招待論文

Research Directions in Evolutionary Multi-Objective Optimization

Current and Future Research Topics

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Summary

After more than 25 years of existence, evolutionary multi-objective optimization has become a mature discipline within evolutionary computation, producing an important flow of publications each year. This paper presents a brief overview of the main topics on which researchers in this area are currently working, as well as some discussion of the areas which, from the author's perspective, constitute promising research directions for the next few years. The topics discussed include algorithmic design, scalability, efficiency, hybridization, parameter control, theory and incorporation of user's preferences. The contents of this paper intends to provide a quick overview of the current state and challenges within evolutionary multi-objective optimization, and is intended to be useful for those interested in pursuing research in this area.

1. Introduction

Evolutionary algorithms (EAs) are stochastic search techniques inspired on Darwin's "survival of the fittest" principle, which apply a set of genetic-inspired operators (e.g., crossover and/or mutation) on a set of solutions (the so-called *population*) with the only aim of increasing the fitness of such solutions [Goldberg 89]. Their simplicity, ease of use and effectivity has made EAs very popular in a wide variety of domains particularly related to classification and optimization tasks.

The popularity of EAs has originated a higher level of specialization, which has given rise to several (more focused) subdisciplines. One of such subdisciplines is evolutionary multi-objective optimization (EMO), which refers to the use of EAs for solving problems that have two or more (often conflicting*1) objectives (the so-called multi-objective problems (MOPs)). The conflicting nature of the objectives of a MOP makes them have not one, but a set of solutions, which represent the best possible trade-offs among all the objectives. The population-based nature of EAs gives them an advantage when solving MOPs, since the proper use of the population allows us to generate several different solutions during a single run of an EA. The stochastic nature of EAs is also an advantage, since they are less susceptible to the specific features of the MOP that we aim to solve than traditional mathematical programming techniques. This has made EAs an increasingly popular choice for solving complex MOPs [Coello 07].

David Schaffer introduced the first implementation of a multi-objective evolutionary algorithm (MOEA) in the mid-1980s [Schaffer 85]. Since then, an important number of MOEAs have been proposed, and the field has experienced a considerable growth [Coello 06, Coello 07].*2

After more than 25 years of activity, the EMO field has reached certain maturity, and, as a consequence, its research topics have become more focused. However, at the same time, this makes look the field a bit intimidating to newcomers who get swamped by the massive volume of information available on most of its research topics. In fact, a quick browsing of the current EMO literature shows that a lot of "work by analogy" is being published these days, and novel ideas seem now more scarce. Thus, some students get the impression that producing original contributions (mainly at the level of a PhD thesis) in this area is now much harder than 10 or 15 years ago. This is perhaps true, but it's part of the typical growing pains of a scientific

*1 If the objectives of a multi-objective problem have no conflict among themselves, then the problem has a single solution which can be obtained by sequentially optimizing all the objectives. Such problems are not of interest for the aims of this paper.

*2 The author maintains the EMO repository, which currently contains over 7170 bibliographical references, plus public-domain software, and a small database of EMO researchers. The EMO repository is located at: http://delta.cs.cinvestav.mx/~ccoello/EMOO discipline. In spite of this, the goal of this paper is to show that the EMO field still has several intriguing and exciting research problems waiting for someone to tackle them. It is only that today is necessary to dig a little bit deeper into the literature than 10 or 15 years ago. Thus, the main aim of this paper is to serve as a quick guideline to these interesting research problems, so that newcomers can easily identify an area of opportunity and select a problem from there for a PhD thesis or for establishing a publications record. It is important to clarify, however, that this paper is not meant to be an introduction to the EMO field, and requires some basic knowledge about this area in order to be understandable. For an introduction to the EMO field, the reader is referred to [Coello 07, Coello 06].

The rest of the paper is organized as follows. In Section Chapter 2, we present the most relevant concepts related to multi-objective optimization, which are meant to make this paper self-contained. Then, Section Chapter 3, we discuss in detail some of the main research trends identified in the current EMO literature. Section Chapter 4 highlights some additional topics that, from the author's perspective, constitute areas of opportunity for future research. Finally, the conclusions of the paper are presented in Section Chapter 5.

2. Basic Concepts

The focus of this paper is the solution of problems of the type*3:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \quad (1)$$

subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1, 2, \ldots, m \qquad (2)$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \ldots, p \qquad (3)$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}, i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}, i = 1, ..., m, j = 1, ..., p$ are the constraint functions of the problem.

Next, we present some definitions that are necessary to make this paper more understandable.

Definition 1. Given two vectors $\vec{u}, \vec{v} \in \mathbb{R}^k$, we say that $\vec{u} \leq \vec{v}$ if $u_i \leq v_i$ for $i = 1, ..., k$, and that $\vec{u} < \vec{v}$ if $\vec{u} \leq \vec{v}$ and $\vec{u} \neq \vec{v}$.

Definition 2. Given two vectors $\vec{u}, \vec{v} \in \mathbb{R}^k$, we say that $\vec{u}$ **dominates** $\vec{v}$ (denoted by $\vec{u} \prec \vec{v}$) iff $\vec{u} < \vec{v}$.

*3 Without loss of generality, we will assume only minimization problems.

Definition 3. We say that a vector of decision variables $\vec{x}^* \in \mathcal{F}$ ($\mathcal{F}$ is the feasible region) is **Pareto optimum** if there does not exist another $\vec{x} \in \mathcal{F}$ such that $\vec{f}(\vec{x}) \prec \vec{f}(\vec{x^*})$.

Definition 4. The **Pareto Optimal Set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{is Pareto optimum}\}$$

The vectors $\vec{x}^*$ corresponding to the solutions included in the Pareto optimal set are called *nondominated*.

Definition 5. The **Pareto Front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^n | \vec{x} \in \mathcal{P}^*\}$$

We thus wish to determine the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy ((2)) and ((3)).

3. Current Research in the EMO Field

Based on his research experience of more than 18 years in the EMO field, the author identified the following list of topics that, in the author's perspective, are representative of the current research trends in this area:

(1) Algorithmic design
(2) Scalability
(3) Efficiency
(4) Hybridization

In the following sections, we will provide a short discussion of each of these topics, providing appropriate references where the interested reader may find more details about them.

3·1 Algorithmic Design

Current MOEAs have two main components: (1) a selection mechanism that aims to favor good trade-off solutions (Pareto ranking [Goldberg 89] and nondominated sorting [Srinivas 94] have been the most common mechanisms adopted for this sake) and (2) a density estimator that aims to distribute solutions along the Pareto front, in order to avoid that they overlap in only one or a few regions of the Pareto front. The notion of elitism in MOEAs, which refers to retaining the nondominated solutions found from one generation to the next one, has been around since the mid-1990s [Husbands 94], but became popular after the introduction of the Strength Pareto Evolutionary Algorithm (SPEA) in the late 1990s [Zitzler 99]). Elitism is important not only for practical reasons (we don't want to lose the nondominated solutions that are obtained during the search process), but also for theoretical reasons (elitism is required to guarantee convergence of a MOEA

[Rudolph 00]. Elitism is normally implemented through the use of an external archive [Knowles 00], but can be also enforced using a plus selection (i.e., by selecting the best individuals from the union of the populations of parents and offspring) [Deb 02]. Interesting enough, the use of external archives has also triggered some interesting research, since they can be used, for example, to enhance the convergence properties of MOEAs (see for example [Laumanns 02, Schutze 10]).

Since the publication of the Nondominated Sorting Genetic Algorithm-II (NSGA-II) [Deb 02], traditional Pareto-based MOEAs did little progress, mainly because of the elegance and effectiveness of this algorithm, which attracted a significant amount of attention from partitioners and motivated numerous improvements during the last few years (see for example [Lin 12, Ishibuchi 08a]). Pareto-based selection mechanisms, however, are now being challenged by the curse of dimensionality (Pareto-based selection does not scale properly as we increase the number of objectives [Farina 02]). This has motivated an important amount of research in which the aim is to provide alternative selection mechanisms that can scale properly. The most popular trend in this regard, is to adopt a selection mechanism based on a performance measure [Zitzler 04]. This is a bit ironic considering that many of the most commonly used performance measures are known to have drawbacks (e.g., many of them are not Pareto-compliant [Zitzler 03]). This idea was first explored in the Indicator-Based Evolutionary Algorithm (IBEA) [Zitzler 04] is intended to be adapted to the user's preferences by formalizing such preferences in terms of continuous generalizations of the dominance relation. This is a very interesting idea, since it avoids the need to provide an explicit diversity preservation mechanism. In order to achieve this aim, the optimization goal of IBEA is defined in terms of a binary performance measure (e.g., the additive ϵ-indicator [Zitzler 03]). In a further paper, the same authors introduced the Set Preference Algorithm for Multiobjective Optimization (SPAM) [Zitzler 08], which consists of a hillclimber based on the same idea of IBEA, but which turns out to be more general, since it is not restricted to a single binary performance measure (several of such performance measures can be used in sequence, and any type of set preference relation is acceptable). Over the years, however, EMO researchers focused their efforts on studying an intriguing indicator called hypervolume [Zitzler 98],*4 which is the only known unary quality indicator which guarantees strict monotonicity regarding the Pareto dominance relation [Zitzler 03]. In fact, it has been proved that the maximization of this performance measure is equivalent to finding the Pareto optimal set [Fleischer 03]. The nice theoretical properties of the hypervolume indicator make it the perfect choice for designing an indicator-based MOEA, and such algorithms started to appear some years ago (see for example the S Metric Selection Evolutionary Multiobjective Optimization Algorithm (SMS-EMOA) [Emmerich 05, Beume 07] and the multi-objective version of CMA-ES (a well-known single-objective optimizer), which also adopts a hypervolume-based selection mechanism [Igel 07]). The use of the hypervolume, however, has some drawbacks, from which the main one is the high computational cost involved in the computation of the hypervolume contributions (which is required when adopting the hypervolume for selecting solutions). This has triggered a lot of research that has produced more efficient algorithms for the exact calculation of the hypervolume [While 12]. More recently, alternative schemes based on iterative procedures [Ishibuchi 09] and Monte Carlo sampling [Bader 11] have been proposed. Such algorithms, however, remain relatively slow when dealing with problems that have more than 5 objectives even if using approximation schemes.

The idea of designing indicator-based selection is, however, interesting and worth exploring. Currently, there is some evidence regarding the possibility of designing selection mechanisms based on other indicators that are computationally inexpensive. For example, in [Rodriguez 12], the recently proposed Δ_p indicator [Schutze 12] is incorporated in the selection mechanism of a MOEA with very promising results, in spite of its theoretical limitations. In fact, the proposed MOEA is shown to produce results similar to thsoe of the SMS-EMOA, but only at a tiny fraction of its computational cost (particularly when dealing with problems having more than 4 objectives). Given the importance of this topic, more research around indicator-based selection mechanisms is certainly expected to occur in the next few years (see for example [Brockhoff 12]).

Density estimators are the other component of MOEAs that also deserves some attention. In the early days of EMO, most MOEAs adopted naive fitness sharing schemes in which an individual was *penalized* for lying on the same niche of other individuals (a niche is defined either in decision or in objective function space by adopting a certain niche radius for each individual, whose value is normally defined by the user) [Goldberg 87, Deb 89]. Over the years, however, density estimators have also changed, and a variety of schemes have been proposed in the specialized literature, including clustering [Zitzler 99, Kukkonen

*4 The **Hypervolume** (also known as the S metric or the Lebesgue Measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively.

06], adaptive grids [Knowles 00, Knowles 03], crowded-comparison operators [Deb 02], and entropy [Farhang 02], among others.

Regarding algorithmic design, there are, however, other ideas that are worth mentioning. For example, the multi-objective evolutionary algorithm based on Decomposition (MOEA/D) [Zhang 07], which is based on the use of scalarizing functions is a very interesting and powerful MOEA that borrows concepts from mathematical programming (it is based on the Normal Boundary Intersection (NBI) method [Das 98]). This same idea of transforming a MOP into several single-objective optimization problems that can then be efficiently solved with a powerful optimizer has been explored by other researchers, using the ε-constraint method, which is another well-known mathematical programming technique (see for example [Ranjithan 01, Laumanns 06, Landa 06]).

In fact, it is possible to adopt other concepts to design new selection schemes for a MOEA. Some interesting ideas are, for example, the maximin method [Balling 01, Solteiro 10, Menchaca 12], game theory [Miyamoto 08, Annamdas 09], and the contact theorem [Osyczka 96], all of which are worth studying in more depth.

3·2 Scalability

The current algorithmic design trends reflect the emphasis that has been placed on scalability in the last few years because of the drawbacks of Pareto-based selection mechanisms (an aspect that was disregarded for many years in the specialized literature) [Wagner 07]. Such Pareto-based selection mechanisms don't scale properly, because the proportion of nondominated solutions in a population increases rapidly with the number of objectives. Indeed, in [Farina 02], it is shown that this number goes to infinity when the number of objectives approaches to infinity. This implies that in the presence of many objectives the selection of new solutions is carried out almost at random since a large number of the solutions are equally good in the Pareto sense [Knowles 07]. This has made scalability an important research topic [Sato 07, Purshouse 07, Ishibuchi 08b].

It is possible to identify two main approaches to deal with the so-called many-objective optimization problems:[*5] 1) to adopt relaxed forms of Pareto optimality by proposing an optimality relation that yields a solution ordering finer than that produced by Pareto optimality (see for example [Sulflow 07]) and 2) to reduce the number of objectives of the original MOP, so that the dimensionality of the problem can be lowered to a reasonable value that a standard MOEA can deal with [Saxena 07, Brockhoff 06, Lopez 09a]. Although this last approach could seem a more attractive choice, it has a number of difficulties (for example, the dimensionality reduction is not always possible). This has made relaxed forms of Pareto optimality more popular in the literature [Lopez 08].

It is also worth mentioning that some researchers consider a third approach to deal with many-objective optimization problems: the incorporation of preference information interactively during the search. By incorporating preferences we can cope with many-objective problems in two aspects. First, the search can be focused on the decision maker's region of interest, thus avoiding the evaluation of a very large number of solutions. Second, the preference relations usually used in interactive methods help to deal with a large number of objectives since they are able to rank incomparable nondominated solutions. Several ranking schemes and preference incorporation methods that fall within this category have been proposed in the specialized literature (see for example [Murata 09, Lopez 09b, Sato 07]).

It is interesting to notice that less attention has been paid to both diversity and visualization in many-objective optimization optimization, in spite of the evident importance of these two topics (see for example [Adra 11, Li 10]). Additionally, it has been until very recently that scalability in decision variable space has been studied in a MOEA's context (see for example [Durillo 10]).

One final issue that is worth mentioning is that the number of nondominated solutions may not be the main source of difficulty in many-objective optimization problems. A recent study presented in [Schutze 11] uses the descent cones to measure the probability to improve a solution through the use of the evolutionary operators of a MOEA. This study shred light into a very intriguing issue: the addition of an objective does not make the problem per se harder. This paper provides both theoretical and empirical evidence regarding the real sources of difficulty in many-objective optimization, and provides some discussions regarding the current research being performed around this topic. One of the main outcomes of this paper is that the emphasis has been on distinguishing solutions in a many-objective context, but without ensuring convergence. Finally, the authors also argue that memetic strategies (which incorporate local search engines) may be particularly useful for dealing with many-objective optimization problems (see for example [Lara 10]).

Clearly, all of the previously discussed topics that are

*5 Although the term *many-objective optimization* is used in an informal way in the literature, it is normally assumed that it refers to problems having four or more objectives.

related to scalability are still worth studying in more depth, and much more research around them is expected to arise in the next few years.

3·3 Efficiency

Efficiency in MOEAs can be considered from two perspectives: (1) in terms of algorithmic complexity and (2) in terms of the number of (objective function) evaluations performed by a MOEA. The first choice, which is the typical computer science notion of efficiency seems to leave little margin in the case of MOEAs, because the computational efficiency bounds of nondominance checking have been known for over 35 years [Kung 75]. Looking at the specialized literature, one would think that this is normally assumed by researchers, but few detailed studies of MOEA's algorithmic complexity and on the algorithms used to extract nondominated solutions from a set are currently available in the specialized literature (see for example [Yukish 04]). Also, few attempts have been made to incorporate the best-known algorithms for nondominance checking into a MOEA (see for example [Jensen 03]).

The second notion of efficiency has been more commonly addressed in the EMO literature. In the early days of the field, some researchers proposed clever approaches that aimed to reduce the number of individuals that were compared, so that a faster nondominance checking could be done. Perhaps the first effort in this direction was the Niched-Pareto Genetic Algorithm (NPGA) [Horn 94], which adopts a binary tournament selection based on nondominance. However, instead of comparing every solution against everybody else, NPGA takes a sample of the population and compares the two individuals selected for the tournament only with respect to such sample. The authors of this MOEA empirically showed that a sample of about 10% of the total population size was enough to provide reasonably good results at a lower computational cost than the other MOEAs available at that time.

There have also been proposals in which a very small population size is adopted, based on the concept of the micro-genetic algorithm [Krishnakumar 89], in which no more than five individuals are used in the population [Coello 01]. This sort of MOEA requires, however, of clever reinitialization schemes in order to avoid getting stuck during the search [Coello 01]. In spite of its possible drawbacks, this idea has been revisited by a few researchers in the last few years [Toscano 03, Tiwari 11].

Nowadays, this topic has attracted a lot of interest, since MOEAs have been increasingly applied to computationally expensive problems (e.g., in aeronautical engineering [Arias 11]). One of the main approaches in this regard has been the use of surrogate models, which have been used for a long time in engineering (see for example [Voutchkov 06, Knowles 06, Ray 06, Carrese 11]). The core idea of surrogate models is to build an approximate model of the problem, which is computationally unexpensive to evaluate. The main problem with surrogate models is that they evidently have errors with respect to the real objective function to be optimized (these differences are periodically checked to adjust the approximation model) and, sometimes, the error may be significant. Additionally, some of the current surrogates available for MOEAs can be applied only to problems of low dimensionality (e.g., parEGO [Knowles 06]).

The use of knowledge extracted during the search to improve the performance of the recombination and mutation operators is another possible choice (this the approach adopted in the cultural algorithms [Reynolds 95] which have been only scarcely explored in a multi-objective optimization context [Coello 03, Best 10]).

Knowledge from past fitness evaluations can also be used to build empirical models that approximate the fitness function that we aim to optimize. Such a model can be used to predict promising new solutions at a smaller computational cost than when using the original problems (see [Knowles 06, Jin 05]).

Another interesting idea is to use fitness inheritance [Smith 95] in order to reduce the number of objective function evaluations. Under this scheme, when assigning fitness to an individual, a certain number of times the evaluation is done as usual, and the rest of the time, the fitness of an offspring is assigned as the average of the fitnesses of its parents. This avoids a fitness function evaluation by exploiting the similarity that an offspring is assumed to have with its parents. Fitness inheritance has been extended to multi-objective optimization by some researchers with interesting results [Bui 05, Reyes 05, Pilato 07, Giannakoglou 10] in spite of the limitations that this sort of approach seems to have in multi-objective optimization [Ducheyne 08].

Hybrid schemes can also be an interesting choice for designing efficient MOEAs. In [Hernandez 06], a MOEA is used to produce a coarse-grained approximation of the Pareto front, and then a local search scheme based on rough sets theory is adopted to rebuild the missing portions of the Pareto front. A similar approach is adopted in [Santana 06], but using scatter search as the local search engine. These schemes can clearly reduce the number of objective function evaluations performed [Wanner 08]. Also, the use of special operators that speed up convergence is very promosing (see for example [Adra 07]).

For more information on the incorporation of knowledge into a MOEA, the interested reader is referred to [Landa 08].

3·4 Hybridization

A very interesting topic that has attracted a lot of interest in recent years is the hybridization of MOEAs with mathematical programming techniques. The main motivation for this sort of hybridization is to speed up convergence of a MOEA. In this direction, the main work has focused on the hybridization of MOEAs (which act as a global search engine, given their coarse-grain nature) with a gradient-based method (which acts as a local search engine that refines the best solutions obtained by the MOEA), giving rise to the so-called multi-objective memetic algorithms [Goh 09].

Fliege [Fliege 00] introduced a method called *Steepest Descent Direction* which uses a definition of the Multiobjective Gradient (MOG). Fliege's method implies solving certain quadratic-programming problem involving the Jacobian matrix of the MOP. This method works for convex Pareto fronts as well as for concave Pareto fronts.

In [Bosman 06, Brown 03, Harada 06], the idea of moving a solution towards a particular improvement direction is analyzed. The goal of these works was to find a descent direction for the multi-objective case, which is equivalent to the one that we can obtain for the single-objective case, when using the gradient. As it turns out, this is indeed a multi-objective problem as well [Bosman 05a], because we aim for a set of descent directions, instead of only one. The authors of [Bosman 05a] propose an analytical way of computing this set of directions, from which one is randomly selected. This method is called *Combined-objectives Repeated Line Search* (CORL) and was further improved in [Bosman 06]. In both cases, CORL was hybridized with $\mathbb{MIDEA}$ [Bosman 05b]. Although the preliminary results obtained with CORL are promising, this approach has some scalability problems [Harada 06].

An alternative to the previous approach was made by Harada et al. [Harada 06]. They use the ideas introduced by Fliege [Fliege 00] to build what they called the *Pareto Descent Method* (PDM). These researchers proposed PDM as an option to deal with special MOPs when the solution lies on the boundary between the feasible and the infeasible regions. In these cases, it is necessary to find a different descent direction. The authors do not propose a hybrid based on PDM; this is left as future work by them [Harada 06].

Brown and Smith [Brown 03] introduced the concept of *directional cone* which is conformed by the intersection of the negative half-spaces (generated by the gradients) over all the objective functions. Brown and Smith proposed that the offspring, in the particular MOEA, must lie inside the directional cone. However, they do not propose a full algorithm. Furthermore, it is not clear how the directional cone could deal with nonconvex Pareto fronts. Finally, they do not complete the hybridization with any particular MOEA. A procedure to approximate the gradient of the objective functions using neighborhood information is also introduced in [Brown 03]. According to its authors, that method reduces the computational cost of calculating the Jacobian Matrix in Fliege's Method. But, it is easy to see that this also increases the number of objective function evaluations required.

The main problem of using an improvement direction is that it is impossible to know beforehand for how long will a certain direction be useful, unless a nonlinear problem is solved (which requires solving another nonlinear single-objective optimization problem). Evidently, it makes sense to follow a promising descent direction as long as it remains as a good search direction. However, in problems with a very irregular geometry in the search space, the optimum descent direction will be constantly changing. Thus, this issue remains as an important drawback when adopting gradient-based information. Another drawback in Fliege's methods – on which [Brown 03], [Harada 06] and [Bosman 05a] are based —is that it has a slow convergence rate [Brown 03], and it is susceptible to get trapped in local (i.e., false) Pareto fronts.

Shukla [Shukla 07] introduced the use of two stochastic gradient-based techniques to improve the mutation mechanism of the NSGA-II: Schäffler's stochastic method [Schaffler 02] and Timmel's method [Timmel 80]. These are relatively straightforward approaches that could be easily improved, but they also illustrate the local nature of the gradient-based information and its possible limitations. Also, Schäffler's method requires a high number of objective function evaluations, which is an important drawback, if we consider that the main aim of using gradient-based information is precisely to reduce the overall computational cost of a MOEA.

More recent results in this direction, indicate that the hybridization of MOEAs with gradient-based methods can be done more efficiently and effectively [Lara 10, Lara 10]. This is indeed a very promising research topic. For a more thorough treatment of this topic the interested reader is referred to [Lara 12].

The hybridization of MOEAs with direct search methods (i.e., those that don't require gradient information) are still relatively scarce. For example, Hu et al. [Hu 03] combined sequential quadratic programming (SQP) and the

ε-constraint method with SPEA [Zitzler 99] and NSGA-II [Deb 02]. This approach transforms a multi-objective problem into several single-objective optimization problems, which are solved using SQP. Koduru et al. [Koduru 07] poposed a hybrid based on particle swarm optimization [Kennedy 01] and Nelder and Mead's method [Nelder 65]. and uses an external archive that contains the fuzzy nondominated solutions that have been found during the search. The *Nonlinear Simplex Search Genetic Algorithm (NSS-GA)* [Zapotecas 08] hybridizes the NSGA-II with one of the following mathematical programming methods (which are used as local search engines): Nelder and Mead's method (which is used for multidimensional optimization) and the golden section (which is used for unidimensional optimization). Zhong et al. [Zhong 10] hybridized the nonlinear simplex search and Differential Evolution (DE) [Storn 97]. The simplex was constructed selecting random solutions from the current population, which were then sorted according to Pareto dominance. At each iteration of the local search, a movement into the simplex was performed for generating new nondominated solutions. Koduru et al. [Koduru 08] have also proposed to adopt Nelder and Mead's method as a local search engine, hybridized with a genetic algorithm. Wei and Zhang [Wei 11] proposed to use a simplex model to predict the position of the optimal positions in the next iteration, when solving a dynamic multi-objective problem. A crossover operator based on particle swarm optimization is used in this case as the global search engine. In more recent work, [Zapotecas 12] proposed a more powerful hybrid that combines MOEA/D [Zhang 07] with Nelder and Mead's method.

Evidently, more work on the hybridization of MOEAs with direct search methods is also required. For example, the potential of other approaches such as Hooke and Jeeves' method (also known as pattern search) [Hooke 61] and the Complex method [Ravindran 06] (which is intended for problems with constraints) has not been only scarcely explored in a multi-objective context (see for example [Zapotecas 10]).

4. Some Future Challenges

Some of the topics that, from the author's perspective, are worth studying during the next few years are the following:

(1) **Parameter control**: The design of mechanisms that allow an automated control of the parameters of an evolutionary algorithm is a topic that has been frequently studied in single-objective optimization (see for example [Eiben 07, Meyer 07]). However, this topic has been only scarcely studied in a multi-objective context (see for example [Toscano 03, Deb 07]). Evidently, this is a very challenging topic, due to the high nonlinear interaction among the parameters of an evolutionary algorithm [Dejong 07]. Thus, topics such as online adaptation and self-adaptation are rarely addressed in EMO (see for example [Zapotecas 11]). In fact, the design of a parameterless MOEA is rarely discussed in the specialized literature, and the microgenetic algorithm 2 [Toscano 03] seems to be the only effort in that direction until now. More recently, some research efforts have focused on specific topics related to parameter control, such as stopping criteria of a MOEA [Wagner 11]. However, there are still few studies on the way in which the parameters of a MOEA affect its performance (see for example [Toscano 05]) or on the use of information from previous runs to improve performance of subsequent runs (see [Dejong 07]).

(2) **Theory**: Although there has been a remarkable progress in this area, there is still relatively little work on theoretical aspects of MOEAs. Most theoretical work has typically focused on proofs of convergence of a MOEA under certain conditions [Villalobos 06, Schutze 07] and on runtime analysis of MOEAs[*6] [Giel 03, Laumanns 04, Neumann 04, Friedrich 08]. More recent work focuses, however, on computational complexity analysis [Neumann 12], on convergence rates of specific MOEAs [Beume 11], on many-objective optimization [Brockhoff 09, Schutze 11] and on the hypervolume indicator [Auger 09, Bringmann 12, Auger 12].

(3) **Incorporation of user's preferences**: An aspect that is frequently disregarded in the EMO literature is the fact that searching for good trade-off solutions is not the only task required when solving a multi-objective problem. The user (or decision maker) is normally interested only in a certain portion of the Pareto front and, most certainly, will be interested in only a few nondominated solutions. Thus, it is required to be able to incorporate the user's preferences into the selection mechanism of our MOEA, such that the search can be conducted in a more efficient way (e.g., biasing the search only to the regions of interest as defined by the user). Although there is some important work done in this area (see for example [Cvetkovic 02, Branke 08, Koksalan 10]), more

*6 Runtime analysis addresses the question of how long a certain algorithm takes to find the optimal solution for a specific problem or a class of problems.

research is still required, since the design of good mechanisms for incorporating user's preferences into a MOEA requires studying first the extensive work done in this regard in the Operations Research literature [Figueira 05].

Other topics that are also worth exploring but that were not discussed here due to space limitations, include constraint-handling techniques for MOEAs [Oyama 07, Harada 07], advanced data structures for implementing external archives (the most common way of incorporating elitism in current MOEAs) [Fieldsend 03, Hernandez 07, Hernandez 11], parallelization techniques for MOEAs [Talbi 08], and visualization of Pareto fronts, particularly when dealing with many-objective problems [Obayashi 03, Hettenhausen 10].

5. Conclusions

This paper has provided a short review of the main topics on which EMO researchers are currently working, as well as those, which, from the author's perspective, seem more promising for doing research within the next few years. Overall, this paper aims to serve as a quick reference for those interested in doing research in the EMO field, since this paper provides a rough but wide view of the current state of the area. Newcomers may be particularly interested in the last part of the paper in which some possible research topics (for example, for developing a PhD thesis) are provided. Hopefully, this paper will motivate interest from several researchers and students, because their work is required in order to maintain active the EMO field for many more years.

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