A Method for Refining the Geometry of Images Collected by the Random Conical Tilt Scheme Based on Common Lines

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A group of I images collected by the random conical tilt scheme [1] is characterized by ϕ as the tilt angle and θ_i as the rotational angles. In this communication we are going to present an approach based on common lines to refine the estimators of these two angles.

It can be proven that between any pair of two-dimensional projections we have a common line, a fact exploited for asymmetrical particles in [2]. Furthermore, for the random conical tilt geometry a further derivation proves that the common-line is at an angle ρ, over the first image, and $-\rho$ over the second one, so that

$$\tan\rho = \frac{\cos\phi(1-\cos\theta)}{\sin\theta} \tag{1}$$

where $\theta_1 = 0$, and $\theta_2 = \theta$, for each pair of images. We use the quadratic distance between unidimensional projections

$$\mathrm{D}(\rho, j, k) = \sum_{r \text{ rays}} [\mathrm{p}_j(\rho - \pi/2, r) - \mathrm{p}_k(-\rho - \pi/2, r)]^2 \tag{2}$$

of images j and k, as the function to locate the common-line by searching around one estimated point calculated using the approximated initial values for all the angles (the initial value of ϕ is taken from the reading of goniometer's microscope, and the initial values of θ's are taken by the alignment of the "zero-tilt-views").

Our method begins by checking if the common-line for all possible pairs of images can be obtained in a reliable way. We then remove those images for which we were unable to find a common-line with any other image. Where the common-line can be detected, we search for the minimum of function D, Eq (2). This minimum gives us the location where the common-line is.

For each pair of images (j,k), we now have ρ_{jk}, and then we calculate ϕ_{jk} from Eq. (1). The next step of our method is to find the "three most typical images" as follows: remove in each step the most atypical image l, that maximizes the distance between the estimated tilt angle for all the group of images, $\overline{\phi}$, and the estimation of the tilt angle of the group without image l. Three orientations can always be described as three rotations around a single tilt.

Once we have selected the three most typical images and the associated ϕ, θ_1, θ_2, θ_3, a new image n is aggregated as follows: (a) Calculate the new $\rho_{i,n}$ using Eq. (1). The quadratic distance among $\rho_{i,n}$, and the location of the common-lines obtained before with Eq. (2) must decrease. (b) Calculate $\theta_{i,n}$, its average will be the new rotational angle for the image n. (c) Update the value of the tilt angle, the new refined value, for all the group of images. All the calculations introduced in this approach can be performed in a modern workstation very fast.

In short, the method described above is a specially derived application of the principles of common lines to the specifics of a random conical tilt data acquisition geometry. It is based on pair-wise comparisons, and it has been shown to be reliable for images with a fractional noise level (fnl) [3] up to 0.2. Cryo-electron microscopy images are still noisier than the ones this method can yet handle, and for that application other methods, such [4] and [5,6], should be used. However, for a wide range of other preservation and imaging modes, the method presented in this contribution could indeed be very valuable.

References

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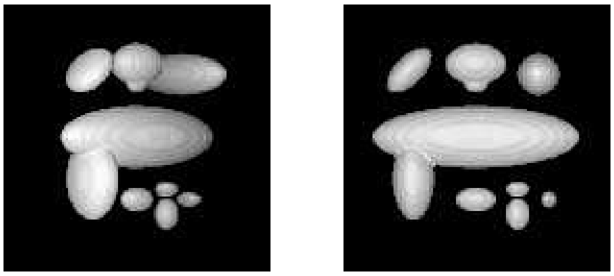

Figure 1: Surface visualization of our test phantom. Its size is $64 \times 64 \times 64$ voxels.

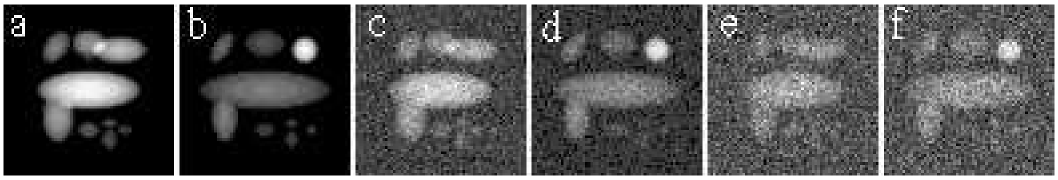

Figure 2: Example of images, a) and b) are projections taken at the same orientation that Fig. 1, without noise. c) and d) the same images than a) and b), respectively, but with a noise at fnl= 0.10 added to them. e) and f) as before, but increasing the fnl up to 0.20.

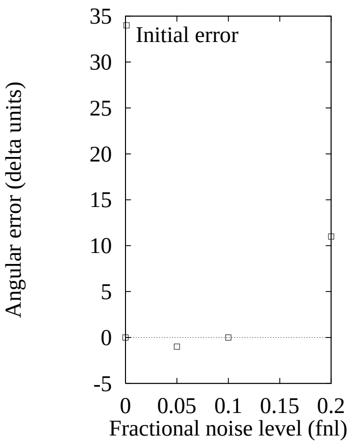

Figure 3: Error in the tilt angle vs. the fnl introduced in the simulated projections. 1 delta unit $= \pi/2048$.

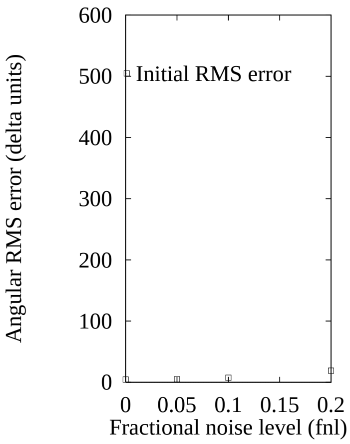

Figure 4: Root-mean-squared error in the rotational angles vs. the fnl

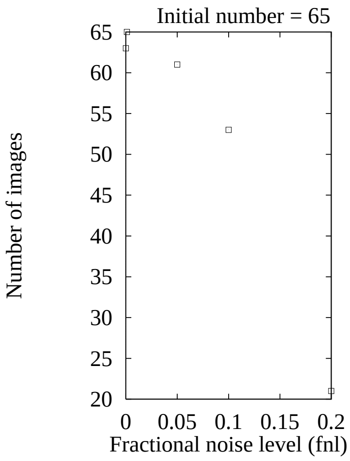

Figure 5: Final number of images in our process vs. the fnl.