USING THE EVOLUTION STRATEGIES' SELF-ADAPTATION MECHANISM AND TOURNAMENT SELECTION FOR GLOBAL OPTIMIZATION

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ABSTRACT
An approach based on a $(\mu+1)$-ES and three simple tournament rules is proposed to solve global optimization problems. The proposed approach does not use a penalty function and does not require any extra parameters other than the original parameters of an evolution strategy. This approach is validated with respect to the state-of-the-art techniques in evolutionary constrained optimization using a well-known benchmark. The results obtained are very competitive with respect to the approaches against which our approach was compared.

INTRODUCTION

Having a strong theoretical background (Schwefel, 1995; Bäck, 1996; Beyer, 2000), Evolution Strategies (ES) have been found efficient in solving a wide variety of optimization problems (Asselmeyer, 1997). However, as other evolutionary Algorithms (EAs), ES lack an explicit mechanism to deal with constrained search spaces.

Several approaches to incorporate constraints into the fitness function of an EA (Michalewicz, 1996; Coello, 2002) have been proposed. The most common approach used to incorporate the constraints of the problem to the fitness function of an EA is the use of penalty functions, where the amount of constraint violation is used to punish or "penalize" an infeasible solution so that feasible solutions are favored by the selection process. Without concerning their simplicity, they have many drawbacks from which the main one is that they require a careful fine tuning of the penalty factors that accurately estimates the degree of penalization to be applied so that we can approach efficiently the feasible region (Smith, 1997; Coello 2002).

In this paper, we argue that the self-adaptation mechanism of a conventional evolution strategy combined with some (very simple) tournament rules based on feasibility can provide us with a highly competitive evolutionary algorithm for constrained optimization.

COMPARING DIFFERENT TYPES OF ES

The motivation of this work is based in the fact that some of the most recent and competitive approaches to incorporate constraints into an evolutionary algorithm use an ES (see for example (Runarsson, 2000; Hamida, 2002)). We then hypothesized that the self-adaptation mechanis of an ES plays an important role when solving numerical optimization problems. In order to verify our hypothesis, we performed a small comparative study among different types of ES. In order to determine the most useful type of ES to deal with constrained spaces, we performed a numerical comparison using five different types of ES. Both a $(\mu+\lambda)$-ES and a (μ,λ)-ES were implemented with and without correlated mutation. Additionally, we also implemented a $(\mu+1)$-ES using the "1/5-success rule". We decided to adopt a very simple constraint-handling approach which shares similarities with some previous proposals (e.g., (Jimenez, 1999; Deb, 2000)). Note however, that all of these previous proposals require an additional mechanism to maintain diversity in the population. In our proposal, however, no extra mechanisms are provided to maintain diversity. The tournament rules adopted in the five types of ES implemented are the following:

- Between 2 feasible solutions, the one with the highest fitness value wins.
- If one solution is feasible and the other one is infeasible, the feasible solution wins.
- If both solutions are infeasible, the one with the lowest sum of constraint violation is preferred.

We decided to use ten (out of 13) of the test functions described in (Runarsson, 2000) to evaluate our five types of ES. The test functions chosen contain characteristics that are representative of what can be considered "difficult" global optimization problems for an evolutionary algorithm. The expressions of the test functions can be seen in (Mezura-Montes, 2003).

TF	optimal	Best	Mean	Median	Worst	St. Dev.
g01	-15	-15	-14.8486	-14.998	-12.9999	0.410082
g02	0.803619	0.793083	0.698932	0.708804	0.576079	0.062927
g03	1	1	1	1	1	0.000014
g04	-30665.539	-30665.539	-30665.442	-30665.539	-30663.496	0.393918
g06	-6961.814	-6961.814	-6961.814	-6961.814	-6961.814	0.00
g07	24.306	24.3681	24.7025	24.7307	25.5167	0.242956
g08	0.095825	0.095825	0.095825	0.095825	0.095825	0.00
g09	680.63	680.6317	680.6736	680.6593	680.9151	0.052483
g11	0.75	0.75	0.7844	0.776296	0.8795	0.037345
g12	1	1	1	1	1	0.00

Table 1. Statistical Results of the $(\mu+1)$-ES after the first set of experiments.

We set a total of 350000 fitness function evaluations and performed 30 runs for each problem and for each type of ES. Equality constraints were transformed into inequalities using a tolerance value of 0.0001. For the $(\mu+1)$-ES the initial values are: σ=4.0, C=0.99, μ=5, and maximum number of generations = 350000. For the $(\mu+\lambda)$-ES and (μ,λ)-ES, we adopted panmictic discrete recombination both for the strategy parameters and for the decision variables. The learning rates values were calculated as indicated in (Schwefel, 1995). The initial values for the standard deviations were 3.0 for all the decision variables. The initial values for the remaining ES are: μ=100, λ=300, and maximum number of generations = 1166.

Due to space limitations we only show the statistical results of the (μ+1)-ES in Table 1 (the complete statistical results can be found in (Mezura-Montes, 2003)), which had the best overall performance, both in terms of the best solution found and in terms of its statistical measures. We found this a little bit surprising, because this is the most simple type of ES implemented in our comparative study. Based on the encouraging results that we obtained from this study, we decided to compare our (μ+1)-ES with respect to other approaches.

The details of the selected approach (we decided to call it Simple Evolution Strategy, or SES) are in the pseudo-code shown in Figure 1. We adopted the "1/5-success rule" to self-adapt the only sigma value used in the algorithm. At each generation μ individuals are created but none of them is evaluated. In fact, we only evaluate the offspring created by the union of the mutated individuals produced. The low computational cost of the approach is owing to the fact that it requires only one sigma value to adapt and one fitness function evaluation per generation.

```
Begin
  t=0
  Create a random initial solution x^0
  Evaluate  f(x^0)
  For t=1 to MAX_GENERATIONS Do
    Produce μ mutations of x^(t-1) using:
          x_i^j=x_i^(t-1)+σ[t]•N_i(0,1)
    forall i ∈ n, j=1,2,…, μ
          Generate one child x^c by the combination of the μ mutations using
          m=randint(1, μ)
          x_i^c= x_i^m, forall i ∈ n
    Evaluate f(x^c)
    Apply comparison criteria to select the best individual x^t between x^(t-1) and x^c
    t=t+1
    If (t mod n = 0)  Then
          If (p_s>1/5) Then
                    σ[t]= σ[t-n]/c
          Else
                    If  (p_s<1/5) Then
                              σ[t]= σ[t-n]•c
                    Else
                              If  (p_s=1/5) Then
                                        σ[t]= σ[t-n]
                              End If
                    End If
          End If
    End If
  End For
End
```

Figure 1. SES algorithm (**n** is the number of decision variables of the problem, **ps** is the success rate of mutations in order to apply the "1/5 success rule")

It is worth emphasizing that our algorithm is based on two mechanisms: (1) the natural self-adaptative mechanism of the ES, which helps the approach to sample the search space well enough as to reach the feasible region reasonably fast and (2) the comparison criteria of our tournaments, which select the most promising solution in order to be used as a new starting point for the search.

COMPARING SES AGAINST OTHER APPROACHES

In the second part of our experiment, we compared the performance of our approach with respect to three techniques that are representative of the state-of-the-art in the area: the homomorphous maps "HM"(Koziel, 1999), the stochastic ranking "SR" (Runarsson, 2000) and the adaptive segregational constraint handling evolutionary algorithm "ASCHEA" (Hamida, 2002). In Tables 2, 3 and 4, we show the results of the comparison.

		Best		Mean		Worst	
TF	**optimal**	**SES**	**HM**	**SES**	**HM**	**SES**	**HM**
g01	-15	-15	-14.7886	-14.8486	-14.7082	-12.9999	-14.6154
g02	0.803619	0.793083	0.79953	0.698932	0.79671	0.576079	0.79119
g03	1	1	0.9997	1	0.9989	1	0.9978
g04	-30665.539	-30665.539	-30664.5	-30665.442	-30655.3	-30663.496	-30645.9
g06	-6961.814	-6961.814	-6952.1	-6961.814	-6342.6	-6961.814	-5473.9
g07	24.306	24.3681	24.62	24.7025	24.826	25.5167	25.069
g08	0.095825	0.095825	0.095825	0.095825	0.08916	0.095825	0.0291438
g09	680.63	680.6317	680.91	680.6736	681.16	680.9151	683.18
g11	0.75	0.75	0.75	0.7844	0.75	0.8795	0.75
g12	1	1	0.99999	1	0.99913	1	0.99195

Table 2. Comparison of results between our approach (SES) and the Homomorphous Maps (HM)

		Best		Mean		Worst	
TF	**optimal**	**SES**	**SR**	**SES**	**SR**	**SES**	**SR**
g01	-15	-15	-15	-14.8486	-15	-12.9999	-15
g02	0.803619	0.793083	0.803515	0.698932	0.781975	0.576079	0.726288
g03	1	1	1	1	1	1	1
g04	-30665.539	-30665.539	-30665.539	-30665.442	-30665.539	-30663.496	-30665.539
g06	-6961.814	-6961.814	-6961.814	-6961.814	-6875.940	-6961.814	-6350.262
g07	24.306	24.3681	24.307	24.7025	24.374	25.5167	24.642
g08	0.095825	0.095825	0.095825	0.095825	0.095825	0.095825	0.095825
g09	680.63	680.6317	680.63	680.6736	680.656	680.9151	680.763
g11	0.75	0.75	0.75	0.7844	0.75	0.8795	0.75
g12	1	1	1	1	1	1	1

Table 3. Comparison of results between our approach (SES) and the Stochastic Ranking (SR)

		Best		Mean		Worst	
TF	optimal	SES	ASCHEA	SES	ASCHEA	SES	ASCHEA
g01	-15	-15	-15	-14.8486	-14.84	-12.9999	NA
g02	0.803619	0.793083	0.785	0.698932	0.59	0.576079	NA
g03	1	1	1	1	0.9999	1	NA
g04	-30665.539	-30665.539	-30665.5	-30665.442	-30665.5	-30663.496	NA
g06	-6961.814	-6961.814	-6961.81	-6961.814	-6961.81	-6961.814	NA
g07	24.306	24.3681	24.3323	24.7025	24.66	25.5167	NA
g08	0.095825	0.095825	0.095825	0.095825	0.095825	0.095825	NA
g09	680.63	680.6317	680.63	680.6736	680.641	680.9151	NA
g11	0.75	0.75	0.75	0.7844	0.75	0.8795	NA
g12	1	1	NA	1	NA	1	NA

Table 4. Comparison of results between our approach (SES) and the Adaptive Segregational Constraint Handling Evolutionary Algorithm (ASCHEA) (NA means not available).

There are several issues derived from this comparison that deserve some discussion:

Our SES was able to converge to the global optimum in 8 of the test 10 functions used (g01, g03, g04, g06, g08, g09, g11 and g12), and it was able to converge very closely to the optimum in the other two.Also, with respect to the homomorphous maps (see Table 2), SES converged to a better "best" solution in 7 problems, and it obtained better average solutions in 8 problems. It also found a better "worst" solution in 6 problems. Thus, it should be clear that SES had a highly competitive performance, even improving the results of the homomorphous maps in some test functions. With respect to stochastic ranking (see Table 3), SES was able to converge to similar "best" solutions in 8 problems and it obtained similar average and worst results in 2 problems. SES found a better average and worst result only in one problem (g06). Finally, with respect to ASCHEA (see Table 4), the SES converged to similar "best" results in 7 problems and found better "best" results in problem g02. SES also obtained better average results in 3 problems and matched ASCHEA's average results in three more.

DISCUSSION OF RESULTS

The results described before indicate a competitive performance of our approach with respect to three techniques representative of the state-of-the-art in constrained evolutionary optimization. Besides being a very simple approach, it is worth reminding that SES does not require any extra parameters (besides those used with an evolution strategy). In contrast, the homomorphous maps require an additional parameter (called v) which has to be found empirically (Koziel, 1999). Stochastic ranking requires the definition of a parameter called P_f, whose value has an important impact on the performance of the approach (Runarsson, 2000). ASCHEA also requires the definition of several extra parameters and in its latest version, it uses niching, which is a process that also has at least one additional parameter (Hamida, 2002). Finally, the number of fitness funtion evaluations of our approach is less or equal to that requiered by the compared approaches: Stochastic ranking performed the same number of evaluations as SES (350000), but the homomorphous maps performed 1,400,000 fitness function evaluations, and ASCHEA performed 1,500,000 fitness function evaluations.

CONCLUSIONS AND FUTURE WORK

The self-adaptation mechanism of an evolution strategy combined with a set of simple tournament rules was used to deal with constrained search spaces. Based on the results obtained, we claim that the proposed approach is a viable alternative for constrained optimization. Its main advantage is that it does not require a penalty function or any extra parameters (other than the original parameters of an evolution strategy) to bias the search towards the feasible region of a problem. Additionally, our approach has a low computational cost and it is easy to implement. Our future work consists on experimenting with a different random numbers distribution (a Gaussian distribution was used in this paper) to avoid convergence to local optima, since this happens in some cases.

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