Reactive power handling by a multi-objective teaching learning optimizer based on decomposition

Miguel A. Medina, *Student Member*, *IEEE*, Carlos A. Coello Coello, *Fellow*, *IEEE*, Juan M. Ramirez, *Member*, *IEEE*

Abstract—The Teaching Learning-Based Optimization (TLBO) is a population-based optimization algorithm suitable for solving complex problems. TLBO imitates the interaction between a teacher and her/his students. The global solution search process of this approach consists of two phases: the Teacher- and the Learner-Phase. This paper proposes a multi-objective teaching learning algorithm based on decomposition (MOTLA/D) for solving a reactive power handling problem. The proposed method is validated on three test systems, and it is compared with respect to a state-of-the-art multi-objective evolutionary algorithm based on decomposition (MOEA/D).

Index Terms--Optimal power flow, Optimization, Reactive power.

I. INTRODUCTION

OPTIMAL reactive power handling (ORP) plays a significant role in the secure operation of power systems. One of the main tasks of a power system operator is to manage the system in such a way that its operation is safe and reliable. Its main aim is to determine the optimal operating capacity and the physical distribution of the compensation devices such as voltage rating of generators, reactive power injection of shunt capacitors/reactors, and tap ratios of the tap setting transformers, in order to ensure a satisfactory voltage profile, while minimizing the transmission losses. Active power line losses are small while reactive power line losses are large. Reducing the reactive power losses enables more active power to be transferred over a single line. Due to the continuous growth in the demand for electricity with unmatched generation and transmission capacity expansion, voltage stability has emerged as a challenge to power system planning and operation. Therefore, a voltage stability index should also be considered as an objective of the ORP problem.

It is important that each system and control area handle capacitive and inductive reactive resources at proper levels to maintain the voltages within established high and low limits.

Reactive generation scheduling, transmission and switching, and load shedding, if necessary, should be implemented to maintain these levels. Likewise, each control area should provide its reactive power requirements, including appropriate reserves to protect the voltage levels for contingency conditions.

The optimal reactive power problem is a nonlinear, non-convex, over-determined system, a large-scale optimization problem with both continuous and discrete variables; additionally, its high dimensionality represents a major difficulty. This problem is quite important for power system security. In this paper, the basic objective is to estimate proper adjustments on the control variables, such as generator bus voltages and tap setting transformers that help to maintain an acceptable voltage profile and minimize the reactive power losses; one voltage stability metric (L_{index}) is also used. Thus, an optimal formulation that contributes to attain these purposes becomes appropriate. In general, it may include several objective functions, possibly in conflict among them.

Such kind of optimization problem has a set of possible solutions (named *Pareto optimal set*), which represents the best commitment (feasible) among the objectives [1]. Several optimization techniques have been proposed to solve such optimal reactive power problems. From them, two major approaches may be identified [2]:

(1) The first approach is based on the use of evolutionary algorithms such as Differential Evolution (DE) [3], Non-dominated Sorting Genetic Algorithm II (NSGA-II) [4], Particle Swarm Optimization (PSO) [5], an Improved Hybrid Evolutionary Programming Technique [6], and Artificial Bee Colony Algorithm (ABC) [7].

(2) The second approach is based on conventional methods. They include Gradient-based Methods, Non-Linear Programming (NLP), Quadratic Programming (QP), Linear Programming (LP) and Interior Point Methods [8-12], the Weighting Method [13], and the ε-Constraint Method [14].

These conventional methods are based on an estimation of the global minimum. However, due to difficulties of differentiability, non-linearity, and non-convexity, these methods do not guarantee reaching the global optimum [15]. Thus, these methods present limitations when dealing with certain types of problems. For instance, they cannot be used when the objective function is not available in an algebraic form. This has motivated the development of alternative

methods, such as metheuristics. Over the years, meta-heuristics (from which evolutionary algorithms is a particular subclass) have become a popular choice for solving complex optimization problems, due to their flexibility, generality (they are less sensitive to the actual shape or continuity of the Pareto front than conventional methods) and ease of use. Additionally, most meta-heuristics require little or no specific domain knowledge.

In this paper, a multi-objective teaching learning algorithm based on decomposition (MOTLA/D) is proposed for solving the reactive power handling problem. In order to minimize the reactive power losses and a voltage stability index [21], the proposed algorithm estimates the following optimal values: (*i*) generator bus voltages; (*ii*) tap setting transformers. The effectiveness of the proposed approach is demonstrated and compared with respect to the multi-objective evolutionary algorithm based on decomposition (MOEA/D) [16], which is representative of the state-of-the-art on the subject; both methods are applied on three test systems: 9-, 26-, and 118-buses.

The rest of the paper is organized as follows. Section II exposes a basic background. In Section III, the general framework of the proposed approach is summarized. Section IV presents the problem formulation and a short description of the test systems. Results of a comparative study are presented in Section V. Finally, conclusions are provided in Section VI.

II. BASIC CONCEPTS

A. Multi-objective optimization

A multi-objective optimization problem (MOP) may be formulated as follows,

$$\min \quad F(x) = \{f_1(x),..., f_k(x)\}$$
$$\text{subject to} \quad x \in \Omega \tag{1}$$

where x is the vector of decision variables, and Ω is the feasible region within decision variable space. $F : \Omega \to \Re^k$ is defined as the mapping of k objective functions.

In multi-objective optimization, the goal is to find the best possible trade off among the objectives since, frequently, one objective can be improved only at the expense of worsening another. To describe the concept of optimality for problem (1) the following definitions are provided [17].

Definition 1. Let $x, y \in \Omega$, such that $x \neq y$, we say that x dominates y (denoted by $x \prec y$) if and only if, $f_i(x) \leq f_i(y)$ for all $i = 1, ..., k$.

Definition 2. Let $x^* \in \Omega$, we say that x^* is a Pareto optimal solution, if there is no other solution $y \in \Omega$ such that $y \prec x^*$.

Definition 3. The Pareto Optimal Set (P^S) is defined by $P^S = \{x \in \Omega \mid x \text{ is Pareto Optimal Solution}\}$, while its image $P^F = \{F(x) \mid x \in P^S\}$ is called the Pareto Optimal Front.

B. Decomposition of a multi-objective optimization problem

There are several approaches for transforming a MOP into a number of scalar optimization problems, which have been described in detail in [18]. Usually, these methods use a weighting vector to define a scalar function and, under certain assumptions, a Pareto optimal solution is achieved by minimizing such function [17]. In this paper, the Tchebycheff's approach is used to decompose a MOP. In this approach, the scalar optimization problem can be stated as [18]:

$$\text{Minimize} \quad g\left(x \mid w, z^*\right) = \max_{i \in \{1,..,k\}} \left\{ w_i \left| f_i(x) - z_i^* \right| \right\} \tag{2}$$
$$\text{Subject to} \quad x \in \Omega$$

where $w = (w_1,..., w_k)$ is a weighting vector and $w_i \geq 0$ for all $i = 1,..., k$, $\sum_{i=1}^{k} w_i = 1$; $z^* = (z_1^*,...., z_k^*)$ represents the reference point, i.e., $z_i^* = \min\{f_i(x) \mid x \in \Omega\}$, for $i = 1,..., k$.

For each Pareto optimal solution x^* there exists a weighting vector w such that x^* is the optimal solution of (2), and each optimal solution is a Pareto optimal solution for (1). Therefore, it is possible to obtain different Pareto optimal solutions using different weighting vectors w [16].

C. Teaching learning based optimization

The original teaching learning based optimization (TLBO) algorithm was proposed by Rao et al. [19], to calculate global solutions for continuous non-linear functions. In optimization algorithms, the population consists of different design variables. In TLBO, the design variables are analogous to different subjects offered to learners. The learners' grade is analogous to the '*fitness*' as in any other evolutionary algorithm, and the teacher is considered to be the best solution obtained so far [19]. Hence, the TLBO is based on two main phases: the *teacher phase*, which involves learning from the teacher, and the *learner phase*, which involves learning through the interaction among learners.

Teacher phase

In TLBO, each class consists of a number of learners with different grades; the learner with the best grade is selected as the teacher. A teacher will try to improve the mean of the class toward their own level, according to his/her capability.

Let M_i be the mean of the class and $T_{best,i}$ be the best solution so far and, therefore, the teacher in the *i-th* iteration. Hence, $T_{best,i}$ will try to move the mean of the class M_i towards its own level. Thus, the new mean will be $T_{best,i}$, designated as $M_{new,i}$. The difference between the mean of the class (M_i) and the new mean ($M_{new,i}$) is expressed by [19]

$$\Delta_i = r_i \left(M_{new,i} - T_F M_i \right) \tag{3}$$

where T_F is a teaching factor that weights the current mean value. The value of T_F can be either 1 or 2, which is determined randomly with equal probability as T_F = round [1 + rand(0,1)]. r_i is a random number within [0, 1]. TLBO uses the current best solution to improve the existing solution, thereby increasing the convergence rate [19]. The difference (3) updates the current solution according to the following expression [19]

$$x_{new} = x_{old} + \Delta \tag{4}$$

x_{new} is accepted if it improves the function value.

Learner phase

A learner interacts randomly with other learners through group discussions, presentations, formal communications, etc., [19]. Thus, each learner may acquire new knowledge if the

others have more knowledge than him/her. The modification of the Learners is expressed as follows [19].

> *for $i = 1$ to number of learners*
> *Randomly* select one learner j, such that $x_i \neq x_j$
>
> *if* $f(x_i) < f(x_j)$
> $\qquad x_{new,i} = x_i + r_i(x_i - x_j)$ $\qquad$ (5)
> *else*
> $\qquad x_{new,i} = x_i + r_i(x_j - x_i)$
> *end* if
> *end for*

x_{new} is accepted if it accomplishes a better objective function value. The new solutions (x_{new}) update the initial learners and the teaching-learning process continues until the stopping criterion is achieved.

Metaheuristics, in general, require parameters that affect their performance. For example, differential evolution (DE) depends on the mutation strategy adopted, and on its intrinsic control parameters such as its scaling factor (F_s) and the crossover rate (P_{cr}); particle swarm optimizers (PSO) require learning factors, the variation of the inertia weight and the velocity's maximum value; ABC requires a limit value. In contrast, TLBO does not require any specific parameters to be tuned, which facilitates its implementation and use.

In this paper, an extension of the TLBO algorithm for multi-objective optimization based on decomposition is proposed, and it is applied to power systems optimization.

III. MULTI-OBJECTIVE TEACHING LEARNING BASED ON DECOMPOSITION

The proposed Multi-Objective Teaching Learning Algorithm based on Decomposition (MOTLA/D) utilizes the Tchebycheff's approach (see eq. (2)), to decompose the MOP into N scalar optimization sub-problems.

Let $w^1,...,w^N$ be a set of evenly spread weighting vectors and z^* be the reference point. Hence, using eq. (2) the objective function of the j-th sub-problem becomes $g(x|w^j,z^*) = \max_{i \in \{1,...,k\}} \{w_i^j |f_i(x) - z_i^*|\}$, with $w^j = \{w_1^j,...,w_k^j\}$ and $j = 1,..., N$. The proposed approach looks for the sequential minimization of these sub-problems. Similar to MOEA/D [16], neighborhood relationships among these sub-problems are defined by computing Euclidean distances between weighting vectors. A neighborhood to the weighting vector w^j is defined as the set of its closest weighting vectors in $\{w^1,...,w^N\}$.

In MOTLA/D, the size of the neighborhood becomes the number of learners in the class. For the j-th sub-problem, this class can be expressed as,

$$C_{jth} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,D} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T_{size},1} & x_{T_{size},2} & \cdots & x_{T_{size},D} \end{bmatrix} \qquad (6)$$

where the subscript D is the number of design variables, and T_{size} is the size of the neighborhood Ω_T. T_{size} is the main control parameter in MOTLA/D. If T_{size} is too small, the algorithm lacks the ability to explore new regions in the searching space. On the other hand, if T_{size} is too large, the exploitation ability of the algorithm is weakened.

Firstly, a randomly distributed initial population within the valid parameters' interval is generated,

$$x_{j,d} = x_d^{min} + rand(0,1) \cdot (x_d^{max} - x_d^{min}), \quad j = 1,..,N$$
$$d = 1,..,D \qquad (7)$$

where N is the number of sub-problems (i.e., number of learners) and D is the number of decision variables (i.e., the number of subjects offered to the learners). x_d^{min} and x_d^{max} are the lower and upper bounds of parameter d, respectively.

The fitness evaluation is based on scaling functions with uniformly distributed weighting vectors. As in [16], the weighting vectors are generated from the following expressions,

$$w_1 + w_2 + ... + w_k = 1 \qquad (8)$$

$$w_i \in \left\{0, \frac{1}{I}, \frac{2}{I}, ..., \frac{I}{I}\right\}, \quad i = 1, 2, ..., k \qquad (9)$$

where I is a user-defined positive integer. For example, for $k = 2$ (i.e., two objective functions), if I is specified as 100, then 101 weighting vectors (0,1), (0.01,0.99),.., (1,0) are used. This method for generating weighting vectors works well for the formulation in this paper. However, other methods can be used [16]. The main phases of the proposed MOTLA/D may be summarized as follows.

Teacher phase

Within the *teacher phase*, the mean of the class for each design variable is evaluated,

$$M = [m_1, m_2, ..., m_D] \qquad (10)$$

For the j-th sub-problem the *teacher* (M_{new}) represents the best learner of the class C$_{jth}$. Thus,

$$M_{new} = \{x_j \Big| \min_{x_j \in \Omega_T} g(x_j|w^j, z^*)\} \qquad (11)$$

Using the difference between two means, eq. (3), the *teacher* will try to improve the mean of the class (M) taking it towards its own level (M_{new}). The difference modifies the j-th learner (x_j) in order to generate a new solution (x_{newT}) as follows:

$$x_{newT} = x_j + \Delta \qquad (12)$$

Learner phase

The *learner phase* generates a new solution (x_{newL}) by randomly selecting another learner x_i, such that $i \neq j$. This may be expressed by,

> *if* $f(x_j) < f(x_i)$
> $\qquad x_{newL} = x_j + r_i(x_j - x_i)$
> *else* $\qquad (13)$
> $\qquad x_{newL} = x_j + r_i(x_i - x_j)$
> *end*

MOTLA/D generates one offspring by recombining the previous solutions: the *teacher phase* and the *learner phase*. Particularly, the algorithm of MOTLA/D crosses each vector as follows,

$$for\ d = 1\ to\ D$$
$$if\ rand \le 0.5$$
$$x_{new,d} = x_{newT,d}$$
$$else \qquad\qquad (14)$$
$$x_{new,d} = x_{newL,d}$$
$$end\ if$$
$$end\ for$$

Crossover is applied for each of the D decision variables. Additionally, a polynomial mutation operator is applied to maintain the solutions' diversity. The operator uses the polynomial distribution,

$$\delta_d = \begin{cases} (2r_d)^{\frac{1}{\mu+1}} - 1, & if\ r_d < 0.5 \\ 1 - [2(1-r_d)]^{\frac{1}{\mu-1}}, & if\ r_d \ge 0.5 \end{cases} \qquad (15)$$

where r_d is a uniformly distributed random number in the interval [0, 1], and μ is a mutation distribution index. The mutated element is given by,

$$x_{new,d} = x_{new,d} + \left[x_{ub,d} - x_{lb,d} \right] \delta_d \qquad (16)$$

where $x_{lb,d}$ and $x_{ub,d}$ are the lower and upper limits for the d-th decision variable, respectively.

For the new solution (x_{new}), if one or more variables lie outside Ω, then the d-th value of x_{new} is reset as follows,

$$x_{new,d} = \begin{cases} x_{lb,d}, & if\ x_{new,d} \le x_{lb,d} \\ x_{ub,d}, & if\ x_{new,d} \ge x_{ub,d} \end{cases} \qquad (17)$$

The new solution (x_{new}) is accepted if it improves the function value and replaces the old one (x_j). The MOTLA/D implementation may be summarized as follows.

Step 1) Initialization

Step1.1) Generate an initial population of N points $\{x_1,...,x_N\}$ and evaluate its individuals $F(x_1), .., F(x_N)$.

Step1.2) Initialize the reference point $z^* = \left(z_1^*,....,z_k^* \right)$, where $z_i^* = \min\{ f_i(x) | x \in \Omega \}$, for $i = 1,...,k$.

Step1.3) Set $t = 1$ and generate a well-distributed set of N weighting vectors $w^j = (w_1^j,...,w_k^j)$, $j = 1,.., N$.

Step1.4) In order to define the neighborhood Ω_T for each vector, compute the Euclidian distances between any two weighting vectors.

Step 2) Teaching learning process

For j = 1 to N do

Step2.1) From the teacher phase, generate a solution x_{newT} according to (12). Then, from the learner phase, generate a solution x_{newL} according to (13).

Step2.2) Recombine solutions x_{newT} and x_{newL} (i.e., apply crossover between them) in order to generate a new solution x_{new} according to (14). Then, apply the mutation operator (16). If an element of x_{new} lies outside Ω, its value is reset according to (17).

Step2.4) Update the reference point z^*: if $f_i(x_{new}) < z_i^*$ then $z_i^* = f_i(x_{new})$ for each $i = 1,.., k$.

Step2.5) Update the population:

if $g(x_{new}|w^j, z^*) \le g(x_j|w^j, z^*)$, then $x_j = x_{new}$ and

$F(x_j) = F(x_{new})$.

Step 3) Stopping Criterion: If $t < N_{gen}$ (number of generations), then $t = t + 1$ and go to **Step 2.** Otherwise, stop MOTLA/D and report as the output of the algorithm: $x_1,...,x_N$ and $F(x_1),...,F(x_N)$.

For the previously described algorithm, the main computational burden lies on Step 2, where MOTLA/D generates N new solutions. Step 2.1 just randomly picks two solutions in the *learner phase*. Step 2.4 performs $O(k)$ comparisons and assignments, where k is the number of objectives. Step 2.5 requires $O(kT_{size})$ basic operations since its major cost is to compute the value of $g(x|w, z^*)$ for T_{size} solutions; the computation of such value requires $O(k)$ basic operations. Therefore, the computational complexity of step 2 is $O(kNT_{size})$.

IV. PROBLEM STATEMENT

In this paper, a reactive power system problem is approached, which may be stated as an optimization problem where two objective functions are minimized, while satisfying a number of equality and inequality constraints. The following objective functions are minimized: (*i*) the reactive power losses; and (*ii*) the voltage stability index L_{index} [21].

A. Objective functions

A.1 Reactive power losses

One important issue in power transmission is the high reactive power losses on the highly loaded lines, with the consequent transmission capacity reduction. Therefore, the reactive power losses minimization is selected as one objective function. Losses are evaluated by the following expression,

Losses for a single line

$$Q_{VAR,i} = X_i\ |I_i|^2 = X_i \left| \frac{(V_{ei} - V_{ri})^2}{X_i^2} \right| \qquad (18)$$

where V_{ei} and V_{ri} are the sending and receiving voltages, respectively; X_i is the line reactance; I_i is the current through the transmission line.

Objective function for the power system losses:

$$f_1 = \sum_{i=1}^{nl} Q_{VAR,i} \qquad (19)$$

where nl is the number of transmission lines. Reducing the reactive power losses enables more active power to be transferred over the line.

A.2 Voltage stability index

A conventional way for the voltage stability assessment is the use of indexes, which estimate the proximity to the voltage collapse and determine which are the buses exhibiting weak stability [20].

Nowadays, there are a variety of indexes that help to assess the steady state voltage stability. In this case, the voltage stability index L_{index} is used [21]. This index is able to evaluate

the steady state voltage stability margin of each bus. The L_{index} value lies between 0 (no load) and 1 (voltage collapse). This value implicitly includes the load effect. The bus with the highest L_{index} value will be the most vulnerable, and therefore, this method helps to identify weak areas that require a critical support of reactive power. The L_{index} is calculated in the following way [21].

The network equations in terms of the bus admittance matrix may be written as,

$$I_{bus} = Y_{bus} V_{bus} \qquad (20)$$

The buses are broken down into two categories: (*i*) the set of load buses (α_L); and (*ii*) the set of generator buses (α_G). Thus, equation (20) becomes,

$$\begin{bmatrix} I^L \\ I^G \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix} \cdot \begin{bmatrix} V^L \\ V^G \end{bmatrix} \qquad (21)$$

It is assumed that the transmission system is linear and allows a representation in terms of a hybrid matrix *H*,

$$\begin{bmatrix} V^L \\ I^G \end{bmatrix} = H \cdot \begin{bmatrix} I^L \\ V^G \end{bmatrix} = \begin{bmatrix} Z^{LL} & F^{LG} \\ K^{GL} & Y^{GG} \end{bmatrix} \cdot \begin{bmatrix} I^L \\ V^G \end{bmatrix} \qquad (22)$$

where V^L and I^L are voltage and current vectors for load buses; V^G and I^G are voltage and current vectors for generator buses; Z^{LL}, F^{LG}, K^{GL}, Y^{GG} are sub-matrices of the hybrid matrix *H*.

Matrix *H* can be evaluated from the admittance matrix (Y_{bus}) by a partial inversion, where the voltage vector associated to the load buses (V^L) is exchanged with the corresponding current vector (I^L). Thus, a voltage stability index for the load buses is defined, namely L_j [21],

$$L_j = \left| 1 - \frac{\sum_{i \in \alpha_G} F_{ji} V_i}{V_j} \right| \qquad (23)$$

For stable conditions, $0 \leq L_j \leq 1$ must not be violated for any *j*. Hence, a global index L_{index} describing the whole system's stability is defined by [21],

$$f_2 = L_{index} = \max_{j \in \alpha_L} \left(L_j \right) \qquad (24)$$

Pragmatically, L_{index} must be lower than a given threshold value. The predetermined threshold value is specified depending on the system configuration and on the utility policy regarding service quality and allowable margin. Thus, the L_{index} in (24) is associated with the worst bus in the sense of voltage stability. The minimization of f_2 implies to move such bus toward a less stressed condition.

B. Constraints

B.1 Equality constraints

The equality constraints are the balance of the active and reactive power described by the set of power flow equations. They may be expressed as follows,

$$P_{gi} - P_{di} - \sum_{j=1}^{N_b} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) = 0 \qquad (25)$$

$$Q_{gi} - Q_{di} - \sum_{j=1}^{N_b} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) = 0 \qquad (26)$$

where, N_b is the number of buses, P_{gi} is the *i-th* active power generation, Q_{gi} is the *i-th* reactive power generation, P_{di} is the *i-th* active power load, Q_{di} is the *i-th* reactive power load, and $|Y_{ij}|$ is the *ij-th* element of the bus admittance matrix. These equality constraints are handled within the power flow calculations.

B.2 Inequality constraints

a) Generators: these constraints are associated to the generator voltage (V_g), active power (P_g), and reactive power (Q_g),

$$V_{gi}^{\min} \leq V_{gi} \leq V_{gi}^{\max}, \quad i = 1, ..., N_g \qquad (27)$$

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, \quad i = 1, ..., N_g \qquad (28)$$

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max}, \quad i = 1, ..., N_g \qquad (29)$$

where, N_g is the number of generators. The reactive power generation (Q_g) is restricted within the power flow program.

b) Transformers: transformer tap setting,

$$T_i^{\min} \leq T_i \leq T_i^{\max}, \quad i = 1, ..., N_t \qquad (30)$$

where N_t is the number of tap changing transformers.

In the test procedure, the generator voltages are allowed to vary within the interval [1.0, 1.05]. Actually, both limits will depend on the operating point, reactive power availability, tap positions, etc., thus, this interval may be modified. The active/reactive minimization losses tend to take the voltages to the upper limits. If this fact actually leads to inconveniences (such as insulations' stress), as indicated above, one alternative could be to limit the generator voltage's upper bound.

C. Decision variables

The decision variables include the generator voltage V_g, and the transformer tap setting (*T*),

$$x = [V_{g_1}, ..., V_{g_{N_g}}, T_1, ..., T_{N_t}] \qquad (31)$$

It is worth noting that the decision variables are self-constrained by the optimization algorithm.

D. Case studies

This paper compares the effectiveness and performance of the proposed algorithm with respect to that of the MOEA/D. Both MOTLA/D and MOEA/D have been applied to three test systems. In the first case study, we consider the nine-bus test system; this system consists of 9 transmission lines and 3 generating units. The system model and data can be found in [22]. The second case study is related to the IEEE-26 bus test system, which has 26 buses, 46 branches, 6 generators, 7 transformers, and 9 shunt capacitors. The detailed data of this problem can be found in [23]. Finally, in the third case study, the IEEE 118-bus test system is used. The system has 54 generator buses, 64 load buses and 186 transmission lines with 9 tap setting transformers. The complete system data are taken from [24]. For each case study, 20 independent runs are performed. The number of sub-problems considered by each algorithm are 100 for cases 1 and 2, and 200 for case 3. It is worth mentioning that the stopping criterion of each algorithm is the number of generations N_{gen}, (120, 180, and 200 generations for cases 1, 2 and 3, respectively). For all test instances, the control parameter settings utilized by the MOTLA/D and MOEA/D are summarized in the following. The neighborhood size (T_{size}), is 30. The distribution index (μ), used in the polynomial mutation, is 20. The parameter of scale factor (F_s) associated with MOEA/D, which represents the amount of perturbation added to the main parent, is 0.5. The Crossover rate (P_{cr}) associated with MOEA/D, which

determines the quantity of elements to be exchanged by the crossover operator, is 1. Finally, a mutation rate $P_m=1/n$ is taken into account, where n is the number of decision variables. This parameter indicates the probability that each decision variable has of being changed.

E. Performance measures

There are two goals in multi-objective optimization: (*a*) to achieve convergence to the Pareto-optimal set; and (*b*) to obtain a well-distributed set of solutions along the Pareto front. These two tasks cannot be measured adequately by one performance measure each. Therefore, in order to assess the algorithms' performance two performance measures are adopted.

E.1 Coverage of two sets

This performance measure was proposed by Zitzler et al. [25]. It compares two sets of non-dominated solutions (*A*, *B*) and outputs the percentage of individuals in one set dominated by the individuals on the other set. This performance measure is defined as,

$$C(A,B) = \frac{\left|\{b \in B | \exists\, a \in A : a \preceq B\}\right|}{|B|} \quad (32)$$

The value $C(A,B) = 1$ means that all points in *B* are dominated by or equal to all points in *A*. $C(A,B) = 0$ represents the situation when none of the solutions in B are covered by the set A. Note that both $C(A,B)$ and $C(B,A)$ have to be considered, since $C(A,B)$ is not necessarily equal to $1 - C(B,A)$. When $C(A,B) = 1$ and $C(B,A) = 0$ then, we say that the solutions in *A* completely dominate the solutions in *B* (i.e., this is the best possible performance for A).

E.2 Spacing

This performance measure was proposed by Schott [26], and it quantifies the spread of solutions (i.e., how uniformly distributed the solutions are) along a Pareto front approximation. This is defined by,

$$S = \sqrt{\frac{1}{|n-1|}\sum_{i=1}^{n}\left(\bar{d}-d_i\right)^2} \quad (33)$$

where n is the number of non-dominated solutions, $d_i = \min_{i,i\neq j}\sum_{m=1}^{k}\left|f_m^i - f_m^j\right|$, $i, j = 1,2,..,n$, where k denotes the number of objectives, and $\bar{d} = \sum_{i=1}^{n} d_i / |n|$. A value of zero implies that all solutions are uniformly spread (i.e., the best possible performance).

V. EXPERIMENTAL RESULTS AND COMPARISON

The advantage of evolutionary algorithms is that they have minimum requirements regarding the problem formulation; objectives can be easily added, removed, or modified. Likewise, in this application, they are well-suited to tackle highly complex problems such as those existing in power systems.

All the algorithms compared were implemented in MATLAB 7.3 and run on a PC with a Pentium core duo processor operating @ 2 GHz with 2 GB RAM. Three test systems were used: the IEEE 9-bus, IEEE 26-bus, and the IEEE 118-bus systems, operating under their corresponding

base case. For each test power system and each algorithm, 20 runs were executed. The following results correspond to the best solution attained by each algorithm, with respect to the coverage of two set performance measure.

A. Case study 1: 9-buses test system

The decision variables are related to the generator voltage V_{gi}, and range in the interval [1.0, 1.05] pu.

Table 1 summarizes the best solution for minimum reactive losses calculated through MOTLA/D and MOEA/D. Notice that for the optimized case, a reduction of the reactive losses and voltage stability index is attained. Both algorithms reduce the losses in 38.54%, which represents an important proportion of the losses with respect to the base case. Regarding the voltage stability index, L_{index}, this has been decreased in 10.99% relative to the base case. In summary, the objective functions become:

$f_{Rloss\,(Base\,Case)} = 0.0755$ p.u $f_{Lindex\,(Base\,Case)} = 0.1673$
$f_{Rloss\,(MOTLA/D)} = 0.0464$ p.u $f_{Lindex\,(MOTLA/D)} = 0.1489$
$f_{Rloss\,(MOEA/D)} = 0.0464$ p.u $f_{Lindex\,(MOEA/D)} = 0.1489$

B. Case study 2: IEEE 26-buses test system

The decision variables are related to the generator voltage V_{gi}, and range in the interval [1.0, 1.05] pu. Likewise, another decision variable is the transformer tap setting T_i, which ranges in the interval [0.95, 1.05].

The best solution for minimum reactive losses (R_{loss}) and voltage stability index (L_{index}) is summarized in Table 2. The minimum R_{loss} and L_{index} for the base case is *0.6302* p.u. and *0.1241*, respectively. As can be noticed, MOTLA/D estimates $R_{loss} = 0.1487$ p.u and $L_{index} = 0.1023$, while MOEA/D attains $R_{loss} = 0.2105$ p.u and $L_{index} = 0.0995$. This means that MOTLA/D reaches 76.4% reduction in losses and 17.56% reduction in L_{index} with respect to the base case. Meanwhile, MOEA/D reaches 66.59% reduction in losses and 19.82% reduction in L_{index} with respect to the base case. It is assumed that the tap positions vary among 32 positions (16 up, and 16 down), and the closest is selected in Table 2. In summary, the objective functions become:

$f_{Rloss\,(Base\,Case)} = 0.6302$ p.u $f_{Lindex\,(Base\,Case)} = 0.1241$
$f_{Rloss\,(MOTLA/D)} = 0.1487$ p.u $f_{Lindex\,(MOTLA/D)} = 0.1023$
$f_{Rloss\,(MOEA/D)} = 0.2105$ p.u $f_{Lindex\,(MOEA/D)} = 0.0995$

C. Case study 3: 118-buses tests system

The decision variables are related to the generator voltage V_{gi}, and range in the interval [0.98, 1.05] pu. Likewise, another decision variable is the transformer tap setting T_i, which ranges in the interval [0.95, 1.05].

Table 3 summarizes the optimal values for the two objective functions (R_{loss}) and (L_{index}) estimated by both algorithms. The minimum R_{loss} and L_{index} for the base case became 7.8223 p.u and 0.0693, respectively. MOTLA/D reduced reactive losses from 7.8223 p.u to 6.9097 p.u (a reduction of approximately 11.66%) and improved the L_{index} from 0.0693 to 0.0630 (a reduction of approximately 9.1%). Meanwhile, MOEA/D reduced reactive losses from 7.8223 p.u to 6.9116 p.u (a reduction of approximately 11.64%) and improved the L_{index} from 0.0693 to 0.0630 (a reduction of approximately 9.1%). It is assumed that taps vary among 32 positions (16 up, and 16 down), and the closest is selected in Table 3. In summary, the objective functions become:

$f_{Rloss\ (Base\ Case)} = 7.8223$ p.u $f_{Lindex\ (Base\ Case)} = 0.0693$ $f_{Rloss\ (MOEA/D)} = 6.9116$ p.u $f_{Lindex\ (MOEA/D)} = 0.0630$

$f_{Rloss\ (MOTLA/D)} = 6.9097$ p.u $f_{Lindex\ (MOTLA/D)} = 0.0630$

TABLE 1. CASE STUDY 1: BEST SOLUTIONS CALCULATED BY BOTH MOTLA/D AND MOEA/D

Decision variables	Base case	f_{Rloss}		f_{Lindex}	
		MOTLA/D	MOEA/D	MOTLA/D	MOEA/D
V_{g1} (p.u)	1.04	1.05	1.05	1.05	1.05
V_{g2} (p.u)	1.02533	1.0376	1.0377	1.05	1.05
V_{g3} (p.u)	1.02536	1.0328	1.0331	1.0499	1.0499

TABLE 2. CASE STUDY 2: BEST SOLUTIONS CALCULATED BY BOTH MOTLA/D AND MOEA/D

Decision variables	Base case	f_{Rloss}		f_{Lindex}	
		MOTLA/D	MOEA/D	MOTLA/D	MOEA/D
V_{g1} (p.u)	1.025	1.0265	1.0466	1.05	1.0498
V_{g2} (p.u)	1.02	1.0109	1.0112	1.0429	1.0365
V_{g3} (p.u)	1.03	1.021	1.0137	1.0498	1.0420
V_{g4} (p.u)	1.045	1.0498	1.0487	1.05	1.0499
V_{g5} (p.u)	1.045	1.0248	1.0445	1.05	1.0379
V_{g26} (p.u)	1.015	1.0459	1.0216	1.0499	1.0500
T_3	0.96 (-13)	1.0135 (4)	0.9705 (-9)	0.95 (-16)	0.9850(-5)
T_6	0.96 (-13)	0.95 (-16)	1.0493 (13)	0.95 (-16)	1.0129(1)
T_8	1.017 (5)	1.0016 (1)	0.9840 (-6)	0.95 (-16)	0.9579(-14)
T_9	1.05 (16)	0.9637 (-12)	0.9703 (-10)	0.95 (-16)	0.9500(-16)
T_{10}	1.05 (16)	0.9735 (-8)	0.9598 (-13)	0.95 (-16)	0.9507(-16)
T_{15}	0.95 (-16)	0.964 (-12)	0.9507 (-16)	0.95 (-16)	0.9515(-16)
T_{18}	0.95 (-16)	0.9768 (-7)	0.9730 (-9)	0.95 (-16)	0.9500(-16)

TABLE 3. CASE STUDY 3: BEST SOLUTIONS CALCULATED BY BOTH MOTLA/D AND MOEA/D

| Decision variables	Base case	f_{Rloss}		f_{Lindex}	
		MOTLA/D	MOEAD	MOTLA/D	MOEAD
V_{g1} (p.u)	0.955	1.0356	1.0334	1.0285	1.0273
V_{g4} (p.u)	0.998	1.05	1.05	1.0497	1.05
V_{g6} (p.u)	0.99	1.0472	1.0451	1.0459	1.0401
V_{g10} (p.u)	1.05	1.0499	1.05	1.0491	1.05
V_{g19} (p.u)	0.963	1.0435	1.037	1.0497	1.0397
V_{g24} (p.u)	0.992	1.0497	1.0436	1.0498	1.047
V_{g27} (p.u)	0.968	1.037	1.0371	1.049	1.0327
V_{g31} (p.u)	0.967	1.033	1.031	1.0362	1.035
V_{g36} (p.u)	0.98	1.0495	1.0493	1.0493	1.0495
V_{g40} (p.u)	0.97	1.0398	1.0351	1.0415	1.0461
V_{g42} (p.u)	0.985	1.0499	1.0435	1.0499	1.0498
V_{g54} (p.u)	0.955	1.0187	1.0122	1.0203	1.0118
T_8	0.985 (-5)	0.9935 (-2)	0.9903 (-3)	0.9871 (-4)	0.9911 (-3)
T_{32}	0.96 (-13)	1.000 (1)	1.0006 (1)	1.0013 (1)	0.9995 (-1)
T_{36}	0.96 (-13)	0.9948 (-2)	0.9965 (-5)	1.0006 (1)	1.0032 (2)
T_{51}	0.955 (-14)	0.983 (-6)	0.9857 (-5)	0.9725 (-9)	0.9814 (-6)
T_{93}	0.96 (-13)	1.0259 (9)	1.0177 (6)	1.05 (16)	1.0223 (8)
T_{95}	0.985 (-5)	1.0179 (6)	1.013 (5)	1.0377 (12)	1.0136 (5)
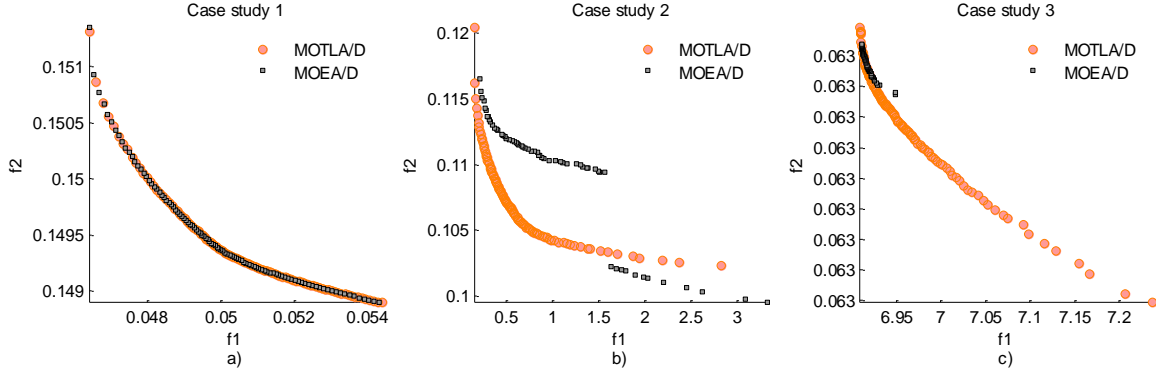
Figure 1. From left to right: Pareto fronts ($f_1 - f_2$) for both the MOTLA/D and MOEA/D (best result): (*a*) Case study 1; (*b*) Case study 2; (*c*) Case study 3.

D. *Comparison of MOTLA/D and MOEA/D*

For each case study, MOTLA/D and MOEA/D are evaluated using the two performance measures (32) and (33). The results are summarized in Tables 4 and 5. Each of these Tables present the average and the standard deviation (in brackets) of each performance measure for each case study. The best results are displayed in **boldface**.

Notice in Table 4 that the proposed approach (MOTLA/D) outperformed MOEA/D in all cases regarding the *Coverage of two sets (C)*. This indicates that the proposed approach produces more solutions that dominate (according to Pareto optimality) the solutions produced by MOEA/D. The difference among the non-dominated solutions produced by MOTLA/D and MOEA/D is more noticeable in cases 2 and 3. According to Table 4, in the case study 2, MOTLA/D produced solutions which dominate to 55% of the solutions generated by MOEA/D. In contrast, MOEA/D produced solutions that dominate only to 30% of the solutions generated by MOTLA/D. In the case study 3, the solutions obtained by MOTLA/D dominate about 41% of the solutions generated by MOEA/D; in contrast, MOEA/D produced solutions that only dominate 25% of the solutions generated by MOTLA/D.

Regarding Spacing (S), MOEA/D attains relatively better results for cases 1 and 2. However, since coverage (which relates to convergence) has precedence over spread, we can conclude that our proposed MOTLA/D outperformed MOEA/D in the analyzed cases of study.

The Pareto's fronts obtained by MOTLA/D and MOEA/D for all cases are depicted in Fig. 1. These curves represent the best case, according to the performance measures defined in (32)-(33). Notice that both algorithms perform similarly for case study 1. The difference between the approximations obtained by MOTLA/D and MOEA/D is more noticeable in cases 2 and 3. It is noteworthy that MOTLA/D is able to achieve more distributed solutions in the case study 3. A distribution of non-dominated solutions as uniform as possible along the Pareto front, ensures that there are not big gaps in the Pareto front and, therefore, all the different types of trade-off solutions are generated. This is relevant, because if big gaps occur, it may happen that the trade-off solution in which we are interested on is not produced (i.e., the solution of concern may be located in the missing portion of the Pareto front).

TABLE 4. RESULTS OF *COVERAGE OF TWO SET(C)* PERFORMANCE MEASURE

TEST	C(MOTLA/D,MOEA/D)	C(MOEA/D,MOTLA/D)
	Average (*Std. Dev.*)	Average (*Std. Dev.*)
Case study 1	**0.014** (*0.009*)	0.011 (*0.011*)
Case study 2	**0.545** (*0.415*)	0.293 (*0.335*)
Case study 3	**0.411** (*0.354*)	0.252 (*0.224*)

TABLE 5. RESULTS OF *SPACING (S)* PERFORMANCE MEASURE

TEST	MOTLA/D	MOEA/D
	Average (*Std. Dev.*)	Average (*Std. Dev.*)
Case study 1	0.0228 (*0.000*)	**0.0197** (*0.002*)
Case study 2	0.0256 (*0.002*)	**0.0184** (*0.003*)
Case study 3	**0.0254** (*0.013*)	0.0338 (*0.014*)

VI. CONCLUSIONS

This paper presented a multi-objective teaching learning algorithm based on decomposition (MOTLA/D) for solving a reactive power system problem. The effectiveness and performance of MOTLA/D were compared with respect to those of MOEA/D, which represents a state-of-the-art algorithm, in three cases of study: 9-, 26-, and 118-buses test systems. The results indicate that the proposed algorithm was able to obtain better solutions than MOEA/D in all the analyzed cases. Thus, it may be concluded that the proposed algorithm is a reliable choice for power systems applications. In this paper, an improvement of both reactive losses and voltage stability were attained. Likewise, some other additional objectives could be taken into account as well.

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VIII. BIOGRAPHIES

Miguel A. Medina (S'2010) was born in Merida. Yuc., MEXICO, on February 5, 1984. He received the B.S. degree in engineering physics from Universidad de Yucatán, Yuc., MEXICO, in 2008 and the M.Sc. degree in electrical engineering from CINVESTAV, Guadalajara, Mexico, in 2010, where he is currently pursuing the Ph.D. degree. His primary area of interest is optimization.

Carlos A. Coello Coello (F'11) received the Ph.D. degree in computer science from Tulane University (in the USA) in 1996. He is a full Professor and Chair of the Computer Science Department of CINVESTAV-IPN, in Mexico City, Mexico. He is associate editor of the *IEEE Transactions on Evolutionary Computation*. His main research interests are evolutionary multiobjective optimization, and constraint-handling techniques for evolutionary algorithms.

Juan M. Ramirez (M'86) received the B.S. degree in electrical engineering from Universidad de Guanajuato, Guanajuato, México, in 1984, the M.Sc. degree in electrical engineering from UNAM, D.F., México, in 1987, and the Ph.D. degree in electrical engineering from UANL, Monterrey, México, in 1992. He joined the Department of Electrical Engineering of CINVESTAV, Guadalajara, in 1999, where he is currently a full-time Professor. His areas of interest are in power electronics and smart grid applications.