column vector $\bar{u} = (s, y_2, \ldots, y_t)$, then $\bar{\lambda} = \mathcal{S}\bar{u}$ is the vector of t shares of the secret s, where λ_i belongs to the attribute $\rho(i)$.

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- Recovering a secret. An LSSS matrix $\mathcal{S}$, is reduced to a square matrix by removing the rows and columns that are unrelated to an authorized set of attributes $\mathcal{H}$. Denote the resulting reduced $v \times v$ matrix as $\tilde{\mathcal{S}} \in \mathbb{F}_r$, with $v \leq u$. Define $\mathcal{I} \subset \{1, 2, \ldots, \ell\}$ as $\mathcal{I} = \{i : \rho(i) \in S\}$. Then, there exist constants $\{\omega_i \in \mathbb{F}_r\}_{i \in I}$ such that

$$\sum_{i \in I} \omega_i \lambda_i = s$$

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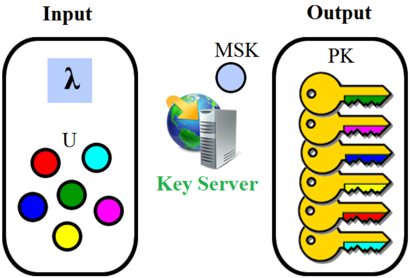
Attribute-based encryption

The four main primitives of Attribute-based encryption are,

- Setup
- Encryption
- Key Generation
- Decryption

Attribute-based encryption: setup

U

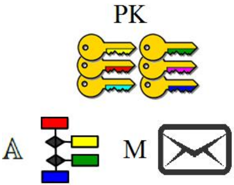


$$\mathbf{PK} = \begin{cases} P, Q, \\ \hat{e}(Q, P)^{\alpha}, \\ aP, \\ H_1, \ldots, H_U \end{cases}$$
$$\mathbf{MSK} = \alpha P.$$

$$\mathbb{G}_1, \mathbb{G}_2$$
$$P \in \mathbb{G}_1, Q \in \mathbb{G}_2$$
$$a, b \in \mathbb{F}_r$$

Attribute-based encryption: encryption

INPUT



OUTPUT

$$\mathrm{PK} = \begin{cases} P, Q, \\ \hat{e}(Q,P)^{\alpha}, \\ aP, \\ H_1, \dots, H_U \end{cases}$$

$\mathbb{A} = (\mathcal{S}, \rho)$
$\mathcal{M}$

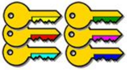
Run a LSSS with t shares (s, λ_i).
Generate $k_1, \dots, k_u \in \mathbb{Z}_r$.

$$\mathrm{CT} = \begin{cases} C = \mathcal{M} \cdot \hat{e}(Q,P)^{\alpha s}, \\ C' = sQ, \\ C_i = \lambda_i(aP) - k_i H_{\rho(i)}, \\ D_i = k_i Q \end{cases}$$

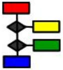
Attribute-based encryption: encryption

INPUT

PK



$\mathbb{A}$



M

Alice

OUTPUT

CT

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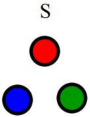
Attribute-based encryption: key generation

INPUT



OUTPUT

$$\mathrm{PK} = \begin{cases} P, Q, \\ \hat{e}(Q, P)^{\alpha}, \\ aP, \\ H_1, \ldots, H_U \end{cases}$$

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Attribute-based encryption: key generation

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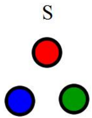
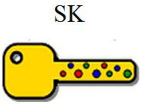


OUTPUT



$$\mathrm{PK} = \begin{cases} P, Q, \\ \hat{e}(Q, P)^{\alpha}, \\ aP, \\ H_1, \ldots, H_U \end{cases}$$

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Select $\tau \in \mathbb{Z}_r$.

$$\mathrm{SK} = \begin{cases} K = \alpha P + \tau(aP), \\ L = \tau Q, \\ \forall x \in S \;\; K_x = \tau H_x \end{cases}$$

Decryption

INPUT

CT

Bob

OUTPUT

$$\text{CT} = \begin{cases} C = \mathcal{M} \cdot e(Q, P)^{\alpha s}, \\ C' = sQ, \\ C_i = \lambda_i(aP) - k_i H_{\rho(i)}, \\ D_i = k_i Q \end{cases}$$

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Decryption

INPUT

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Bob

OUTPUT

$$\frac{\left(e(L, \sum_{i\in\mathcal{I}} \omega_i C_i)\prod_{i\in\mathcal{I}} e(D_i, \omega_i K_{\rho(i)})\right)}{e(C', K)}$$

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OUTPUT

M

$\mathcal{M}$

Table with the timing costs of the Attribute-based protocol

LSSS ABE Protocol	10^3 clock cycles			
	Attributes in $\mathbb{G}_1$		Attributes in $\mathbb{G}_2$	
	Six attributes	Twenty attributes	Six attributes	Twenty attributes
encryption	2,384	7,150	2,921	9,129
key generation	652	1,699	1,326	3,994
decryption ($\Delta = 1$)	4,606	12,776	3,515	9,528
overall cost	7,642	21,625	7,762	22,651
pairing cost	4,378	11,168	3,123	7,043
Pairing cost (%)	57.3	51.6	40.2	31.1

Table: Performance of the ABE protocol primitives (all the timings are given in 10^3 clock cycles)

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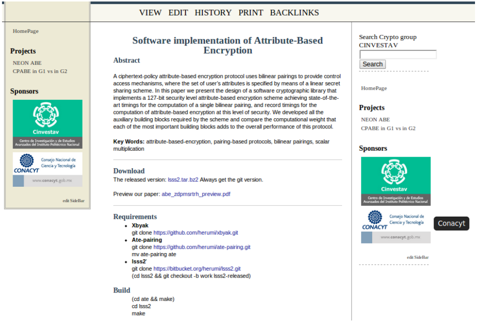
Attribute-based encryption: project web page



Available at: http://sandia.cs.cinvestav.mx/Site/CPABE

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4. To produce a high performance symmetric pairing library with fields of large characteristic and elliptic curves with very low embedding degree

Merci-Gracias-Arigato-Thanks
for your attention

borrowed from Quino. Questions?