Generation of New Scalarizing Functions Using Genetic Programming⋆

Amín V. Bernabé Rodríguez and
Carlos A. Coello Coello[0000−0002−8435−680X]

CINVESTAV-IPN (Evolutionary Computation Group)
Av. IPN 2508, San Pedro Zacatenco
Ciudad de México, 07360, MEXICO
abernabe@computacion.cs.cinvestav.mx
ccoello@cs.cinvestav.mx

Abstract. In recent years, there has been a growing interest in multi-objective evolutionary algorithms (MOEAs) with a selection mechanism different from Pareto dominance. This interest has been mainly motivated by the poor performance of Pareto-based selection mechanisms when dealing with problems having more than three objectives (the so-called many-objective optimization problems). Two viable alternatives for solving many-objective optimization problems are decomposition-based and indicator-based MOEAs. However, it is well-known that the performance of decomposition-based MOEAs (and also of indicator-based MOEAs designed around $R2$) heavily relies on the scalarizing function adopted. In this paper, we propose an approach for generating novel scalarizing functions using genetic programming. Using our proposed approach, we were able to generate two new scalarizing functions (called $AGSF1$ and $AGSF2$), which were validated using an indicator-based MOEA designed around $R2$ (MOMBI-II). This validation was conducted using a set of standard test problems and two performance indicators (hypervolume and s-energy). Our results indicate that $AGSF1$ has a similar performance to that obtained when using the well-known *Achievement Scalarizing Function* (ASF). However, $AGSF2$ provided a better performance that ASF in most of the test problems adopted. Nevertheless, our most remarkable finding is that genetic programming can indeed generate novel (and possible more competitive) scalarizing functions for multi-objective optimization.

Keywords: Multi-objective Optimization · Genetic Programming · Scalarizing Functions.

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1 Introduction

A great variety of real-world problems require the simultaneous optimization of two or more (often conflicting) objective functions. These are known as Multi-objective Optimization Problems (MOPs) and are mathematically defined as follows:

$$\min_{x \in \Omega} \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}))^T \tag{1}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the vector of decision variables, $\Omega \subset \mathbb{R}^n$ is the decision variable space and $\boldsymbol{F}(\boldsymbol{x})$ is the vector of m objective functions, with $m \geq 2$.

When solving an MOP, the goal is to find the set of points that yield the best possible trade-offs among the objective functions. These points are known as Pareto optimal solutions, and they form the Pareto Optimal Set ($\mathcal{P}^*$) of the problem. Its image in objective space is known as the Pareto Optimal Front ($\mathcal{PF}^*$).

The use of evolutionary algorithms for solving MOPs (the so-called Multi-Objective Evolutionary Algorithms, or MOEAs) has become increasingly popular in recent years. MOEAs are population-based methods that allow obtaining a set of different Pareto optimal solutions in a single run, in contrast with traditional mathematical programming techniques, which normally generate a single element of the Pareto optimal set per run [3].

Many MOEAs have been proposed in the literature, but they can be broadly classified into 3 categories: (1) Pareto-based, (2) indicator-based and (3) decomposition-based MOEAs [13]. The work reported in this paper is particularly relevant for decomposition-based MOEAs, but it is also applicable for some indicator-based MOEAs that rely on scalarizing functions (e.g., those based on the $R2$ indicator). Decomposition-based MOEAs decompose a MOP into several single-objective optimization problems, which are simultaneously solved [14]. In order to perform this decomposition, a scalarizing function is adopted. A scalarizing function (also known as utility function or aggregation function), transforms the original MOP into a single-objective problem using a predefined target direction or weights vector. There is empirical evidence that indicates that the performance of MOEAs that rely on scalarizing functions strongly depends on the particular scalarizing function adopted [12]. Consequently, it is relevant to find new scalarizing functions which should have a comparable performance or even better (at least in certain types of MOPs) than the scalarizing functions that are currently being used.

This paper proposes a strategy to evolve scalarizing functions combining two heuristics: genetic programming (GP) to create new functions and an MOEA to evaluate their corresponding fitness. Using the proposed approach, we were able to generate two new scalarizing functions and we compared their performance with respect to that obtained using the well-known *Achievement Scalarizing Function* (ASF). As will be seen later in this paper, our experimental results show that the scalarizing functions generated by our proposed approach have a similar performance, and that one of them outperforms ASF in more than half of the test problems adopted.

The remainder of this paper is organized as follows. Section 2 describes our approach for generating new scalarizing functions using genetic programming. Section 3 presents the experimental results obtained when assessing performance of a MOEA using the new scalarizing functions generated by our proposed approach. Section 4 provides our conclusions and some potential paths for future work.

2 Our Proposed Approach

Genetic programming (GP) is a well-established evolutionary algorithm proposed by Koza [10], in which individuals encode computer programs [1]. Although trees are the most traditional data structure adopted by GP, over the years a variety of other data structures have been adopted as well (e.g., arrays, lists and graphs).

Algorithm 1: Main procedure of our proposed approach

Input : MOP, t_{max};
Output: Final population $\boldsymbol{P}$;

$t \leftarrow 1$;
Randomly initialize the population $\boldsymbol{P} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n\}$;
foreach $\boldsymbol{x}_i \in \boldsymbol{P}$ **do**
    $sf_i \leftarrow$decode genotype from $\boldsymbol{x}_i$;
    $\boldsymbol{x}_i.fitness \leftarrow$ MOEAFitness(MOP,sf_i);
end
while *the stopping criterion is not met* **do**
    $\boldsymbol{P}' \leftarrow$ select and recombine parents from $\boldsymbol{P}$;
    foreach $\boldsymbol{x}_i \in \boldsymbol{P}'$ **do**
        $sf_i \leftarrow$decode genotype from $\boldsymbol{x}_i$;
        $\boldsymbol{x}_i.fitness \leftarrow$ MOEAFitness(MOP, sf_i);
    end
    $\boldsymbol{P} \leftarrow \boldsymbol{P}'$;
    $t \leftarrow t + 1$;
end
return P;

Epigenetic Linear Genetic Programming (ELGP)[1] is an implementation of GP coupled with a local search mechanism that was proposed in [11]. ELGP was originally used for the solution of symbolic regression problems. Individuals in ELGP are stored using a linear representation, which is decoded using stacks.

[1] Source code for ELGP is available at: `https://github.com/lacava/ellen`

Programs coded in the population are essentially mathematical functions, and the user can specify their number of variables (known as *terminals set*), as well as the operators used to manipulate them (known as the *functions set*). This is the GP implementation that we adopted to automatically generate scalarizing functions. However, we evidently had to modify the fitness function originally provided in ELGP, since it was designed to perform symbolic regression.

Algorithm 1 shows the main procedure of our proposal, which follows the essential steps of a generic GP algorithm. After the population of n individuals has been initialized (lines 1-2), the genotype of each individual $\boldsymbol{x}_i$ is decoded to obtain a scalarizing function sf_i, which is in turn used to calculate the fitness of $\boldsymbol{x}_i$ (lines 3-6). Then, the main loop is executed until one of the following stopping criteria is met: either the best fitness found in $\boldsymbol{P}$ is under some threshold or the maximum number of generations t_{max} has been reached. The steps in this loop include the generation of a new population $\boldsymbol{P}'$ using recombination and mutation (line 8), the evaluation of the new individuals (lines 9-12), as well as updating the population $\boldsymbol{P}$ (line 13). Finally, the last population is returned as the output of the algorithm.

The major modification made to ELGP was the way of evaluating the fitness of the individuals. In order to measure the quality of the new scalarizing functions generated by our GP-based approach, we use them to solve an MOP adopting an MOEA and then we employ the hypervolume indicator [15] to assess the quality of the PFs obtained. For this sake, we used the *Many-Objective Metaheuristic Based on the* $R2$ *Indicator-II* (MOMBI-II)[2], which is a metaheuristic that was originally proposed in [7]. MOMBI-II was developed to solve many-objective problems using scalarizing functions and it was able to outperform state-of-the-art MOEAs such as NSGA-III and MOEA/D in both real-world problems and benchmark problems [6]. This is, indeed, the reason why we selected MOMBI-II as our baseline algorithm to validate the new scalarizing functions generated by our proposed approach.

By default, MOMBI-II uses the Achievement Scalarizing Function (ASF) which is defined as follows:

$$ASF(\boldsymbol{f}', \boldsymbol{w}) := \max_i \left(\frac{f'_i}{w_i} \right) \tag{2}$$

where $\boldsymbol{f}' := \boldsymbol{F}(\boldsymbol{x}) - \boldsymbol{z}$ is the image of $\boldsymbol{x}$ in objective space modified by some given reference point $\boldsymbol{z} \in \mathbb{R}^m$ and $\boldsymbol{w} \in \mathbb{R}^m$ is a weights vector.

Our modified version of the fitness evaluation is outlined in Algorithm 2. The main loop calls MOMBI-II to solve the MOP given using the scalarizing function sf to be evaluated (line 3). Then, the hypervolume of the PF obtained is computed and stored (lines 4-5). This is repeated n times, in order to obtain an average value of the hypervolumes generated using sf. Finally, fitness is computed as HV_{max} minus the average hypervolume (lines 7-8). This adjustment

[2] The source code of MOMBI-II is available at: `https://www.cs.cinvestav.mx/~EVOCINV/software/MOMBI-II/MOMBI-II.html`

using HV_{max} is needed since ELGP minimizes fitness, while we aim to maximize hypervolume values.

Algorithm 2: Procedure MOEAFitness

Input : MOP, sf;
Output: $fitness$;
$fitness \leftarrow 0$;
for $i \in \{1, 2, \ldots, n\}$ **do**
    $PF_i \leftarrow$MOMBI2(MOP, sf);
    $HV \leftarrow$ compute hypervolume value of PF_i;
    $fitness \leftarrow fitness + HV$;
end
$fitness \leftarrow HV_{max} - fitness/n$;
return $fitness$;

MOMBI-II uses the $R2$ indicator to guide its search process, which is a weakly Pareto-compliant indicator with a low computational cost [2]. However, in spite of this, our strategy is indeed very time-consuming since we use the hypervolume to guide the search process of our approach. In order to improve this, we adopted the approach reported in [6] to compute the hypervolume, which is one of the most computationally efficient algorithms currently available.

It is also worth emphasizing that we aim to generate scalarizing functions that can be as general as possible, in the sense of being able to attain a reasonably good performance over a wide range of test problems, rather than generating highly specialized scalarizing functions that can provide an outstanding performance in a single test problem. Thus, we argue that the high computational cost of our proposed approach is, consequently, justified.

Both ELGP and MOMBI-II require several parameters to be executed. However, for the sake of simplicity, we don't include them in the pseudocodes of the algorithms here presented. Nonetheless, the final implementation[3] of our proposed strategy includes all of the configuration files we used.

Using our proposed strategy we were able to perform multiple experiments. In this paper we present the results obtained in one of them. We used a population size of 30 individuals and a maximum number of generations of 50. Functions were initialized completely at random, considering two decision variables ($\boldsymbol{f}'$ and $\boldsymbol{w}$) and basic arithmetic operators (addition, subtraction, multiplication, and division). In the `MOEAFitness` procedure, we incorporated DTLZ4 with two objectives as the MOP to be solved. MOMBI-II was set to use a population size of 100 with a maximum number of objective function evaluations of 15,000. The reference point used to calculate hypervolume values was $(1, 1)$. Consequently,

[3] The source code of our approach is available at:
`http://www.computacion.cs.cinvestav.mx/~abernabe/scalarizing_functions`

HV_{max} was set to 1. The running time, using the setup previously described, was nearly one week, using a personal computer with an Intel Core i5-5200U processor and 8GB of RAM.

Table 1. Scalarizing functions stored in last population.

Individual	Decodified scalarizing function	Fitness
1	$sf_1(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((f'_i - (w_i - (f'_i - w_i))) + (((f'_i * w_i) + f'_i)/w_i)) + f'_i)$	0.789438
2	$sf_2(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((((f'_i/f'_i) * (((f'_i + f'_i)/w_i) - f'_i)) * f'_i) - w_i) + f'_i)$	0.78944
3	$sf_3(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((((f'_i/f'_i) * (((f'_i + f'_i)/w_i) - f'_i)) * f'_i) - w_i) + f'_i)$	0.78944
4	$sf_4(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((((f'_i/f'_i) * (((f'_i + f'_i)/w_i) - f'_i)) * f'_i) - w_i) + f'_i)$	0.78944
5	$sf_5(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((f'_i * f'_i) - w_i) + f'_i) + (((w_i - w_i) + f'_i)/w_i))$	0.789522
6	$sf_6(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((f'_i * ((f'_i/f'_i)/w_i)) * w_i)/w_i) + w_i)$	0.78953
7	$sf_7(\boldsymbol{f}', \boldsymbol{w}) := \max_i (f'_i + ((f'_i + (w_i * (((f'_i + f'_i) - w_i) - f'_i))) + (f'_i/w_i)))$	0.789554
8	$sf_8(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((f'_i/w_i) + w_i + f'_i + 3)) + (f'_i/(2w_i)))$	0.789556
9	$sf_9(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i - (w_i - (f'_i - w_i))) + (((f'_i * w_i) + f'_i)/w_i))$	0.789559
10	$sf_{10}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (f'_i * (((((w_i/(2/f'_i)) * f'_i) + w_i) - w_i)/f'_i))$	0.789568
11	$sf_{11}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((2f'_i)/w_i) * f'_i) + (((w_i * f'_i) - (f'_i + w_i)) + f'_i)))$	0.789568
12	$sf_{12}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((w_i + f'_i)/f'_i) * f'_i)/(2w_i))$	0.789577
13	$sf_{13}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (f'_i/((f'_i - (3w_i * 2f'_i + f'_i))/(f'_i + (w_i * f'_i) - w_i)))$	0.789596
14	$sf_{14}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((f'_i * f'_i) + (f'_i - w_i))/(f'_i + (w_i/(w_i + (f'_i/f'_i)))))$	0.789617
15	$sf_{15}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((f'_i * f'_i) + (f'_i - w_i))/(f'_i + (w_i/(w_i + (f'_i/f'_i)))))$	0.789617
16	$sf_{16}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((f'_i * w_i) * f'_i) - w_i) + w_i)$	0.789637
17	$sf_{17}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((((f'_i * (-w_i)) * f'_i) - w_i) + w_i)$	0.789647
18	$sf_{18}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((((w_i - f'_i) - f'_i) + f'_i) - (f'_i * w_i))/((w_i * w_i) + f'_i))$	0.789789
19	$sf_{19}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((f'_i * f'_i) + w_i) + ((f'_i - (f'_i - w_i))/w_i))$	0.789846
20	$sf_{20}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i/(((w_i + w_i) - f'_i) + (f'_i * ((f'_i - f'_i) + f'_i)))) * f'_i)$	0.790033
21	$sf_{21}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((w_i - (f'_i/w_i)) - f'_i)$	0.790152
22	$sf_{22}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i/w_i) - ((((f'_i * (-w_i)) * f'_i) - w_i) * (w_i/f'_i)))$	0.790771
23	$sf_{23}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (w_i + (f'_i/(((w_i * w_i) * f'_i) - w_i)))$	0.792996
24	$sf_{24}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i/f'_i) - (((f'_i * ((f'_i - f'_i) - w_i)) * f'_i) - w_i))$	0.796269
25	$sf_{25}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i/(w_i + w_i)) + f'_i)$	0.797929
26	$sf_{26}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (f'_i + (f'_i/(w_i + w_i)))$	0.797929
27	$sf_{27}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i/w_i) + f'_i)$	0.797983
28	$sf_{28}(\boldsymbol{f}', \boldsymbol{w}) := \max_i ((f'_i/(f'_i + (w_i - f'_i))) * f'_i)$	0.798007
29	$sf_{29}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((((f'_i + w_i) + f'_i) + f'_i) * f'_i)/(w_i * f'_i))$	0.798761
30	$sf_{30}(\boldsymbol{f}', \boldsymbol{w}) := \max_i (((f'_i * ((f'_i - f'_i) - w_i)) * f'_i) - w_i)$	0.804673

At the end of the execution, the algorithm reports the last population as well as each individual's fitness (shown in Table 1). At this point, we performed a second phase of the experiment, where we hand-picked the most promising scalarizing functions obtained to analyze their performance. To do so, we used each of the final 30 functions to solve 7 test problems (DTLZ1 through DTLZ7) with 2 and 3 objectives. Also, we raised the limit of objective function evaluations up to 100,000. Then, we used the average hypervolume values obtained by each function in these problems. The main motivation behind this was to identify the best functions in terms of their generalization capabilities. Since DTLZ4 was the MOP used in the search process, all of the new scalarizing functions found are able to solve it relatively well, which can be seen from how similar their fitness values are. However, we are interested in finding functions that are able to solve a variety of MOPs, and not just one. Therefore, using this preliminary validation, we were able to identify solutions coded in individuals 8 and 21 as the most promising functions. We denoted these two newly found functions as *Artificially Generated Scalarizing Functions* (called **AGSF1** and **AGSF2**). They are defined as:

$$\mathbf{AGSF1}(\boldsymbol{f}', \boldsymbol{w}) := \max_i \left(|f'_i + w_i + \frac{f'_i}{w_i} + \frac{f'_i}{2w_i} + 3| \right) \tag{3}$$

$$\mathbf{AGSF2}(\boldsymbol{f}', \boldsymbol{w}) := \max_i \left(|w_i - \frac{f'_i}{w_i} - f'_i| \right) \tag{4}$$

3 Experimental Results

To evaluate the performance of **AGSF1** and **AGSF2**, we used a total of 23 test problems, including the Deb-Thiele-Laumanns-Zitzler (DTLZ) test suite [4], the Walking-Fish-Group (WFG) test suite [8], and the IDTLZ [9] test suite. The latter consists of a modification of the DTLZ test problems in which the Pareto Fronts are inverted in objective space.

In order to assess the scalability of the two generated scalarizing functions, each of the aforementioned problems was solved with 2, 3, 4, 5, 6 and 10 objectives, setting a limit of 150,000 objective function evaluations. Since we aimed to measure the improvement generated exclusively by the scalarizing functions, we used the same algorithm (MOMBI-II) in the solution of all problems, as well as the same parameters, and we only varied the scalarizing function used.

In [7] a quick scalability test was performed comparing three scalarizing functions commonly used in the area: **ASF**, the Weighted Tchebycheff Scalarizing Function (**WT**) and Penalty-based Boundary Intersection (**PBI**). Our results showed that when using more than tree objectives, **ASF** clearly outperformed **WT** and **PBI**. For this same reason, we compared **AGSF1** and **AGSF2** with respect to **ASF**, since scalability is an important desirable feature for a new scalarizing function.

We performed 30 independent runs, with each of the three scalarizing functions, on all the test problems mentioned. For assessing performance, we adopted the hypervolume and the s-energy [5] indicators. The hypervolume is used to assess convergence (larger values indicate a better performance), while s-energy is used to measure how uniformly distributed the solutions generated are (smaller values indicate a better performance). In both cases, the values obtained were normalized within the range [0,1] to allow an easier comparison of results.

Tables 2 to 7 show the mean hypervolume values (along with their corresponding standard deviations) obtained by **AGSF1** and **AGSF2** with respect to **ASF**. Tables 8 to 13 show the corresponding s-energy values. The best values obtained are represented using **boldface**. Values shown in grayscale indicate that the best value is significantly better according to the Wilcoxon rank-sum test with a significance level of 5%.

We say that a given scalarizing function outperforms another one when the mean value is better and the differences are statistically significant. From the results obtained using the hypervolume, **AGSF1** outperformed **ASF** in 36.23% of the problems, while **ASF** outperformed **AGSF1** in 29.72% of the problems. Regarding **AGSF2**, it outperformed **ASF** in 55.07% of the problems, while **ASF** only outperformed **AGSF2** in 7.25% of the problems. We can observe that

Table 2. Comparison of results in test problems with 2 objectives using the hypervolume.

*

Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.94021**	5.39E-02	0.27743	6.65E-02
DTLZ2	0.5726	1.09E-01	**0.98881**	4.13E-02
DTLZ3	0.64588	1.18E-01	**0.81145**	1.93E-01
DTLZ4	0.96411	1.79E-01	**0.96656**	1.79E-01
DTLZ5	0.5726	1.09E-01	**0.98881**	4.13E-02
DTLZ6	0.53421	2.04E-01	**0.60970**	1.86E-01
DTLZ7	**0.95753**	3.06E-02	0.21037	7.59E-02
WFG1	**0.57355**	2.12E-01	0.16113	9.55E-02
WFG2	**0.42430**	1.12E-01	0.11345	5.32E-02
WFG3	**0.65929**	1.65E-01	0.32225	1.76E-01
WFG4	0.31754	1.47E-01	**0.52158**	2.61E-01
WFG5	0.15006	1.17E-01	**0.51551**	1.69E-01
WFG6	**0.43905**	1.82E-01	0.39697	1.46E-01
WFG7	0.27839	1.41E-01	**0.63681**	2.03E-01
WFG8	**0.65718**	1.18E-01	0.26518	1.34E-01
WFG9	**0.47164**	4.71E-01	0.45408	4.49E-01
IDTLZ1	**0.93325**	2.49E-01	0.93085	2.49E-01
IDTLZ2	**0.89975**	3.00E-01	0.89422	2.98E-01
IDTLZ3	**0.95714**	1.59E-03	0.02772	7.92E-03
IDTLZ4	**0.93309**	2.49E-01	0.92736	2.48E-01
IDTLZ5	**0.89975**	3.00E-01	0.89422	2.98E-01
IDTLZ6	**0.96640**	1.79E-01	0.96046	1.78E-01
IDTLZ7	0.96648	1.79E-01	**0.96654**	1.77E-01

*

Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	0.94021	5.39E-02	**0.95971**	2.64E-02
DTLZ2	0.5726	1.09E-01	**0.74690**	2.33E-03
DTLZ3	**0.64588**	1.18E-01	0.62899	1.58E-01
DTLZ4	0.96411	1.79E-01	**0.99829**	2.33E-05
DTLZ5	0.5726	1.09E-01	**0.74690**	2.33E-03
DTLZ6	**0.53421**	2.04E-01	0.50781	1.85E-01
DTLZ7	0.95753	3.06E-02	**0.97201**	2.30E-02
WFG1	**0.57355**	2.12E-01	0.51501	2.04E-01
WFG2	0.4243	1.12E-01	**0.43835**	6.26E-02
WFG3	0.65929	1.65E-01	**0.68387**	1.61E-01
WFG4	0.31754	1.47E-01	**0.56102**	1.62E-01
WFG5	0.15006	1.17E-01	**0.49791**	1.47E-01
WFG6	**0.43905**	1.82E-01	0.40587	2.15E-01
WFG7	0.27839	1.41E-01	**0.59735**	1.26E-01
WFG8	0.65718	1.18E-01	**0.67604**	1.50E-01
WFG9	0.47164	4.71E-01	**0.51543**	4.79E-01
IDTLZ1	0.93325	2.49E-01	**0.93326**	2.49E-01
IDTLZ2	0.89975	3.00E-01	**0.89998**	3.00E-01
IDTLZ3	0.95714	1.59E-03	**0.99624**	1.32E-03
IDTLZ4	0.93309	2.49E-01	**0.93333**	2.49E-01
IDTLZ5	0.89975	3.00E-01	**0.89998**	3.00E-01
IDTLZ6	0.9664	1.79E-01	**0.96666**	1.79E-01
IDTLZ7	0.96648	1.79E-01	**0.97006**	1.55E-01

Table 3. Comparison of results in test problems with 3 objectives using the hypervolume.

*

Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.78115**	6.67E-02	0.20439	1.36E-01
DTLZ2	0.36573	8.54E-02	**0.51208**	1.53E-01
DTLZ3	0.45864	1.46E-01	**0.57649**	1.60E-01
DTLZ4	**0.94960**	1.66E-01	0.92533	2.39E-01
DTLZ5	0.10259	7.18E-02	**0.82681**	3.98E-02
DTLZ6	0.49945	2.04E-01	**0.50294**	2.02E-01
DTLZ7	**0.51209**	6.11E-02	0.16349	1.29E-01
WFG1	**0.72011**	2.76E-01	0.61325	2.75E-01
WFG2	0.66584	4.19E-01	**0.71352**	3.45E-01
WFG3	**0.60044**	1.76E-01	0.3563	1.42E-01
WFG4	0.31721	1.56E-01	**0.45204**	1.86E-01
WFG5	**0.48179**	1.80E-01	0.46342	1.62E-01
WFG6	**0.46938**	2.00E-01	0.46405	1.65E-01
WFG7	0.37713	1.12E-01	**0.42341**	1.78E-01
WFG8	0.50481	8.66E-02	**0.64052**	2.04E-01
WFG9	0.40139	4.27E-01	**0.54778**	4.09E-01
IDTLZ1	0.83644	2.24E-01	**0.93314**	2.49E-01
IDTLZ2	0.92708	2.48E-01	**0.93223**	2.49E-01
IDTLZ3	0.89394	2.98E-01	**0.89884**	3.00E-01
IDTLZ4	0.96078	1.78E-01	**0.96613**	1.79E-01
IDTLZ5	0.93026	2.48E-01	**0.93283**	2.49E-01
IDTLZ6	0.8629	3.38E-01	**0.86650**	3.40E-01
IDTLZ7	**0.89981**	3.00E-01	0.89971	3.00E-01

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Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	0.78115	6.67E-02	**0.79505**	6.61E-02
DTLZ2	0.36573	8.54E-02	**0.52165**	1.57E-01
DTLZ3	0.45864	1.46E-01	**0.57568**	1.69E-01
DTLZ4	0.9496	1.66E-01	**0.98814**	3.01E-03
DTLZ5	0.10259	7.18E-02	**0.57040**	1.24E-01
DTLZ6	0.49945	2.04E-01	**0.51185**	2.48E-01
DTLZ7	0.51209	6.11E-02	**0.79868**	8.35E-02
WFG1	**0.72011**	2.76E-01	0.63706	2.96E-01
WFG2	0.66584	4.19E-01	**0.67507**	4.22E-01
WFG3	0.60044	1.76E-01	**0.65851**	2.11E-01
WFG4	0.31721	1.56E-01	**0.46655**	1.62E-01
WFG5	0.48179	1.80E-01	**0.59633**	1.84E-01
WFG6	0.46938	2.00E-01	**0.47637**	1.91E-01
WFG7	0.37713	1.12E-01	**0.54767**	1.36E-01
WFG8	0.50481	8.66E-02	**0.63066**	1.08E-01
WFG9	0.40139	4.27E-01	**0.58829**	4.23E-01
IDTLZ1	0.83644	2.24E-01	**0.90195**	2.41E-01
IDTLZ2	0.92708	2.48E-01	**0.93242**	2.49E-01
IDTLZ3	0.89394	2.98E-01	**0.89892**	3.00E-01
IDTLZ4	0.96078	1.78E-01	**0.96635**	1.79E-01
IDTLZ5	0.93026	2.48E-01	**0.93365**	2.47E-01
IDTLZ6	0.8629	3.38E-01	**0.86641**	3.40E-01
IDTLZ7	**0.89981**	3.00E-01	0.88463	2.95E-01

Table 4. Comparison of results in test problems with 4 objectives using the hypervolume.

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Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.79463**	8.72E-02	0.42196	1.59E-01
DTLZ2	0.40867	1.62E-01	**0.71332**	1.37E-01
DTLZ3	**0.82567**	4.75E-02	0.44948	1.68E-01
DTLZ4	0.27359	1.27E-01	**0.75469**	1.41E-01
DTLZ5	0.49497	2.86E-02	**0.89203**	4.60E-02
DTLZ6	**0.60878**	8.49E-02	0.25492	1.17E-01
DTLZ7	**0.36283**	1.20E-01	0.18684	1.26E-01
WFG1	0.41544	2.94E-01	**0.41699**	2.60E-01
WFG2	**0.85445**	3.04E-01	0.69125	3.94E-01
WFG3	**0.61315**	1.02E-01	0.44094	1.57E-01
WFG4	**0.50368**	1.85E-01	0.50028	2.03E-01
WFG5	0.50659	1.84E-01	**0.59780**	1.44E-01
WFG6	0.49412	1.65E-01	**0.54457**	2.17E-01
WFG7	**0.33741**	1.37E-01	0.33588	1.51E-01
WFG8	0.37295	1.50E-01	**0.66888**	2.29E-01
WFG9	0.11596	2.08E-01	**0.39254**	3.68E-01
IDTLZ1	0.65308	1.22E-01	**0.70751**	1.58E-01
IDTLZ2	0.88085	1.64E-01	**0.94438**	1.76E-01
IDTLZ3	0.88272	1.64E-01	**0.91043**	2.43E-01
IDTLZ4	0.8524	2.28E-01	**0.91060**	2.43E-01
IDTLZ5	0.92851	1.72E-01	**0.98888**	4.59E-03
IDTLZ6	0.77558	3.47E-01	**0.82205**	3.68E-01
IDTLZ7	0.86633	3.40E-01	**0.86660**	3.40E-01

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Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.79463**	8.72E-02	0.79315	8.68E-02
DTLZ2	0.40867	1.62E-01	**0.66507**	1.28E-01
DTLZ3	**0.82567**	4.75E-02	0.80508	6.61E-02
DTLZ4	0.27359	1.27E-01	**0.52208**	1.16E-01
DTLZ5	0.49497	2.86E-02	**0.75741**	2.70E-02
DTLZ6	**0.60878**	8.49E-02	0.5973	9.86E-02
DTLZ7	0.36283	1.20E-01	**0.75108**	1.04E-01
WFG1	**0.41544**	2.94E-01	0.40384	2.41E-01
WFG2	0.85445	3.04E-01	**0.92206**	2.25E-01
WFG3	**0.61315**	1.02E-01	0.59961	1.15E-01
WFG4	0.50368	1.85E-01	**0.59369**	1.83E-01
WFG5	0.50659	1.84E-01	**0.67177**	1.57E-01
WFG6	**0.49412**	1.65E-01	0.48852	2.21E-01
WFG7	0.33741	1.37E-01	**0.44829**	1.63E-01
WFG8	0.37295	1.50E-01	**0.75818**	1.69E-01
WFG9	0.11596	2.08E-01	**0.16571**	2.26E-01
IDTLZ1	0.65308	1.22E-01	**0.68065**	1.33E-01
IDTLZ2	0.88085	1.64E-01	**0.96325**	1.09E-02
IDTLZ3	0.88272	1.64E-01	**0.89657**	2.40E-01
IDTLZ4	0.8524	2.28E-01	**0.90002**	2.41E-01
IDTLZ5	0.92851	1.72E-01	**0.95289**	1.77E-01
IDTLZ6	0.77558	3.47E-01	**0.90936**	2.43E-01
IDTLZ7	**0.86633**	3.40E-01	0.84093	3.30E-01

Table 5. Comparison of results in test problems with 5 objectives using the hypervolume.

Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.86779**	5.45E-02	0.46538	1.78E-01
DTLZ2	0.44663	1.55E-01	**0.76285**	1.49E-01
DTLZ3	**0.47940**	1.18E-01	0.39361	1.76E-01
DTLZ4	0.90638	1.29E-02	**0.98160**	1.30E-02
DTLZ5	**0.53888**	3.07E-01	0.43925	3.32E-01
DTLZ6	**0.78701**	1.06E-01	0.5533	2.12E-01
DTLZ7	0.158	7.64E-02	**0.40469**	1.14E-01
WFG1	**0.45271**	1.74E-01	0.282	1.72E-01
WFG2	**0.94906**	1.63E-01	0.81943	3.11E-01
WFG3	**0.46443**	2.08E-01	0.4261	1.82E-01
WFG4	**0.73234**	1.19E-01	0.4263	1.61E-01
WFG5	0.55916	2.16E-01	**0.60429**	1.93E-01
WFG6	0.50864	2.21E-01	**0.51723**	1.59E-01
WFG7	**0.39129**	1.57E-01	0.21911	1.46E-01
WFG8	**0.48954**	1.50E-01	0.43107	1.23E-01
WFG9	0.46108	1.49E-01	**0.84287**	1.14E-01
IDTLZ1	**0.07390**	1.77E-01	0.03128	3.32E-02
IDTLZ2	0.58141	1.94E-01	**0.78339**	3.92E-01
IDTLZ3	0.56714	2.23E-01	**0.84305**	3.31E-01
IDTLZ4	0.57595	1.92E-01	**0.94902**	1.76E-01
IDTLZ5	**0.81135**	2.17E-01	0.75983	4.19E-01
IDTLZ6	0.63062	2.47E-01	**0.95735**	1.78E-01
IDTLZ7	0.89739	2.99E-01	**0.89966**	3.00E-01

Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.86779**	5.45E-02	0.84491	8.39E-02
DTLZ2	0.44663	1.55E-01	**0.76262**	1.08E-01
DTLZ3	0.4794	1.18E-01	**0.54441**	1.52E-01
DTLZ4	0.90638	1.29E-02	**0.90696**	1.69E-01
DTLZ5	0.53888	3.07E-01	**0.78200**	2.39E-01
DTLZ6	0.78701	1.06E-01	**0.81507**	9.60E-02
DTLZ7	0.158	7.64E-02	**0.85507**	7.38E-02
WFG1	0.45271	1.74E-01	**0.52692**	2.05E-01
WFG2	0.94906	1.63E-01	**0.95750**	1.65E-01
WFG3	0.46443	2.08E-01	**0.51382**	2.04E-01
WFG4	**0.73234**	1.19E-01	0.67221	1.31E-01
WFG5	0.55916	2.16E-01	**0.63777**	1.68E-01
WFG6	0.50864	2.21E-01	**0.58072**	1.74E-01
WFG7	0.39129	1.57E-01	**0.46373**	1.68E-01
WFG8	0.48954	1.50E-01	**0.52675**	1.55E-01
WFG9	0.46108	1.49E-01	**0.61813**	1.43E-01
IDTLZ1	**0.07390**	1.77E-01	0.04525	4.84E-02
IDTLZ2	0.58141	1.94E-01	**0.69048**	3.81E-01
IDTLZ3	0.56714	2.23E-01	**0.77850**	3.05E-01
IDTLZ4	0.57595	1.92E-01	**0.81559**	2.72E-01
IDTLZ5	0.81135	2.17E-01	**0.89972**	2.40E-01
IDTLZ6	0.63062	2.47E-01	**0.80709**	3.17E-01
IDTLZ7	**0.89739**	2.99E-01	0.8315	3.24E-01

Table 6. Comparison of results in test problems with 6 objectives using the hypervolume.

Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.77527**	1.01E-01	0.22056	1.14E-01
DTLZ2	0.34124	1.47E-01	**0.75911**	1.23E-01
DTLZ3	**0.69014**	1.01E-01	0.33642	1.68E-01
DTLZ4	0.29561	1.09E-01	**0.89387**	7.15E-02
DTLZ5	**0.38145**	2.03E-01	0.2731	2.08E-01
DTLZ6	**0.70644**	1.10E-01	0.67136	2.26E-01
DTLZ7	**0.63776**	9.70E-02	0.26638	1.10E-01
WFG1	**0.52651**	2.12E-01	0.47232	2.51E-01
WFG2	**0.92164**	2.32E-01	0.85487	2.80E-01
WFG3	0.48992	2.51E-01	**0.49084**	2.08E-01
WFG4	**0.67214**	1.75E-01	0.52001	2.11E-01
WFG5	0.42035	2.09E-01	**0.50445**	2.26E-01
WFG6	0.47871	2.56E-01	**0.51014**	2.64E-01
WFG7	**0.77093**	1.47E-01	0.61142	2.41E-01
WFG8	0.42047	1.98E-01	**0.65890**	1.78E-01
WFG9	0.44598	1.43E-01	**0.70867**	1.70E-01
IDTLZ1	0.92238	1.72E-01	**0.92276**	1.72E-01
IDTLZ2	0.65039	1.75E-01	**0.66423**	1.79E-01
IDTLZ3	0.78181	2.12E-01	**0.79616**	2.18E-01
IDTLZ4	**0.89270**	1.68E-01	0.82396	2.75E-01
IDTLZ5	0.90936	2.43E-01	**0.91405**	2.44E-01
IDTLZ6	0.91392	1.70E-01	**0.92086**	1.70E-01
IDTLZ7	0.82823	3.70E-01	**0.83264**	3.72E-01

Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.77527**	1.01E-01	0.73828	1.05E-01
DTLZ2	0.34124	1.47E-01	**0.66103**	1.09E-01
DTLZ3	0.69014	1.01E-01	**0.70567**	8.26E-02
DTLZ4	0.29561	1.09E-01	**0.61672**	9.31E-02
DTLZ5	0.38145	2.03E-01	**0.66591**	1.83E-01
DTLZ6	0.70644	1.10E-01	**0.77577**	1.03E-01
DTLZ7	**0.63776**	9.70E-02	0.53999	9.49E-02
WFG1	0.52651	2.12E-01	**0.53469**	2.15E-01
WFG2	0.92164	2.32E-01	**0.92320**	2.30E-01
WFG3	**0.48992**	2.51E-01	0.4749	2.54E-01
WFG4	0.67214	1.75E-01	**0.69414**	2.12E-01
WFG5	0.42035	2.09E-01	**0.60843**	1.65E-01
WFG6	0.47871	2.56E-01	**0.54992**	2.18E-01
WFG7	0.77093	1.47E-01	**0.77884**	1.65E-01
WFG8	0.42047	1.98E-01	**0.61073**	1.89E-01
WFG9	0.44598	1.43E-01	**0.66831**	1.33E-01
IDTLZ1	0.92238	1.72E-01	**0.92356**	1.72E-01
IDTLZ2	0.65039	1.75E-01	**0.69323**	1.41E-01
IDTLZ3	0.78181	2.12E-01	**0.80746**	2.18E-01
IDTLZ4	**0.89270**	1.68E-01	0.8922	1.67E-01
IDTLZ5	0.90936	2.43E-01	**0.91376**	2.44E-01
IDTLZ6	0.91392	1.70E-01	**0.92140**	1.69E-01
IDTLZ7	**0.82823**	3.70E-01	0.79901	3.55E-01

Table 7. Comparison of results in test problems with 10 objectives using the hypervolume.

Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.81294**	1.10E-01	0.58344	1.34E-01
DTLZ2	**0.65066**	1.41E-01	0.65001	1.53E-01
DTLZ3	**0.66770**	8.45E-02	0.43772	1.57E-01
DTLZ4	0.65936	7.43E-02	**0.84957**	1.40E-01
DTLZ5	**0.30730**	2.18E-01	0.25256	2.74E-01
DTLZ6	0.44166	2.43E-01	**0.57816**	2.36E-01
DTLZ7	**0.56313**	1.12E-01	0.29824	1.36E-01
WFG1	**0.54699**	1.98E-01	0.52031	2.20E-01
WFG2	**0.93733**	1.69E-01	0.92103	1.73E-01
WFG3	0.91771	1.72E-01	**0.95130**	2.43E-02
WFG4	0.59836	1.91E-01	**0.61382**	2.03E-01
WFG5	**0.59335**	1.85E-01	0.56807	1.96E-01
WFG6	**0.59188**	1.79E-01	0.51932	1.74E-01
WFG7	**0.56534**	1.29E-01	0.55873	1.62E-01
WFG8	0.71276	1.43E-01	**0.73562**	1.81E-01
WFG9	0.46988	1.73E-01	**0.62335**	1.81E-01
IDTLZ1	**0.60202**	2.27E-01	0.57452	2.12E-01
IDTLZ2	0.63038	2.07E-01	**0.65898**	2.07E-01
IDTLZ3	**0.70399**	1.86E-01	0.69497	1.77E-01
IDTLZ4	0.52129	2.71E-01	**0.63378**	2.38E-01
IDTLZ5	**0.86257**	2.32E-01	0.86257	2.32E-01
IDTLZ6	**0.78459**	2.17E-01	0.77397	2.16E-01
IDTLZ7	**0.71568**	4.32E-01	0.61684	4.69E-01

Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.81294**	1.10E-01	0.76191	1.63E-01
DTLZ2	0.65066	1.41E-01	**0.75830**	1.02E-01
DTLZ3	0.6677	8.45E-02	**0.70229**	1.37E-01
DTLZ4	0.65936	7.43E-02	**0.70648**	2.05E-01
DTLZ5	0.3073	2.18E-01	**0.36810**	2.60E-01
DTLZ6	**0.44166**	2.43E-01	0.43486	2.15E-01
DTLZ7	**0.56313**	1.12E-01	0.46371	1.65E-01
WFG1	**0.54699**	1.98E-01	0.5019	1.85E-01
WFG2	0.93733	1.69E-01	**0.94825**	1.72E-01
WFG3	0.91771	1.72E-01	**0.95036**	1.77E-02
WFG4	0.59836	1.91E-01	**0.67055**	1.68E-01
WFG5	**0.59335**	1.85E-01	0.54048	1.85E-01
WFG6	0.59188	1.79E-01	**0.61110**	2.19E-01
WFG7	0.56534	1.29E-01	**0.58656**	1.14E-01
WFG8	0.71276	1.43E-01	**0.72063**	2.06E-01
WFG9	0.46988	1.73E-01	**0.57258**	1.58E-01
IDTLZ1	**0.60202**	2.27E-01	0.54422	2.37E-01
IDTLZ2	**0.63038**	2.07E-01	0.61508	2.04E-01
IDTLZ3	0.70399	1.86E-01	**0.72820**	1.87E-01
IDTLZ4	0.52129	2.71E-01	0.52129	2.71E-01
IDTLZ5	**0.86257**	2.32E-01	0.84874	2.29E-01
IDTLZ6	**0.78459**	2.17E-01	0.7841	2.19E-01
IDTLZ7	**0.71568**	4.32E-01	0.71295	4.30E-01

Table 8. Comparison of results in test problems with 2 objectives using *s*-energy.

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Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.03396**	5.34E-02	0.04773	6.30E-02
DTLZ2	0.0425	1.80E-01	**0.00436**	4.57E-03
DTLZ3	0.10072	2.22E-01	**0.09638**	1.77E-01
DTLZ4	1.38E-05	5.83E-05	**1.19E-05**	5.78E-05
DTLZ5	0.0425	1.80E-01	**0.00436**	4.57E-03
DTLZ6	0.08795	1.86E-01	**0.07449**	1.55E-01
DTLZ7	**0.00444**	6.50E-03	0.03115	3.50E-02
WFG1	0.07402	1.16E-01	**0.05840**	5.61E-02
WFG2	0.16328	1.88E-01	**0.11361**	1.66E-01
WFG3	**0.05228**	5.60E-02	0.0613	5.80E-02
WFG4	0.07741	1.53E-01	**0.02509**	4.12E-02
WFG5	0.13371	1.13E-01	**0.10734**	1.12E-01
WFG6	0.07397	1.51E-01	**0.01898**	1.80E-02
WFG7	0.04333	6.10E-02	**0.01828**	1.86E-02
WFG8	**0.00512**	1.48E-02	0.03646	1.79E-01
WFG9	0.14668	1.50E-01	**0.09273**	5.06E-02
IDTLZ1	0.0347	1.79E-01	**0.00231**	9.86E-03
IDTLZ2	**0.00010**	1.71E-04	0.05152	2.01E-01
IDTLZ3	**0.00004**	1.27E-04	5E-05	1.89E-04
IDTLZ4	**0.00016**	8.28E-05	0.00026	7.96E-05
IDTLZ5	**0.00010**	1.71E-04	0.05152	2.01E-01
IDTLZ6	**0.00001**	3.54E-05	0.03335	1.79E-01
IDTLZ7	**0.02281**	1.52E-02	0.09989	2.19E-01

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Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	0.03396	5.34E-02	**0.02294**	4.12E-02
DTLZ2	0.0425	1.80E-01	**0.02872**	1.31E-01
DTLZ3	0.10072	2.22E-01	**0.04910**	4.88E-02
DTLZ4	1E-05	5.83E-05	**2.45E-06**	7.05E-06
DTLZ5	0.0425	1.80E-01	**0.02872**	1.31E-01
DTLZ6	0.08795	1.86E-01	**0.06997**	7.05E-02
DTLZ7	**0.00444**	6.50E-03	0.02013	7.77E-02
WFG1	0.07402	1.16E-01	**0.06070**	7.74E-02
WFG2	0.16328	1.88E-01	**0.13370**	1.51E-01
WFG3	**0.05228**	5.60E-02	0.08831	1.32E-01
WFG4	**0.07741**	1.53E-01	0.10605	2.06E-01
WFG5	**0.13371**	1.13E-01	0.18122	1.68E-01
WFG6	0.07397	1.51E-01	**0.02855**	3.87E-02
WFG7	0.04333	6.10E-02	**0.02975**	2.49E-02
WFG8	0.00512	1.48E-02	**0.00295**	2.15E-03
WFG9	**0.14668**	1.50E-01	0.15609	1.66E-01
IDTLZ1	0.0347	1.79E-01	**0.01802**	9.57E-02
IDTLZ2	**0.00010**	1.71E-04	0.01968	5.99E-02
IDTLZ3	**0.00004**	1.27E-04	0.0002	9.49E-04
IDTLZ4	**0.00016**	8.28E-05	0.03446	1.79E-01
IDTLZ5	**0.00010**	1.71E-04	0.01968	5.99E-02
IDTLZ6	**0.00001**	3.54E-05	0.00042	2.11E-03
IDTLZ7	**0.02281**	1.52E-02	0.03939	9.28E-02

Table 9. Comparison of results in test problems with 3 objectives using *s*-energy.

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Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.02023**	1.09E-01	0.03334	1.79E-01
DTLZ2	**0.01456**	2.15E-02	0.05342	1.82E-01
DTLZ3	**6.50E-07**	2.04E-06	0.00174	9.37E-03
DTLZ4	1.70E-02	9.12E-02	**0.00198**	7.25E-03
DTLZ5	0.13761	1.82E-01	**0.12105**	1.74E-01
DTLZ6	0.30194	2.65E-01	**0.10278**	1.58E-01
DTLZ7	0.22586	3.13E-01	**0.21113**	2.96E-01
WFG1	0.03947	1.55E-01	**0.00124**	3.93E-03
WFG2	0.05319	2.01E-01	**0.00678**	1.50E-02
WFG3	0.11941	2.48E-01	**0.04173**	1.33E-01
WFG4	0.00771	4.31E-03	**0.00630**	9.99E-04
WFG5	**2.06E-06**	3.44E-06	0.03333	1.79E-01
WFG6	0.09337	1.69E-01	**0.05733**	1.39E-02
WFG7	**0.00103**	2.49E-04	0.00137	1.81E-03
WFG8	0.06543	2.05E-01	**0.04181**	1.80E-01
WFG9	0.03334	1.79E-01	**2.04E-06**	5.73E-06
IDTLZ1	0.23695	3.04E-01	**0.05408**	1.43E-01
IDTLZ2	0.04852	1.37E-01	**0.02185**	1.17E-01
IDTLZ3	0.1014	2.99E-01	**0.00520**	2.79E-02
IDTLZ4	0.07753	2.01E-01	**0.07705**	2.19E-01
IDTLZ5	0.06258	1.97E-01	**0.00042**	2.24E-03
IDTLZ6	0.04757	1.60E-01	**0.03524**	1.51E-01
IDTLZ7	0.04302	1.80E-01	**0.00234**	2.76E-03

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Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	0.02023	1.09E-01	**1.96E-06**	2.95E-06
DTLZ2	0.01456	2.15E-02	**8.00E-03**	5.06E-03
DTLZ3	6.50E-07	2.04E-06	**1.51E-07**	1.39E-07
DTLZ4	0.01702	9.12E-02	**0.00047**	2.35E-03
DTLZ5	**0.13761**	1.82E-01	0.24996	2.50E-01
DTLZ6	0.30194	2.65E-01	**0.10464**	1.88E-01
DTLZ7	**0.22586**	3.13E-01	0.26026	3.15E-01
WFG1	0.03947	1.55E-01	**0.02587**	1.11E-01
WFG2	0.05319	2.01E-01	**0.00968**	3.16E-02
WFG3	0.11941	2.48E-01	**0.05415**	1.79E-01
WFG4	0.00771	4.31E-03	**0.00670**	1.29E-03
WFG5	2.06E-06	3.44E-06	**1.66E-06**	3.74E-06
WFG6	0.09337	1.69E-01	**0.05543**	8.35E-03
WFG7	**0.00103**	2.49E-04	0.0352	1.79E-01
WFG8	0.06543	2.05E-01	**0.00137**	2.44E-03
WFG9	0.03334	1.79E-01	**1.31E-06**	3.61E-06
IDTLZ1	0.23695	3.04E-01	**0.00233**	5.04E-03
IDTLZ2	0.04852	1.37E-01	**0.00023**	1.14E-03
IDTLZ3	0.1014	2.99E-01	**0.00004**	5.93E-05
IDTLZ4	**0.07753**	2.01E-01	0.011	5.71E-02
IDTLZ5	0.06258	1.97E-01	**0.02224**	5.09E-02
IDTLZ6	**0.04757**	1.60E-01	0.06766	2.14E-01
IDTLZ7	0.04302	1.80E-01	**0.01335**	5.57E-02

Table 10. Comparison of results in test problems with 4 objectives using *s*-energy.

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Problem	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.01848**	4.51E-02	0.08178	2.25E-01
DTLZ2	**0.00002**	6.24E-05	0.00031	1.02E-03
DTLZ3	0.00901	4.84E-02	**0.00155**	8.12E-03
DTLZ4	**0.00289**	9.64E-03	0.00679	3.53E-02
DTLZ5	0.30118	2.32E-01	**0.29328**	2.34E-01
DTLZ6	0.26745	1.39E-01	**0.23423**	1.51E-01
DTLZ7	**0.06111**	7.91E-02	0.10539	1.24E-01
WFG1	0.15004	2.32E-01	**0.10807**	2.24E-01
WFG2	**0.00208**	8.78E-03	0.07611	2.45E-01
WFG3	0.35852	1.94E-01	**0.22826**	1.74E-01
WFG4	1.72E-06	5.49E-06	**1.44E-07**	3.37E-07
WFG5	**0.00145**	7.81E-03	0.03484	1.79E-01
WFG6	0.03196	7.74E-02	**0.03058**	7.08E-02
WFG7	**0.00061**	1.36E-03	0.0035	1.16E-02
WFG8	0.09748	2.05E-01	**0.01868**	5.73E-02
WFG9	**7.11E-06**	3.71E-05	0.03355	1.79E-01
IDTLZ1	**0.20303**	1.62E-01	0.20585	1.16E-01
IDTLZ2	0.22802	2.40E-01	**0.20800**	1.67E-01
IDTLZ3	**0.20221**	2.00E-01	0.21904	1.66E-01
IDTLZ4	0.04939	6.27E-02	**0.04081**	4.84E-02
IDTLZ5	0.03609	1.79E-01	**0.00245**	2.16E-03
IDTLZ6	0.06322	2.10E-01	**0.05788**	1.99E-01
IDTLZ7	**0.00485**	1.22E-02	0.03573	1.79E-01

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Problem	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.01848**	4.51E-02	0.02254	8.34E-02
DTLZ2	**0.00002**	6.24E-05	7E-05	2.39E-04
DTLZ3	**0.00901**	4.84E-02	0.03406	1.45E-01
DTLZ4	0.00289	9.64E-03	**0.00185**	5.63E-03
DTLZ5	0.30118	2.32E-01	**0.24575**	1.73E-01
DTLZ6	0.26745	1.39E-01	**0.25200**	1.46E-01
DTLZ7	0.06111	7.91E-02	**0.06099**	7.26E-02
WFG1	**0.15004**	2.32E-01	0.17604	2.58E-01
WFG2	0.00208	8.78E-03	**0.00072**	2.74E-03
WFG3	0.35852	1.94E-01	**0.35003**	2.04E-01
WFG4	**1.72E-06**	5.49E-06	1E-05	2.52E-05
WFG5	0.00145	7.81E-03	**1.52E-06**	4.53E-06
WFG6	**0.03196**	7.74E-02	0.05567	1.94E-01
WFG7	**0.00061**	1.36E-03	0.03797	1.30E-01
WFG8	0.09748	2.05E-01	**0.02312**	6.83E-02
WFG9	**7.11E-06**	3.71E-05	1.25E-05	6.07E-05
IDTLZ1	0.20303	1.62E-01	**0.19638**	1.26E-01
IDTLZ2	0.22802	2.40E-01	**0.16250**	1.57E-01
IDTLZ3	**0.20221**	2.00E-01	0.21936	2.07E-01
IDTLZ4	0.04939	6.27E-02	**0.04817**	4.72E-02
IDTLZ5	0.03609	1.79E-01	**0.00498**	1.38E-02
IDTLZ6	0.06322	2.10E-01	**0.01442**	6.83E-02
IDTLZ7	0.00485	1.22E-02	**0.00308**	7.18E-03

Table 11. Comparison of results in test problems with 5 objectives using s-energy.

Problem*	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.09211**	9.95E-02	0.11242	1.52E-01
DTLZ2	**0.02271**	4.83E-02	0.03855	1.39E-01
DTLZ3	0.04598	1.79E-01	**0.01271**	2.40E-02
DTLZ4	**0.02071**	8.48E-02	0.1125	2.81E-01
DTLZ5	**0.47548**	1.11E-01	0.58109	1.64E-01
DTLZ6	**0.35530**	2.14E-01	0.46193	1.94E-01
DTLZ7	**0.35466**	1.95E-01	0.36159	2.07E-01
WFG1	0.23667	1.87E-01	**0.14168**	1.15E-01
WFG2	**0.02767**	6.89E-02	0.18631	2.19E-01
WFG3	0.45411	2.05E-01	**0.33911**	1.54E-01
WFG4	0.03362	1.79E-01	**0.00001**	3.28E-05
WFG5	**0.00493**	1.97E-02	0.0105	5.57E-02
WFG6	**0.00718**	3.01E-02	0.01238	6.40E-02
WFG7	**0.00969**	3.69E-02	0.01125	5.81E-02
WFG8	**0.00264**	1.21E-02	0.00435	1.54E-02
WFG9	**0.00004**	7.81E-05	0.00044	2.04E-03
IDTLZ1	0.42105	2.27E-01	**0.36516**	1.35E-01
IDTLZ2	0.42158	1.76E-01	**0.39433**	1.65E-01
IDTLZ3	0.0042	7.40E-03	**0.00271**	2.01E-03
IDTLZ4	**0.39638**	1.50E-01	0.41305	1.42E-01
IDTLZ5	**0.00404**	1.32E-03	0.08645	2.50E-01
IDTLZ6	**0.00106**	4.92E-04	0.03432	1.79E-01
IDTLZ7	0.00634	3.55E-03	**0.00541**	8.90E-03

Problem*	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.09211**	9.95E-02	0.09653	1.32E-01
DTLZ2	**0.02271**	4.83E-02	0.02979	6.78E-02
DTLZ3	**0.04598**	1.79E-01	0.06622	1.51E-01
DTLZ4	**0.02071**	8.48E-02	0.05334	1.16E-01
DTLZ5	**0.47548**	1.11E-01	0.48196	1.40E-01
DTLZ6	**0.35530**	2.14E-01	0.39761	1.90E-01
DTLZ7	0.35466	1.95E-01	**0.22952**	1.45E-01
WFG1	0.23667	1.87E-01	**0.21233**	2.60E-01
WFG2	0.02767	6.89E-02	**0.01192**	2.51E-02
WFG3	**0.45411**	2.05E-01	0.46662	1.79E-01
WFG4	0.03362	1.79E-01	**0.00024**	8.47E-04
WFG5	**0.00493**	1.97E-02	0.04316	1.85E-01
WFG6	0.00718	3.01E-02	**0.00305**	9.02E-03
WFG7	**0.00969**	3.69E-02	0.04716	1.86E-01
WFG8	**0.00264**	1.21E-02	0.02136	5.34E-02
WFG9	**0.00004**	7.81E-05	0.03835	1.79E-01
IDTLZ1	0.42105	2.27E-01	**0.37583**	1.87E-01
IDTLZ2	**0.42158**	1.76E-01	0.44151	1.86E-01
IDTLZ3	0.0042	7.40E-03	**0.00340**	2.17E-03
IDTLZ4	**0.39638**	1.50E-01	0.42975	2.17E-01
IDTLZ5	0.00404	1.32E-03	**0.00388**	1.43E-03
IDTLZ6	**0.00106**	4.92E-04	0.00319	1.23E-02
IDTLZ7	**0.00634**	3.55E-03	0.06161	1.90E-01

Table 12. Comparison of results in test problems with 6 objectives using s-energy.

Problem*	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	0.21791	1.32E-01	**0.18048**	8.55E-02
DTLZ2	0.23022	2.21E-01	**0.20171**	2.32E-01
DTLZ3	**0.14514**	1.76E-01	0.15298	2.21E-01
DTLZ4	0.08846	1.72E-01	**0.06157**	9.75E-02
DTLZ5	0.32194	1.56E-01	**0.25564**	1.86E-01
DTLZ6	**0.39549**	1.86E-01	0.49549	2.07E-01
DTLZ7	**0.24238**	1.54E-01	0.29417	1.34E-01
WFG1	**0.23283**	1.60E-01	0.30558	2.35E-01
WFG2	**0.06166**	1.11E-01	0.26726	2.55E-01
WFG3	**0.40026**	1.38E-01	0.46579	2.02E-01
WFG4	0.03638	1.25E-01	**0.00205**	5.01E-03
WFG5	**0.00806**	2.09E-02	0.03415	1.20E-01
WFG6	0.05204	1.89E-01	**0.00049**	1.05E-03
WFG7	**0.06101**	1.95E-01	0.07192	1.94E-01
WFG8	0.4212	1.56E-01	**0.30553**	1.54E-01
WFG9	**0.05270**	1.19E-01	0.0866	2.44E-01
IDTLZ1	**0.27139**	1.24E-01	0.30806	1.41E-01
IDTLZ2	0.28669	1.59E-01	**0.25054**	7.92E-02
IDTLZ3	**0.00005**	1.98E-04	9E-05	3.34E-04
IDTLZ4	0.5303	1.89E-01	**0.46159**	1.41E-01
IDTLZ5	**0.06674**	1.14E-01	0.08063	1.72E-01
IDTLZ6	0.41202	1.81E-01	**0.41163**	1.63E-01
IDTLZ7	0.01611	5.50E-02	**0.00762**	2.75E-02

Problem*	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	0.21791	1.32E-01	**0.20438**	1.19E-01
DTLZ2	0.23022	2.21E-01	**0.17225**	1.85E-01
DTLZ3	0.14514	1.76E-01	**0.10612**	1.93E-01
DTLZ4	0.08846	1.72E-01	**0.05743**	1.30E-01
DTLZ5	0.32194	1.56E-01	**0.21800**	9.58E-02
DTLZ6	**0.39549**	1.86E-01	0.41407	1.42E-01
DTLZ7	**0.24238**	1.54E-01	0.2849	2.07E-01
WFG1	**0.23283**	1.60E-01	0.24275	1.47E-01
WFG2	0.06166	1.11E-01	**0.06107**	8.89E-02
WFG3	**0.40026**	1.38E-01	0.4131	1.36E-01
WFG4	0.03638	1.25E-01	**0.02977**	9.42E-02
WFG5	**0.00806**	2.09E-02	0.06099	1.90E-01
WFG6	0.05204	1.89E-01	**0.02432**	1.28E-01
WFG7	**0.06101**	1.95E-01	0.06444	1.98E-01
WFG8	0.4212	1.56E-01	**0.40457**	1.96E-01
WFG9	**0.05270**	1.19E-01	0.10652	2.08E-01
IDTLZ1	**0.27139**	1.24E-01	0.31907	1.91E-01
IDTLZ2	0.28669	1.59E-01	**0.25578**	1.16E-01
IDTLZ3	**0.00005**	1.98E-04	6E-05	2.13E-04
IDTLZ4	0.5303	1.89E-01	**0.51955**	2.37E-01
IDTLZ5	0.06674	1.14E-01	**0.06600**	9.65E-02
IDTLZ6	**0.41202**	1.81E-01	0.42941	2.05E-01
IDTLZ7	**0.01611**	5.50E-02	0.18035	2.40E-01

Table 13. Comparison of results in test problems with 10 objectives using s-energy.

Problem*	ASF		AGSF1	
	MED	STD	MED	STD
DTLZ1	**0.33773**	1.72E-01	0.53746	1.79E-01
DTLZ2	0.44721	1.97E-01	**0.35538**	1.49E-01
DTLZ3	0.39859	1.97E-01	**0.34639**	1.46E-01
DTLZ4	**0.16349**	1.51E-01	0.19282	1.27E-01
DTLZ5	**0.18951**	1.65E-01	0.22913	8.11E-02
DTLZ6	**0.23592**	1.03E-01	0.2532	1.01E-01
DTLZ7	**0.11122**	1.07E-01	0.18736	1.95E-01
WFG1	**0.05130**	2.01E-02	0.05535	2.49E-02
WFG2	**0.26651**	1.58E-01	0.39618	2.16E-01
WFG3	**0.18477**	1.93E-01	0.20172	1.34E-01
WFG4	0.10415	1.86E-01	**0.08143**	8.58E-02
WFG5	0.10478	1.60E-01	**0.08112**	1.77E-01
WFG6	0.21392	2.81E-01	**0.08966**	1.73E-01
WFG7	**0.14863**	1.76E-01	0.14944	2.00E-01
WFG8	0.31137	1.97E-01	**0.23653**	1.78E-01
WFG9	0.21529	2.31E-01	**0.12855**	2.01E-01
IDTLZ1	2.23E-07	1.20E-06	**4.29E-10**	2.30E-09
IDTLZ2	0.15901	1.52E-01	**0.11532**	1.32E-01
IDTLZ3	**2.53E-23**	1.36E-22	1.30E-16	6.98E-16
IDTLZ4	0.15096	2.51E-01	**0.13664**	2.10E-01
IDTLZ5	0.03375	1.79E-01	**0.01551**	5.64E-02
IDTLZ6	**0.00493**	2.60E-02	0.03346	1.79E-01
IDTLZ7	**0.00088**	4.21E-03	0.0517	2.01E-01

Problem*	ASF		AGSF2	
	MED	STD	MED	STD
DTLZ1	**0.33773**	1.72E-01	0.36237	1.56E-01
DTLZ2	0.44721	1.97E-01	**0.37708**	1.83E-01
DTLZ3	**0.39859**	1.97E-01	0.45988	2.24E-01
DTLZ4	**0.16349**	1.51E-01	0.24254	2.50E-01
DTLZ5	**0.18951**	1.65E-01	0.21848	1.01E-01
DTLZ6	**0.23592**	1.03E-01	0.57038	2.38E-01
DTLZ7	**0.11122**	1.07E-01	0.13158	1.28E-01
WFG1	**0.05130**	2.01E-02	0.05754	3.70E-02
WFG2	**0.26651**	1.58E-01	0.28387	1.53E-01
WFG3	0.18477	1.93E-01	**0.12599**	5.67E-02
WFG4	0.10415	1.86E-01	**0.08544**	9.81E-02
WFG5	**0.10478**	1.60E-01	0.1169	2.17E-01
WFG6	0.21392	2.81E-01	**0.15141**	2.29E-01
WFG7	0.14863	1.76E-01	**0.12281**	2.07E-01
WFG8	0.31137	1.97E-01	**0.21146**	2.22E-01
WFG9	0.21529	2.31E-01	**0.14267**	1.42E-01
IDTLZ1	**2.23E-07**	1.20E-06	3.33E-07	1.29E-06
IDTLZ2	**0.15901**	1.52E-01	0.18295	2.08E-01
IDTLZ3	**2.53E-23**	1.36E-22	2.86E-17	1.54E-16
IDTLZ4	0.15096	2.51E-01	**0.14840**	2.43E-01
IDTLZ5	0.03375	1.79E-01	**0.01226**	6.28E-02
IDTLZ6	0.00493	2.60E-02	**0.00011**	2.04E-04
IDTLZ7	**0.00088**	4.21E-03	0.00364	1.36E-02

AGSF1 significantly improved performance in the DTLZ test problems with 2 objectives. However, with an increasing number of objectives, this improvement begins to decay. A similar behavior can be seen in the WFG test problems. But, as the number of objectives increases, both **AGSF1** and **ASF** exhibit a similar performance. Finally, **AGSF1** clearly performs better than **ASF** in most of the IDTLZ test problems (with the exception of the bi-objective instances). In contrast, **AGSF2** improves the results obtained by **ASF** in most than half of the test problems. And in the worst cases, it performs similarly to **ASF**. Again, the improvement observed decreases as the number of objectives increases. Regarding the s-energy indicator, **AGSF1** outperformed **ASF** in 21.74% of the problems, while **ASF** outperformed **AGSF1** in 12.32% of the problems. Regarding **AGSF2**, it outperformed **ASF** in 18.84% of the problems, while **ASF** outperformed **AGSF2** in only 4.35% of the problems.

From this data, we can observe that, on average, **AGSF1** performs similarly or marginally better than **ASF**. However, **AGSF2** performs better than **ASF** in more than half of the cases when comparing hypervolume values and gets a similar performance when comparing s-energy values.

4 Conclusions and Future Work

We have proposed a strategy to generate new scalarizing functions using a combination of genetic programming and an MOEA. Using this strategy we were able to develop several scalarizing functions, from which we chose two (**AGSF1** and **AGSF2**) to perform an experimental evaluation of their performance. We used MOMBI-II to evaluate both of them against **ASF**, which is the scalarizing function used by default in MOMBI-II. Results obtained using a set of test problems and two performance indicators (namely hypervolume and s-energy) showed that **AGSF1** has a similar performance to that of **ASF**, while **AGSF2** outperforms **ASF** in more than half of the test problems adopted. It is interesting to note that the two new scalarizing functions were obtained trying to solve a specific MOP (DTLZ4 with 2 objectives), in which both of them outperformed **ASF**. However, these two new scalarizing functions were able to generalize their good performance in problems with a completely different Pareto Front geometry, or even with an increasing number of objectives.

There are several possible paths for future research. To the authors' best knowledge, this is the first proposal for the automatic generation of scalarizing functions, but there are obviously many other modifications that are worth exploring. For example, our proposed strategy can be run for a longer number of generations, aiming to produce better scalarizing functions. In the experiments reported in this paper, we adopted a maximum number of generations of 50 due to the high computational cost of our proposed approach, but if more computational power is available, a more thorough exploration of the search space could be conducted. Additionally, other operators can be considered in the functions set (e.g., trigonometric functions). Furthermore, the fitness evaluation procedure can be modified to either use another MOP (or even a combination of MOPs), or to use another performance indicator different from the hypervolume (or in addition to it). It is worth noting that the two scalarizing functions generated by our system share the same term ($\frac{f_i'}{w_i}$) found in **ASF**. This suggests that this term could be a good starting seed for future executions of the algorithm. Finally, it would also be interesting to modify our proposed approach so that it can be used, for example, to generate performance indicators to be adopted in the selection mechanism of an (indicator-based) MOEA.

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