

Multiobjective Optimization

Carlos A. Coello Coello
CINVESTAV-IPN
Departamento de Computación
México D.F., MÉXICO
`carlos.coellocoello@cinvestav.mx`

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Abstract

This chapter provides a short overview of multiobjective optimization using metaheuristics. The chapter includes a description of some of the main metaheuristics that have been used for multiobjective optimization. Although special emphasis is made on evolutionary algorithms, other metaheuristics, such as particle swarm optimization, artificial immune systems and ant colony optimization are also briefly discussed. Other topics such as applications and recent algorithmic trends are also included. Finally, some of the main research trends that are worth exploring in this area are briefly discussed.

1 Introduction

Metaheuristics have been widely used for solving different types of optimization problems (see for example [49, 73, 100]).

One particular class of optimization problems involves having two or more (often conflicting) objectives which we aim to optimize at the same time. In fact, such problems, which are called “multi-objective” are quite common in real-world applications, and their solution has triggered an important amount of work within Operations Research [129].

During the last 45 years, a large number of mathematical programming techniques have been developed to solve certain specific classes of multiobjective optimization problems. However, such techniques have a relatively limited applicability (e.g., some of them require the first or even the second derivative of the objective functions and the constraints, others can only deal with convex Pareto fronts, etc.). Such limitations have motivated the development of alternative optimization methods, from which metaheuristics have become a very popular alternative [26, 69].

From the many metaheuristics currently available, evolutionary algorithms have been, without doubt, the most popular choice for dealing with any sort of

optimization problem [43] and multiobjective optimization is, by no means, an exception. Thus, in this chapter, we will focus our discussion mainly on the use of evolutionary algorithms for solving multiobjective optimization problems.

The use of evolutionary algorithms for solving multiobjective optimization problems was originally hinted in 1967 [150], but the first actual implementation of what is now called a “**multi-objective evolutionary algorithm** (MOEA)” was not produced until the mid-1980s [158, 157]. However, this area, which is now called “evolutionary multi-objective optimization,” or EMO) has experienced a very important growth, mainly in the last 20 years [10, 14, 24]. The author maintains the EMOO repository, which, as of May 6th, 2024, contains over 13,600 bibliographic entries, as well as public-domain implementation of some of the most popular MOEAs. The EMOO repository is located at: <https://emoo.cs.cinvestav.mx/>.

The remainder of this chapter is organized as follows. In Section 2, we provide some basic concepts related to multiobjective optimization, which are required to make this chapter self-contained. The use of evolutionary algorithms in multiobjective optimization is motivated in Section 3. A short discussion on other bio-inspired metaheuristics that have also been used for multiobjective optimization is provided in Section 5. Some of the main research topics which are currently attracting a lot of attention in the EMO field are briefly discussed in Section 4. A set of sample applications of MOEAs is provided in Section 6. Some of the main topics of research in the EMO field that currently attract a lot of attention are briefly discussed in Section 7. Such topics include the use of other metaheuristics. Finally, some conclusions are provided in Section 8.

2 Basic Concepts

In this chapter, we focus on the solution of multiobjective optimization problems (MOPs) of the form:

$$\text{minimize } [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \quad (1)$$

subject to the m inequality constraints:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \dots, m \quad (2)$$

and the p equality constraints:

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \dots, p \quad (3)$$

where k is the number of objective functions $f_i : \mathbb{R}^n \rightarrow \mathbb{R}$. We call $\vec{x} = [x_1, x_2, \dots, x_n]^T$ the vector of decision variables. We wish to determine from among the set \mathcal{F} of all vectors which satisfy (2) and (3) the particular set of values $x_1^*, x_2^*, \dots, x_n^*$ which yield the optimum values of all the objective functions.

2.1 Pareto optimality

It is rarely the case that there is a single point that simultaneously optimizes all the objective functions. In fact, this situation only arises when there is no conflict among the objectives, which would make unnecessary the development of special solution methods, since this single solution could be reached after the sequential optimization of all the objectives, considered separately. Therefore, we normally look for “trade-offs”, rather than single solutions when dealing with multiobjective optimization problems. The notion of “optimality” normally adopted in this case is the one originally proposed by Francis Ysidro Edgeworth [50] and later generalized by the French economist Vilfredo Pareto [137]. Although some authors call this notion *Edgeworth-Pareto optimality*, we will use the most commonly adopted term: *Pareto optimality*.

We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ (i.e., a feasible solution) is *Pareto optimal* if there does not exist another $\vec{x} \in \mathcal{F}$ such that $f_i(\vec{x}) \leq f_i(\vec{x}^*)$ for all $i = 1, \dots, k$ and $f_j(\vec{x}) < f_j(\vec{x}^*)$ for at least one j (assuming that all the objectives are being minimized).

In words, this definition says that \vec{x}^* is a Pareto optimal solution if there exists no feasible vector of decision variables $\vec{x} \in \mathcal{F}$ which would decrease some criterion without causing a simultaneous increase in at least one other criterion. Assuming the inherent conflict normally present among (at least some) objectives, the use of this concept normally produces several solutions. Such solutions constitute the so-called *Pareto optimal set*. The vectors \vec{x}^* corresponding to the solutions included in the Pareto optimal set are called *nondominated*. The image of the Pareto optimal set under the objective functions (i.e., the objective function values corresponding to the decision variables contained in the Pareto optimal set) is called *Pareto front*.

3 Multi-Objective Evolutionary Algorithms

The core ideas related to the development of search techniques that simulate the mechanism of natural selection (Darwin’s survival of the fittest principle) can be traced back to the early 1930s [58]. However, the three main techniques based on this notion were developed during the 1960s: genetic algorithms [86], evolution strategies [162] and evolutionary programming [59]. These approaches, which are now generically denominated “evolutionary algorithms,” have been found to be very effective for solving single-objective optimization problems [70, 163, 60].

The basic operation of an evolutionary algorithm (EA) is described next. First, a set of potential solutions (called “population”) to the problem being solved is randomly generated. Each solution in the population (called “individual”) encodes all the decision variables of the problem (i.e., each individual contains all the decision variables of the problem to be solved). The user needs to define a measure of performance for each of the solutions. Such a measure of performance is called “fitness function” and will allow us to know how good is a solution with respect to the others. Such a fitness function is normally a

variation of the objective function of the problem that we wish to solve (e.g., the objective function that we aim to optimize). Then, a selection mechanism must be applied in order to decide which individuals will “mate.” This selection process is generally stochastic and is normally based on the fitness contribution of each individual (i.e., the fittest individuals have a higher probability of being selected). Upon mating, a set of “offspring” (or children) are generated. Such offspring are “mutated” (this operator produces a small random change, with a low probability, on the contents of an individual), and constitute the new population to be evaluated at the next iteration (each iteration is called a “generation”). This process is repeated until reaching a stopping condition (normally, a maximum number of generations defined by the user) [51].

The main motivation for using EAs for solving multiobjective optimization problems relies on their population-based nature, which allows them to generate (if properly manipulated) several elements of the Pareto optimal set, in a single run. In contrast, mathematical programming techniques normally generate a single element of the Pareto optimal set per run. Additionally, the so-called multi-objective evolutionary algorithms (MOEAs) are less susceptible to the shape and continuity of the Pareto front and require less specific domain information to operate [24].

MOEAs extend a traditional (single-objective) EA in two main aspects:

- **The selection mechanism:** In this case, the aim is to select nondominated solutions, and to consider all the nondominated solutions in a population to be equally good (unless there is some specific preference from the user, all the elements of the Pareto optimal set are equally good).
- **A diversity maintenance mechanism:** Because of stochastic noise, EAs tend to converge to a single solution if run for a sufficiently long time [70]. In order to avoid this, it is necessary to block the selection mechanism in a MOEA, favoring the diversity of solutions, so that several elements of the Pareto optimal set can be generated in a single run.

Regarding selection, early MOEAs relied on the use of aggregating functions (mainly linear) [76] and relatively simple population-based approaches [157]. However, such approaches have evident drawbacks (i.e., the use of linear aggregating functions does not allow the generation of non-convex portions of the Pareto front regardless of the weights combination that is adopted [34]). Towards the mid-1990s, MOEAs started to adopt variations of the so-called *Pareto ranking* selection mechanism. This approach was originally proposed by David E. Goldberg in his seminal book on genetic algorithms [70], and it consists of sorting the population of an EA based on Pareto optimality, such that all non-dominated individuals are assigned the same rank (or importance). The aim is that all nondominated individuals get the same probability of being selected, and that such probability is higher than the one corresponding to individuals which are dominated. Although conceptually simple, this sort of selection mechanism allows for a wide variety of possible implementations [24, 38].

Regarding diversity maintenance, a wide variety of methods have been proposed in the specialized literature to maintain diversity in a MOEA. Such approaches include fitness sharing and niching [72, 39], clustering [179, 205], geographically-based schemes [103], the use of entropy [102, 30] and parallel coordinates [84], among others. In all cases, the core idea behind diversity maintenance mechanisms is to penalize solutions that are too close from each other in some space (i.e., decision variable or objective function space or even both). Most MOEAs penalize solutions that are too close from each other in objective function space, because it is normally aimed to have solutions well-distributed along the Pareto front.

Additionally, some researchers have proposed the use of mating restriction schemes (which impose rules on the individuals that can be recombined) [93, 114, 202]. Furthermore, the use of relaxed forms of Pareto dominance became relatively popular some years ago, mainly as an archiving technique which encourages diversity, while allowing the archive to regulate convergence (the most popular of such mechanisms is, with no doubt, ϵ -dominance [107], which has been adopted in some approaches such as ϵ -MOEA [41]).

A third component of modern MOEAs is elitism, which normally consists of using an external archive (also called “secondary population” [88]) that may (or may not) interact in different ways with the main (or “primary”) population of the MOEA during selection. The main purpose of this archive is to store *all* the nondominated solutions generated throughout the search process, while removing those that become dominated later in the search (called local nondominated solutions). The approximation of the Pareto optimal set produced by a MOEA is thus the final contents of this archive. It is important to emphasize that the use of elitism is not only advisable (the lack of elitism could make us lose nondominated solutions generated during the search), but it is also required because of theoretical reasons (elitism is required in order to guarantee convergence of a MOEA to the Pareto optimal set as proved in [152]).

It is worth noticing that, in practice, external archives are normally bounded to a certain maximum number of solutions [104]. This was originally done in some MOEAs that used the external archive during the selection stage (see for example [205]). In such a case, allowing the size of the archive to grow too much, dilutes the selection pressure, which has a negative effect on the performance of the MOEA. However, most modern MOEAs bound the size of the external archive, even if the archive is not used during the selection process, mainly because of practical reasons (this makes easier to compare results with respect to other MOEAs).

An important remark is that the use of a plus (+) selection is another possible elitist mechanism. Under this sort of selection scheme, the population of parents competes with the population of offspring (both populations are of the same size) and then we keep only the best half. This sort of selection scheme has been relatively popular in single-objective optimization, and has been also adopted in some modern MOEAs (see for example [42]), but it’s less popular than the use of external archives.

4 Multi-Objective Evolutionary Algorithms

In spite of the very large number of publications related to MOEAs that can be found in the literature, there is only a handful of algorithms that are actually used by a significant number of researchers and/or practitioners around the world.

1. **Strength Pareto Evolutionary Algorithm 2** (SPEA2): This is an updated version of the **Strength Pareto Evolutionary Algorithm** (SPEA) proposed in the late 1990s [205] whose main features are the following: It adopts an external archive (called the external nondominated set), which stores the nondominated solutions previously generated, and participates in the selection process (together with the main population). For each individual in this archive, a *strength* value is computed. This strength value is proportional to the number of solutions that a certain individual dominates. In SPEA, the fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. As the size of the external nondominated set grows too much, this significantly reduces the selection pressure and slows down the search. In order to avoid this, SPEA adopts a clustering technique that prunes the contents of the external nondominated set so that its size remains below a certain (pre-defined) threshold. SPEA standardized the use of external archives as the elitist mechanism of a MOEA, although this sort of mechanism had been used before by other researchers (see for example [91]). SPEA2 has three main differences with respect to the original SPEA [208]: (1) it incorporates a fine-grained fitness assignment strategy which takes into account, for each individual, both the number of individuals that dominate it and the number of individuals by which it is dominated; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently (i.e., a more efficient clustering algorithm is adopted), and (3) it uses an enhanced archive truncation method that guarantees the preservation of boundary solutions (this fixes a bug from the original SPEA).
2. **Pareto Archived Evolution Strategy** (PAES): This is perhaps the most simple MOEA than can be possibly designed. It was proposed by Knowles and Corne [105], and it consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring through the application of mutation to the parent) in combination with a historical archive that stores the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. Such (external) archive adopts a crowding procedure that divides objective function space in a recursive manner. Then, each solution is placed in a certain grid location based on the values of its objectives (which are used as its “coordinates” or “geographical location”). A map of such a grid is maintained, indicating the number of solutions that reside

in each grid location. When a new nondominated solution is ready to be stored in the archive, but there is no room for it (the size of the external archive is bounded), a check is made on the grid location to which the solution would belong. If this grid location is less densely populated than the most densely populated grid location, then a solution (randomly chosen) from this heavily populated grid location is deleted to allow the storage of the newcomer. This aims to redistribute solutions, favoring the less densely populated regions of the Pareto front. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space).

3. **Nondominated Sorting Genetic Algorithm II** (NSGA-II): This is a heavily revised version of the Nondominated Sorting Genetic Algorithm (NSGA), which was originally proposed in the mid 1990s [171] as a straightforward implementation of the Pareto ranking algorithm described by Goldberg in his book [70]. NSGA was, however, slow, and produced poorer results than other non-elitist MOEAs available in the mid-1990s, such as MOGA [61] and NPGA [89]. NSGA-II adopts a more efficient ranking procedure than its predecessor. Additionally, it estimates the density of solutions surrounding a particular one in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called *crowding distance*. During selection, NSGA-II uses a crowded-comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). NSGA-II does not use an external archive as most of the modern MOEAs in current use. Instead, the elitist mechanism of NSGA-II consists of combining the best parents with the best offspring obtained (i.e., this is a $(\mu + \lambda)$ -selection used in evolution strategies). Due to its clever mechanisms, NSGA-II is much more efficient (computationally speaking) than its predecessor, and its performance is so good that it became very popular, triggering a significant number of applications, and becoming some sort of landmark against which new MOEAs were compared during more than 20 years, in order to merit publication. More recently, the **Nondominated Sorting Genetic Algorithm III** (NSGA-III) [40] was introduced. NSGA-III still adopts the same NSGA-II framework (i.e., it still performs a classification of the population in nondominated levels). However, its mechanism to maintain diversity is based on the use of a number of well-spread reference points which are adaptively updated. The NSGA-III doesn't require any additional parameters (other than those associated to the genetic algorithm that is used as its search engine), and it is designed to deal with many-objective optimization problems (i.e., problems having 4 or more objectives).

4. **Pareto Envelope-based Selection Algorithm** (PESA): This algorithm was proposed by Corne et al. [28], and uses a small internal population and a larger external (or secondary) population. PESA adopts the same adaptive grid from PAES to maintain diversity. However, its selection mechanism is based on a crowding measure. This same crowding measure is used to decide what solutions to introduce into the external population (i.e., the archive of nondominated vectors found along the evolutionary process). Therefore, in PESA, the external memory plays a crucial role in the algorithm since it determines not only the diversity scheme, but also the selection performed by the method. There is also a revised version of this algorithm, called PESA-II [27], which is identical to PESA, except for the fact that region-based selection is used in this case. In region-based selection, the unit of selection is a hyperbox rather than an individual. The procedure consists of selecting (using any of the traditional selection techniques [71]) a hyperbox and then randomly selecting an individual within such hyperbox. The main motivation of this approach is to reduce the computational costs associated with traditional MOEAs (i.e., those based on Pareto ranking).

5. **Multiobjective Evolutionary Algorithm based on Decomposition** (MOEA/D): This approach was proposed by Zhang and Li [201]: The main idea of this algorithm is to decompose a multiobjective optimization problem into several scalar optimization sub-problems which are simultaneously optimized. The decomposition process requires the use of weights, but the authors provide a method to generate them. During the optimization of each sub-problem, only information from the neighboring sub-problems is used, which allows this algorithm to be effective and efficient. It is worth indicating that the decomposition process is based on the use of a scalarizing function [138], which plays a key role in the performance of the algorithm. MOEA/D is considered one of the most powerful MOEAs currently available, as has been evidenced by several comparative studies and many extensions of the algorithm are currently available (see for example [125, 180, 189]).

4.1 Recent Algorithmic Trends

Many other MOEAs have been proposed in the specialized literature based on concepts such as the use of very small population sizes [178, 177], the use of the maximin expression [128, 124], a transformation of a multiobjective problem into a linear assignment problem [182], among many others. However, they will not be discussed here due to obvious space limitations. A more interesting issue, however, is to try to predict which sort of MOEA will become predominant in the next few years.

Efficiency is, for example, a concern nowadays, and several approaches have been developed in order to improve the efficiency of MOEAs (see for example [95, 46, 111]). Also, the use of fitness approximation, fitness inheritance, surrogates and other similar techniques has become more common in recent years, which is a clear indication of the more frequent use of MOEAs for the solution of computationally expensive problems (see for example [20, 122, 198, 121]).

However, the most promising research line within algorithmic design seems to be the use of a performance measure in the selection mechanism of a MOEA. This research trend formally started with the **Indicator-Based Evolutionary Algorithm** (IBEA) [206], although this idea had been already formulated and used (in different ways) by other authors before (see for example [90, 103]). Nevertheless, the most representative MOEA within this family is perhaps the **S metric selection Evolutionary Multi-Objective Algorithm** (SMS-EMOA) [52, 11]. SMS-EMOA was originally proposed by Emmerich et al. [52] and is based on NSGA-II. SMS-EMOA creates an initial population and then, it generates only one solution by iteration (i.e., it uses steady state selection) using the operators (crossover and mutation) of the NSGA-II. After that, it applies nondominated sorting. When the last front has more than one solution, SMS-EMOA uses the hypervolume contribution to decide which solution will be removed. The **Hypervolume** (also known as the S metric or the Lebesgue Measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively. This is the only unary performance indicator which is known to be fully Pareto compliant [209]. Beume et al. [11] proposed not to use the contribution to the hypervolume indicator when in the nondominated sorting process we obtain more than one front. In that case, they proposed to use the number of solutions which dominate to one solution (the solution that is dominated by more solutions is removed). The authors argued that the motivation for using the hypervolume indicator is to improve the distribution in the nondominated front and then it is not necessary in fronts different to the nondominated front.

The hypervolume indicator has attracted a lot of attention from researchers due to its interesting theoretical properties. For example, it has been proved that the maximization of the hypervolume is equivalent to finding the Pareto optimal set [57]. Empirical studies have shown that (for a certain number of points previously determined) the maximization of the hypervolume does indeed produce subsets of the Pareto front which are well-distributed [103, 52].

However, there are also practical reasons for being interested in indicator-based selection. The main one is that MOEAs such as SMS-EMOA seem to continue working as usual as we increase the number of objectives, as opposed to Pareto-based selection mechanisms which are known to degrade quickly in problems having more than 3 objectives (this research area is known as *many-objective optimization*). Although the reasons for the poor scalability of Pareto-based MOEAs requires further study (see for example [161]), the need for scalable selection mechanisms has triggered an important amount of research around indicator-based MOEAs. The main drawback of adopting the hypervolume in the selection mechanism of a MOEA is its extremely high computational cost.

One possible alternative for dealing with this high computational cost is to estimate the hypervolume contribution. However, MOEAs designed around this idea (e.g., HyPE [7]) seem to have a poor performance with respect to those that adopt exact hypervolume contributions. An alternative is to rely on other performance indicators. In this regard, several researchers have proposed the design of selection mechanisms based on performance measures such as Δ_p [160, 149], and $R2$ [78, 15, 83, 181]. The use of the maximin [8, 127, 124] is another intriguing alternative, as this expression seems to be equivalent to the use of the ϵ -indicator [207].

5 Use of Other Metaheuristics

A wide variety of other bio-inspired metaheuristics have become popular in the last few years for solving optimization problems [26]. Multiobjective extensions of many of these metaheuristics already exist [24], but few efforts have been made to actually exploit the main specific features of each of them. There are also few efforts in trying to understand the types of problems in which each of these metaheuristics can be more suitable.

Next, we briefly review three popular bio-inspired metaheuristics that are good candidates for being used as multiobjective optimizers, but many other choices also exist (see for example [24]).

5.1 Artificial Immune Systems (AIS)

Our natural immune system has provided a fascinating metaphor for developing a new bio-inspired metaheuristic. Indeed, from a computational point of view, our immune system can be considered as a highly parallel intelligent system that is able to learn and retrieve previously acquired knowledge (i.e., it has “memory”), when solving highly complex recognition and classification tasks. This motivated the development of the so-called **artificial immune systems** (AISs) during the early 1990s [35, 132].

AISs were identified in the early 1990s as a useful mechanism to maintain diversity in the context of multimodal optimization [62, 168]. Smith et al. [169] showed that fitness sharing can emerge when their emulation of the immune system is used. Furthermore, the approach that they proposed turns out to be more efficient (computationally speaking) than traditional fitness sharing [39], and it does not require additional information regarding the number of niches to be formed.

Over the years, a wide variety of multi-objective extensions of AISs have been proposed (see for example [23, 140, 118, 115, 116, 197]). However, most of the algorithmic trends in MOEAs, have had a delayed arrival in multiobjective AISs. Also, the high potential of multiobjective AISs for pattern recognition and classification tasks has been scarcely exploited.

For more information on multiobjective AISs, the interested reader is referred to [16, 63].

5.2 Particle Swarm Optimization (PSO)

This metaheuristic is inspired on the movements of a flock of birds seeking food, and it was originally proposed in the mid-1990s [101]. In the *particle swarm optimization* algorithm, the behavior of each particle (i.e., individual) is affected by either the best local (within a certain neighborhood) or the best global (i.e., with respect to the entire swarm, or population) individual. PSO allows particles to benefit from their past experiences (a mechanism that doesn't exist in traditional evolutionary algorithms) and uses neighborhood structures that can regulate the behavior of the algorithm.

The similarity of PSO with EAs, has made possible the quick development of an important number of multiobjective variants of this metaheuristic (see for example [106, 146, 166, 136]). Unlike AISs, most algorithmic trends adopted with MOEAs have quickly been incorporated into multiobjective particle swarm optimizers (MOPSOs). Nevertheless, few PSO models have been used for multiobjective optimization, and the study of the specific features that could make MOPSOs advantageous over other metaheuristics in some specific classes of problems is still a pending task.

For more information on MOPSOs, the interested reader is referred to [24, 9, 48, 22].

5.3 Ant Colony Optimization (ACO)

This metaheuristic was inspired on the behavior observed in colonies of real ants seeking for food. Ants deposit a chemical substance on the ground, called pheromone [45], which influences the behavior of the ants: they tend to take those paths in which there is a larger amount of pheromone. Therefore, pheromone trails can be seen as an indirect communication mechanism used by the ants (which can be seen as agents that interact to solve complex tasks). This interesting behavior of ants gave rise to a metaheuristic called *ant system*, which was originally applied to the travelling salesperson problem. Nowadays, the several variations of this algorithm that have been developed over the years, are collectively denominated *ant colony optimization* (ACO), and they have been applied to a wide variety of domains, including continuous optimization problems.

Several multiobjective versions of ACO are currently available (see for example [54, 203, 183, 147]). However, the algorithmic trends developed in MOEAs have had a slow delay in being incorporated into multiobjective ACO algorithms. Additionally, the use of alternative ACO models for multiobjective optimization has been relatively scarce.

For more information on multiobjective ACO, the interested reader is referred to [68, 24, 4].

5.4 Other Metaheuristics

Many other metaheuristics have been extended to deal with multiobjective optimization problems, including: simulated annealing [165, 77, 82], differential evolution [191, 47, 110], tabu search [204, 2, 36], scatter search [155, 109, 85], and artificial bee colony [153, 193, 194], among many others. However, their discussion was omitted due to obvious space constraints.

6 Some Applications

Multiobjective metaheuristics have been extensively applied to a wide variety of domains. Next, we will provide a short list of sample applications classified in three large groups: (1) engineering, (2) industrial and (3) scientific. Specific areas within each of these large groups are also identified.

By far, engineering applications are the most popular in the current literature on multiobjective metaheuristics. This is not surprising if we consider that engineering disciplines normally have problems with better understood mathematical models. A representative sample of engineering applications is the following:

- Electrical engineering [33, 96]
- Hydraulic engineering [64, 195]
- Structural engineering [200, 94]
- Aeronautical engineering [142, 173]
- Robotics [196, 99]
- Control [172, 156]
- Telecommunications [170, 53]
- Civil engineering [126, 119]
- Transport engineering [174, 17]

Industrial applications are the second most popular in the literature on multiobjective metaheuristics. A representative sample of industrial applications of multiobjective metaheuristics is the following:

- Design and manufacture [74, 3]
- Scheduling [97, 31]
- Management [6, 66]

Finally, there are several publications devoted to scientific applications. For obvious reasons, computer science applications are the most popular in the literature on multiobjective metaheuristics. A representative sample of scientific applications is the following:

- Chemistry [56, 144]
- Physics [154, 185]
- Medicine [110, 156]
- Computer science [151, 196]

This sample of applications should give at least a rough idea of the increasing interest of researchers for adopting multiobjective metaheuristics in practically all kinds of disciplines.

7 Some Current Challenges

The existence of challenging, but solvable problems, is a key issue to preserve the interest in a research discipline. Although multiobjective optimization using metaheuristics is a discipline in which a very important amount of research has been conducted, mainly within the last fifteen years, several interesting problems still remain open. Additionally, the research conducted so far has also led to new, and intriguing topics. The following is a small sample of open problems that currently attract a significant amount of research within this area:

- **Large-scale problems:** Although scalability in objective function space (the so-called many-objective problems [131, 184]) has been studied for more than 15 years, large scale multi-objective optimization problems (i.e., problems having 100 or more decision variables) is a more recent research area (see for example [87, 176, 123]). Initially, most of the proposed approaches relied on cooperative co-evolution [5] or on the analysis of the decision variables [18]. However, over the years, the use of approaches based on dimensionality reduction (this is also called problem reformulation) [81] and specialized mechanisms to generate offspring [80] became popular. Recent approaches contain mechanisms that allow solving problems with over one million decision variables, relying on clever offspring generation mechanisms (see [113]).
- **Incorporation of user's preferences:** It is normally the case, that the user does not need the entire Pareto front of a problem, but only a certain portion of it. For example, solutions lying at the extreme parts of the Pareto front are normally unnecessary since they represent the best value for one objective, but the worst for the others. Thus, if the user has at least a rough idea of the sort of trade-offs that he/she aims to find, it is desirable to be able to explore in more detail only the non-dominated solutions within the neighborhood of such trade-offs. This is

possible, if we use, for example, biased versions of Pareto ranking [32] or some multi-criteria decision making technique, from the many developed in Operations Research [134, 21, 13]. In spite of the importance of preference incorporation in real-world applications, the use of these schemes in multiobjective metaheuristics is still relatively scarce [112, 148].

- **Dealing with expensive problems:** There is an important number of real-world multi-objective problems for which a single evaluation of an objective function may take several minutes, hours or even days [142]. Evidently, such problems require special techniques to be solvable using MOEAs [79]. The main approaches that have been developed in this area can be roughly divided into three main groups:

1. **Use of parallelism:** This is the most obvious approach given the current access to low-cost parallel architectures (e.g., GPUs [167, 92]). It is worth noting, however, that in spite of the existence of interesting proposals (see for example [55]), the basic research in this area has remained scarce, since most publications involving parallel MOEAs focus on specific applications or on parallel extensions of specific MOEAs, but rarely involves basic research.
2. **Surrogates:** In this case, knowledge of past evaluations of a MOEA is used to build an empirical model that approximates the fitness functions to be optimized. This approximation can then be used to predict promising new solutions at a smaller evaluation cost than that of the original problem [122, 143]. Current functional approximation models include Polynomials (response surface methodologies [164]), neural networks (e.g., multi-layer perceptrons (MLPs) [136]), radial-basis function (RBF) networks [37], support vector machines (SVMs) [25], Gaussian processes [192], and Kriging [143] models. The use of statistical models developed in the mathematical programming literature in combination with MOEAs is also an interesting alternative (see for example, the multi-objective P-algorithm [188, 135]).
3. **Fitness approximation:** The idea of these approaches is to predict the fitness value of an individual without performing its actual evaluation [141, 65]. From the several techniques available [20], fitness inheritance has been the one of the most popular (in relative terms) in multi-objective optimization [145, 190] but the research in this area has been very scarce in recent years. However, other ideas, such as the use of transformation methods that allow to approximate the fitness landscape (see for example [19]) or incremental learning [120] are certainly worth pursuing.

- **Theoretical Foundations:** Although an important effort has been made in recent years to provide a solid theoretical foundation to this field, a lot of work is still required and the work done in the Operations Research community can provide some useful hints (see for example [187, 135]). The most relevant theoretical work in this area includes topics such as convergence [186], archiving [159, 1], algorithm complexity [133], and run-time analysis [108, 44].
- **Constraint-handling:** There has been a lot of research on constraint-handling techniques for MOEAs in recent years (see for example [29, 67, 98, 130]). However, there has been some criticism to state-of-the-art constraint-handling techniques developed for MOEAs, because there is evidence indicating that they cannot properly solve some real-world problems (e.g., in mechanical design [139]) in spite of the fact that such methods have a good performance in the currently available benchmarks. This triggered an interesting discussion, since it is evident that the current benchmarks for constrained MOEAs are fairly limited. For example, these test problems are, in general, not properly characterized and, in general, the only information available about them is: the number of decision variables, the number of constraints, the true Pareto front and the feasibility rate. However, there are studies available which indicate that this information is insufficient [175].
- **Evolutionary Multitasking:** Gupta et al. [75] proposed the so-called *evolutionary multitasking* as a new paradigm in evolutionary optimization in which the population of a sequential evolutionary algorithm is used to solve multiple optimization problems simultaneously. This idea is inspired on bio-cultural models of multifactorial inheritance. When introduced, its authors also proposed a cross-domain optimization platform that allows to solve diverse problems concurrently and by allowing knowledge transfer, its use can speed up convergence in complex optimization problems. Multitasking has become relatively popular in multiobjective optimization in recent years (see for example [199, 12, 117]) and it is indeed a very promising research area.

8 Conclusions

In this chapter, we have provided some basic concepts related to multiobjective optimization using metaheuristics. This overview has included basic concepts related to multiobjective optimization in general, as well as some algorithmic details, with a particular emphasis on multiobjective evolutionary algorithms.

Some of the recent algorithmic trends have also been discussed, and some sample applications have been addressed. In general, breadth has been favored over depth, but a significant number of bibliographic references are provided for

those interested in gaining an in-depth knowledge of any of the topics discussed herein.

The information provided in this chapter aims to serve as a general overview of the field and to motivate the interest of the reader for pursuing research in this area. As could be seen, several research opportunities are still available for newcomers.

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