A Parallel Island Model for Hypervolume-Based Many-Objective Optimization

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In recent years, the solution of many-objective optimization (i.e., multi-objective optimization problems with more than 3 objectives) has attracted a lot of attention in the evolutionary computation community. This is mainly due to the fact that Pareto-based Multi-Objective Evolutionary Algorithms (MOEAs) cannot properly solve these problems [10]. The reason is that, as we increase the number of objectives, many more solutions will become nondominated unless the population size is considerably increased. Since most MOEAs (mainly because of practical reasons) tend to adopt relatively small population sizes for many-objective optimization problems, their selection pressure will quickly dilute, which makes a MOEA to behave similarly to a random search algorithm. Although a density estimator can help to increase the selection pressure, most Pareto-based MOEAs adopt mechanisms that do not properly work in many-objective problems (e.g., the crowding-comparison operator adopted in NSGA-II [4]).

In order to deal with many-objective optimization problems, modern MOEAs normally adopt one of the two following strategies: (1) Use of a scalarization method, or (2) use of an indicator-based approach.

Scalarization methods transform a multi-objective optimization problem into several single-objective optimization problems that are simultaneously solved. The Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [12] is the most representative MOEA based on a scalarization method. MOEA/D is known to perform well in many-objective optimization problems, but it relies on a set of weight vectors, whose distribution in objective space influences the diversity

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of the final solutions. The weight vectors are used to guide the search of individuals in the population and, therefore, the number of weight vectors is directly related to the population size of MOEA/D. As the number of objectives increases, more weights are required and, consequently, a larger population size must be adopted.

When adopting an indicator-based approach, the idea is to use a performance indicator either in the density estimator (as done, for example in the $\mathscr{S}$ Metric Selection Evolutionary Multi-Objective Algorithm (SMS-EMOA) [2], which adopts the hypervolume for its density estimator) or directly in the selection mechanism of a MOEA (as done, for example, in the Indicator-Based Evolutionary Algorithm (IBEA) [14], which can use any performance indicator). The performance indicator that has been most frequently adopted is the hypervolume [13], mainly because it is the only unary indicator which is known to be Pareto-compliant and because it has been proved that maximizing it is equivalent to reaching the true Pareto optimal set [5]. However, its high computational cost (which increases exponentially with the number of objectives) has led to the use of other performance indicators such as IGD+ [8] and $R2$ [3]. Nevertheless, the use of these other performance indicators involves some issues. For example, IGD+ requires a reference set which is not easy to provide and $R2$ also relies on weight vectors (as MOEA/D).

Some attempts have been made to approximate the hypervolume contribution values at an affordable computational cost (see for example [1] where Monte Carlo sampling is adopted). However, the performance of these approaches quickly deteriorates as we lower the quality of the hypervolume contribution values.

This chapter focuses on the recently proposed parallelization of SMS-EMOA, which was original introduced by the authors of this chapter [6]. Although a few other authors have parallelized SMS-EMOA [9, 11], those works focus on the parallelization of objective function evaluations (which are computationally expensive in some real-world applications), which makes such approaches inappropriate for dealing with many-objective problems.

The common factor of the aboved-mentioned approaches is that they parallelize the operations of SMS-EMOA using several slave processors and being coordinated by a master process. In fact, this paradigm is better known as master-slave model. On the other hand, our proposal, named PArallel MICRo Optimizer based on the $\mathscr{S}$ metric ($\mathscr{S}$-PAMICRO), reduces the execution time of SMS-EMOA, through the asynchronous island model.

In our proposed approach, the population is parallelized, thus the overall population is partitionated in subpopulations, in which each one evolves a serial SMS-EMOA in semi-isolation. Occasionally, few individuals are exchanged between subpopulations (i.e., they migrate). In order to deal with the prohibitive computational cost of SMS-EMOA, we used micro-populations (i.e., subpopulations with no more than 11 individuals). In our original study [6], we observed that the computational cost of calculating the hypervolume in our proposed approach seemed to be dominated by the polynomial terms and not by the exponential terms. Furthermore, $\mathscr{S}$-PAMICRO maintains diversity through the use of external archives that are pruned to a fixed size, employing a technique based on the Parallel-Coordinates graph [7].

In [6], $\mathscr{S}$-PAMICRO was compared only with respect to hypervolume-based algorithms in one benchmark. In this chapter, we provide more evidence of its applicability in many-objective optimization problems, deepening into its implementation details and on the effects that its most relevant migration parameters have on performance.

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