

A Proposal for Using an Evolutionary Algorithm with a History Mechanism for Tuning a Chess Evaluation Function

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Abstract

Here, we propose an evolutionary algorithm (i.e., evolutionary programming) for tuning the weights of a chess engine. Most of the previous work in this area has normally adopted co-evolution (i.e., tournaments among virtual players) to decide which players will pass to the following generation, depending on the outcome of each game. In contrast, our method uses evolution to decide which virtual players will pass to the next generation based on the number of positions solved from a number of chess grandmaster games. Using a search depth of 1-ply our method can solve 40.81% of the positions evaluated from chess grandmaster games (this value is higher than reported in the previous related work). Additionally, our method is capable to solve 53.10% of the positions using a historical mechanism that keeps a record of the “good” virtual players found during the evolutionary process. Our proposal has also been able to increase the competition level of our search engine, when playing against the program *Chessmaster* (grandmaster edition), our chess engine reach a rating of 2404 points for the best virtual player with supervised learning, and 2442 rating points for the best virtual player with unsupervised learning. Finally, it is also worth mentioning that our results indicate that the piece material values obtained by our approach are similar to the values known from chess theory.

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1. Introduction

Computer chess is considered to be had started with the work of Alan Turing and Claude Shannon in the mid and late 1940s. In 1947, Alan Turing [25] designed a pioneer program to play chess, and in 1949 Claude Shannon [22] proposed two strategies to implement a chess engine. The first of them, called “Type A,” considered all possible moves to a fixed depth of the search tree, and the second, called “Type B,” used chess knowledge to explore the main lines to a greater depth.

During the 1950s, chess programs played at a very basic level, but by the 1960s, chess programs could defeat amateur chess players. During the 1970s, chess programs began to use heuristics and specialized hardware to improve their rating. During the 1980s, tournaments between chess programs and humans started.

During the 1990s, chess programs became powerful enough to start challenging chess masters. In 1997, *Deep Blue* defeated the world chess champion, Garry Kasparov, with a final score of 3.5 to 2.5 (Deep blue was capable of evaluating 200 million positions per second). In 2002, Garry Kasparov had a six games match against the chess program that was the world champion at that time, *Deep Junior*. The final score was 3 to 3. In 2006, the world chess champion Vladimir Kramnik, from Russia, was defeated by the program *Deep Fritz*. The final score was 4 to 2 in favor of *Deep Fritz*.

The current paper is an extended and improved contribution of our previous work [27] in which we initially provided a preliminary approach. In this work, we go into the addressed topic and propose an evolutionary algorithm (properly evolutionary programming) to tune the weights of chess evaluation function through a database of chess grandmaster games. The main difference with the previous work is that here, we tune a larger number of weights. Furthermore, we also add a historic mechanism that allows to retain “good” virtual players during the evolutionary process. In this work, we used 33 weights and David-Tabibi et al. [8] uses 35 weights.

The remainder of this paper is organized as follows. In Section 2, we briefly review the previous related work that makes use of co-evolution (tournaments among virtual players), and the only two works that we could find, which make use of evolution through a database of grandmaster games. In Section 3, we describe the characteristics of our chess engine, and the mathematical expression adopted for the evaluation function of our chess engine. In Section 4, we show the methodology that we adopted to tune the weights of our chess engine. In Section 5, we present our experimental results. Finally, our conclusions and some possible paths for future research are provided in Section 6.

2. Previous Related Work

First, we will briefly discuss works that do not make use of an evolutionary algorithm for tuning the weights of the evaluation function of a chess engine. Thrun [24] developed the program NeuroChess which learned to play chess from final outcomes with an evaluation function represented by neural networks. This work also included both temporal difference learning [23] and explanation-based learning [9]. Hsu et al. [16] tuned the weights of their evaluation function for the computer *Deep Thought* (later

called *Deep Blue*) using a database of grandmaster-level games. Beal and Smith [1] determined the values of the pieces using temporal-difference learning. In further work, the concept of piece-square values was introduced into their work [2].

Evolutionary algorithms have also been used before for tuning the evaluation function of a chess engine. Kendall and Whitwell [18] used them for tuning the evaluation function of their chess program. Nasreddine et al. [21] proposed a real-coded evolutionary algorithm that incorporated the so-called “dynamic boundary strategy” where the boundaries of the interval of each weight are dynamic.

Bošković presented three works that make use of differential evolution to adjust the weights of the evaluation function of their chess program. In their first work [4], they tuned the chess material values and the mobility factor of the evaluation function. The weights obtained matched the values known from chess theory. In a second work, Bošković et al. [5] employed adaptation and opposition-based optimization mechanisms with co-evolution to improve the rating of their chess program. In a third work, Bošković et al. [3] improved their opposition-based optimization mechanisms with a new history mechanism which uses an auxiliary population containing competent individuals. This mechanism ensures that skilled individuals are retained during the evolutionary process.

Genetic programming has also been used for tuning the weights of the chess evaluation function. Hauptman and Sipper [14] evolved strategies for playing chess endgames. Their evolved program could draw against CRAFTY which is a state-of-the-art chess engine with a rating of 2614 points. In a second work, Hauptman and Sipper [15] evolved entire game-tree search algorithms to solve mate-in- N problems in which the opponent cannot avoid being mated in at most N moves. It is worth noticing that this work does not adopt the alpha-beta pruning algorithm.

Genetic algorithms have also been used for tuning the weights of the chess evaluation function. David-Tabibi et al. [7] used reverse engineering to adjust the weights of an evaluation function. Basically, they used a grandmaster-level chess program for tuning the weights of their chess program. In a second work, David-Tabibi et al. [8] combined supervised and unsupervised learning to build a grandmaster-level program. This work presented the first attempt to adjust the weights of a chess program by learning only from a database of games played by humans. Evolutionary programming has been used before for tuning the weights of the chess evaluation function. Fogel et al. [10] used this sort of algorithms to improve the rating of a chess program by 400 points. They tuned the material values of the pieces, the piece-square values, and the weights of three neural networks. Their computer programs learned chess by playing games against itself. In a second work, Fogel et al. [11] incorporated co-evolution, but this time, they evolved their program during 7462 generations, reaching a rating of 2650. The resultant program was called *Blondie25*. In a third work, Fogel et al. [12], used rules for managing the time allocated per move inside their program *Blondie25*. They achieved a rating of 2635 points against the program *Fritz8.0*, who was rated #5 in the world. It is noteworthy that *Blondie25* was also the first machine learning based chess program able to defeat a human chess master.

Vázquez-Fernández et al. [26] used a database of typical chess problems to adjust the weights of the evaluation function of their chess engine. Using evolutionary programming as their search engine, they mutated only those weights involved in the

solution of the current problem and adapted the mutation mechanism through the number of problems solved by each virtual player. With their algorithm they obtained the “theoretical” values of the pieces and achieved an increase in the strength of their chess engine of 335 points. In a further paper, Vázquez-Fernández et al. [27] used a database of games played by chess grandmasters to adjust the weights of the material values and the mobility factor of the pieces. In this case, they obtained the “theoretical” values of the pieces with their evolutionary algorithm.

There are only two works in which the selection mechanism of an evolutionary algorithm used to play chess is based on databases of chess grandmaster to decide which virtual players will pass to the following generation (in the remaining works in which evolutionary algorithms were adopted in some way, the proposed approaches used the final results of a game: win, loss or draw). In the first work [8], the authors carried out the learning (called supervised) using a genetic algorithm. Additionally, in that paper, the authors use co-evolution (called unsupervised learning) to improve the adjust of the weights of their chess engine. In the second work [27], the authors adjusted the weights of both the material values of the pieces and the mobility factor through an evolutionary algorithm. Our work differs from this last paper in that we adopt a larger number of weights. Additionally, we also adopt a historic mechanism to allow the best virtual players to survive throughout the evolutionary process.

3. Our Chess Engine

To carry out our experiments, we developed a chess engine with the following characteristics:

- The search depth adopted by our engine is
- We use the alpha-beta pruning search algorithm [19].of one ply (which corresponds to the movement of one side) for the training phase as used in [8], and of six ply for the games among virtual players as recommended in [16].
- We incorporated a mechanism to stabilize positions through the Quiescence algorithm, which takes into account the exchange of material and the king’s checks.
- We use hash tables and iterative deepening [6].

Our chess program evaluates the position of the virtual player A with the following expression:

$$eval = \sum_{i=1}^{22} Weight_i * f_i \quad (1)$$

where $Weight_i$ is one of the weights shown in Table 1, and f_i is the feature of the weight $Weight_i$ for the virtual player A . The description of the features are the following:

- f_{PAWN_VALUE} is the number of pawns of the virtual player A .

- f_{KNIGHT_VALUE} is the number of knights of the virtual player A .
- f_{BISHOP_VALUE} is the number of bishops of the virtual player A .
- f_{ROOK_VALUE} is the number of rooks of the virtual player A .
- f_{QUEEN_VALUE} is the number of queens of the virtual player A .
- $f_{PAWN_DOUBLED_PENALTY_VALUE}$ denotes the number of doubled pawns of the virtual player A .
- $f_{PAWN_ISOLATED_PENALTY_VALUE}$ denotes the number of isolated pawns of the virtual player A .
- $f_{PAWN_BACKWARD_PENALTY_VALUE}$ denotes the number of backward pawns of the virtual player A .
- f_{PAWN_PASSED} denotes the number of passed pawns of the virtual player A .
- $f_{PAWN_CENTRAL}$ denotes the number of central pawns of the virtual player A .
- $f_{KNIGHT_SUPPORTED}$ denotes the number of knights (of the virtual player A) supported or defended by one of his pawns.
- $f_{KNIGHT_OPERATIONS_BASE}$ denotes the number of knights (of the virtual player A) in an operation's base (it is when a knight cannot be evicted from its position by an opponent's pawn).
- $f_{KNIGHT_PERIPHERY_0}$ denotes the number of knights (of the virtual player A) in the squares $a_1, \dots, a_8, b_1, \dots, g_1, h_1, \dots, h_8$, and b_8, \dots, g_8 .
- $f_{KNIGHT_PERIPHERY_1}$ denotes the number of knights (of the virtual player A) in the squares $b_2, \dots, b_7, c_2, \dots, f_2, g_2, \dots, g_7$, and c_7, \dots, f_7 .
- $f_{KNIGHT_PERIPHERY_2}$ denotes the number of knights (of the virtual player A) in the squares $c_3, \dots, c_6, d_3, e_3, f_3, \dots, f_6$, and d_6, \dots, e_6 .
- $f_{KNIGHT_PERIPHERY_3}$ denotes the number of knights (of the virtual player A) in the squares d_4, e_4, d_5, e_5 .
- $f_{ROOK_OPEN_COLUMN}$ denotes the number of rooks (of the virtual player A) in an open column.
- $f_{ROOK_SEMIOPEN_COLUMN}$ denotes the number of rooks (of the virtual player A) in a semi-open column (a column that contains only opponent's pawns).
- $f_{ROOK_CLOSED_COLUMN_BEHIND}$ denotes the number of rooks (of the virtual player A) in a closed column behind of its pawns. We defined a closed column as the column that contains pawns of both players.
- $f_{ROOK_CLOSED_COLUMN_AHEAD}$ denotes the number of rooks (of the virtual player A) in a closed column ahead of its pawns.

- $f_{ROOK_SEVEN_RANK}$ denotes the number of rooks (of the virtual player A) in the seventh rank.
- $f_{KNIGHT_MOBILITY}$ is the number of knight's moves of the virtual player A .
- $f_{BISHOP_MOBILITY}$ is the number of bishops' moves of the virtual player A .
- $f_{ROOK_MOBILITY}$ is the number of rooks' moves of the virtual player A .
- $f_{QUEEN_MOBILITY}$ is the number of queens' moves of the virtual player A .
- $f_{KING_MOBILITY}$ is the number of king's moves of the virtual player A .
- $f_{KING_ATTACKING_MATERIAL}$ is the sum of the material value of the pieces that are attacking the opposite king. We mean those pieces whose movements act on its opposite king's square or on its opposite king's adjacent squares.
- $f_{KING_DEFENDING_MATERIAL}$ is the sum of the material value of the pieces that are defending its king. We mean those pieces whose movements act on its opposite king's square or on its opposite king's adjacent squares.
- $f_{KING_CASTLING}$ is a binary value. It is true if the king is castled; otherwise, it is false.
- f_{KING_PAWNS} is the number of pawns located on its king's adjacent squares.
- f_{BISHOP_AHEAD} is the number of pawns which are in front of its bishop and obstructing its movement.
- $f_{BISHOP_PAWNS_MOBILITY}$ is the number of movements of the pawns which obstruct the movement of the bishop.
- f_{BISHOP_PAIR} is a binary value. It is true if the player A has the bishop pair; otherwise, it is false.

The main aim of the work reported here is to show that the weights of the evaluation function from eq. (1) can be tuned using evolutionary programming [13]. In our approach, the training of the virtual players is conducted using a database of games from chess grandmaster games. Additionally, our work proposes the use of a historic mechanism which allows good virtual players to survive throughout the evolutionary process.

4. Our Proposed Approach

As indicated before, our proposed approach is based on an evolutionary algorithm (evolutionary programming [13]) which has a selection mechanism based on supervised learning through a database of chess grandmaster games. The selection mechanism allows that the virtual players which find the largest number of movements proposed by chess grandmaster pass to the next generation. Also, our evolutionary algorithm has a historic mechanism that allows to recover good virtual players which have temporarily left the evolutionary process.

The evolutionary algorithm is shown in Algorithm 1. Line 1 initializes the weights of N virtual players with random values within their corresponding boundaries. Line 2 sets the generations counter equal to zero. In lines 3 to 15 we carry out the tuning of the weights for the N virtual players during $Gmax$ generations. In line 4, a virtual player's score is incremented in one for each movement of the p positions on the database for which the virtual player did the same action as the human chess master. Line 5 applies the selection mechanism so that only the best $N/2$ virtual players pass to the following generation. Line 6 updates the historical mechanism of virtual players in a way that maintains in an array the best virtual players who have left the evolutionary process. In lines 7 to 13 we obtain the second half of virtual players needed. If $100 * rand(0, 1) < P_r$ (where P_r is a control parameter defined by the user), we obtain the virtual player i from the historical mechanism, and if not, we mutate the virtual player $i - N/2$ to obtain the virtual player i . Finally, line 14 increases the generation counter in 1.

The procedure for computing the score of each virtual player is described in Algorithm 2. In lines 1 to 3, we establish the score counter to zero for each virtual player. Line 4 chooses p training positions from database S . Line 5 chooses chess grandmaster movements for position p . Line 6 sets the position p (this allows to each virtual player to calculate its next movement). Finally, each virtual player calculates its next move n , and if this movement matches movement m , this virtual player increases its score by 1.

Algorithm 1 EvolutionaryAlgorithm()

```

1: initializePopulation();
2: g = 0;
3: while g++ < Gmax do
4:   scoreCalculation();
5:   selection();
6:   updateHistoricalMechanism();
7:   for i = N/2 → N - 1 do
8:     if 100 * rand(0, 1) < Pr then
9:       VP[i] ← historicalMechanism();
10:    else
11:      VP[i] ← mutate(VP[i-N/2]);
12:    end if
13:  end for
14:  g++;
15: end while

```

Algorithm 2 scoreCalculation()

```
1: for  $i = 0 \rightarrow N - 1$  do
2:   score[i] = 0;
3: end for
4: for each position  $p$  in database  $S$  do
5:    $m = \text{grandmasterMovement}(p)$ ;
6:   setPosition( $d$ );
7:   for each virtual player  $i$  do
8:      $n = \text{nextMovement}(i)$ ;
9:     if  $m == n$  then
10:      score[i]++;
11:    end if
12:   end for
13: end for
```

4.1. Initialization

The population of our evolutionary algorithm was initialized with 20 virtual players (10 parents and 10 offspring in subsequent generations). The weight values for these virtual players were randomly generated with a uniform distribution within their allowable bounds. The allowable bounds for each weight is shown in Table 1.

4.2. Mutation

If the condition $100 * \text{rand}(0, 1) < P_r$ is true (in Algorithm 1), one offspring is taken from the historical mechanism; otherwise, it is mutated from its corresponding parent (the parent $i - N/2$ is mutated to generate the offspring i , for $i = N/2, \dots, N - 1$).

The values that were mutated are shown in Table 1.

In our implementation, we adopted Michalewicz's non-uniform mutation operator [20]. The expression to obtain the mutated weight V'_k from the previous weight V_k is the following:

$$V'_k = \begin{cases} V_k + \Delta(t, UB - V_k) & \text{if } R = \text{TRUE} \\ V_k - \Delta(t, V_k - LB) & \text{if } R = \text{FALSE} \end{cases} \quad (2)$$

where $[LB, UB]$ is the range of the weight V_k , and $R = \text{flip}(0.5)$ (the function $\text{flip}(p)$ returns TRUE with a probability p). Michalewicz suggests using:

$$\Delta(t, y) = y * (1 - r^{(1-t/T)^b}) \quad (3)$$

where r is a random real number in the range $(0, 1)$, T is the maximum number of generations and b is a user-defined parameter. In our case, $b = 5$, which is the value recommended by Michalewicz [20].

It should be noted that no crossover operator is employed in our case, since we adopted evolutionary programming (this paradigm models the evolutionary process at the species level and, therefore, it does not incorporate any crossover operator).

5. Experimental Results

Our experiments were carried out on a personal computer with a 64-bits architecture, with 3 GBytes in RAM, having two cores running at 2.8 GHz. The programs were compiled using *g++* in the OpenSuse 11.4 operating system. For our virtual players we used the opening book *Olympiad.abk* included with the graphical user interface *Arena*.¹

For the training of our experiments we used a database of 4000 games from grandmasters in chess having a rating above 2600 Elo. These experiments uses the chess engine described in Section 3.

5.1. Experiment A

In this case, we tuned the first 24 weights shown in Table 1. We carried out 31 runs using $Pr = 5, 10, \dots, 35, 40$ and found that $Pr = 10, 15, 20$ produced the highest number of positions properly solved. We carried out the following experiments:

1. 31 runs with $Pr = 0$ and 24 weights (without the historical mechanism). These runs are shown in Table 2.
2. 31 runs with $Pr = 10$ and 24 weights. These runs are shown in Table 3.
3. 31 runs with $Pr = 15$ and 24 weights. These runs are shown in Table 4.
4. 31 runs with $Pr = 20$ and 24 weights. These runs are shown in Table 5.

The description of these tables is the following. In the first four rows we describe the number of training generations (200), the population size (20), the number of training positions (4000), and the value of Pr employed. Next, the first and second column describe the percentage of positions solved by the best virtual player at generation 0 and generation 200, respectively. The third and fourth column describe the average positions solved by all virtual players at generation 0 and generation 200, respectively. The fifth column describes the execution time for each run. Text in black represents the run corresponding to the median of the fourth column (with the legend median). Finally, the penultimate row describes the standard deviation of the fourth column, and the last row describes the average execution time for all runs. In these tables we can see that the positions solved for the median run at generation 200 were 33.50%, 46.30%, 42.29% and 40.69% for $Pr = 0$, $Pr = 10$, $Pr = 15$, $Pr = 20$, respectively. In Table 2 and Table 3, the weights obtained at generation 200 for the best virtual player was called $VP_{200,0}^{24}$ and $VP_{200,10}^{24}$, respectively.

In Figure 1 we can see the percentage of positions solved for the best virtual player and the average positions solved by all virtual players for the median run in Table 2. At generation 0, the average positions solved by all virtual players was 18.07%, and 21.01% for the best virtual player. At generation 200, the average positions solved by all virtual players was 33.50%, and 33.53% for the best virtual player. Note that this value is competitive with that reported in [8] after the supervised evolution with 32% of positions solved.

¹<http://www.playwitharena.com/>

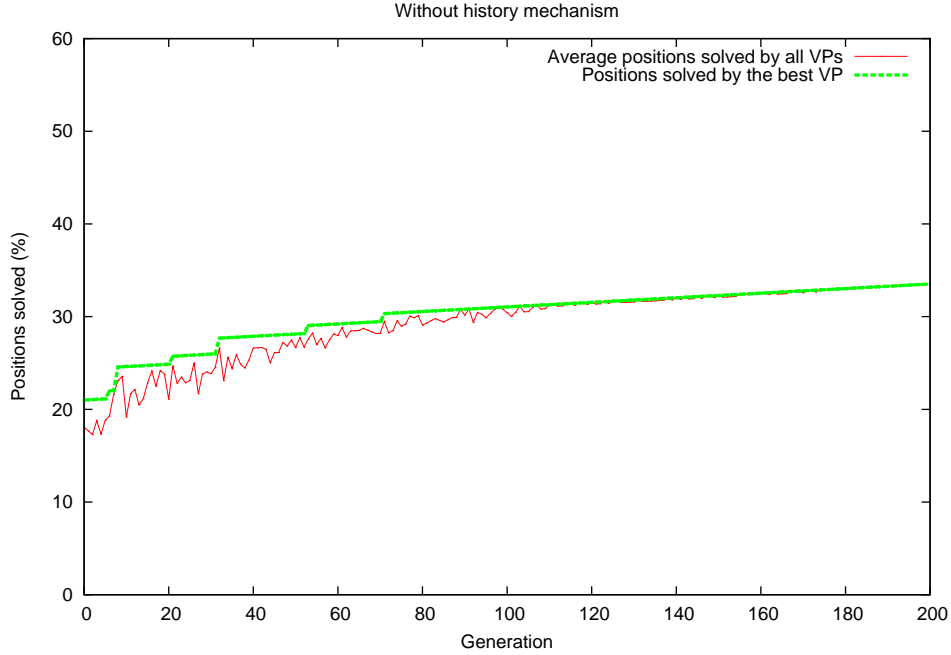


Figure 1: Evolutionary process without the historical mechanism for $P_r = 0$ with 24 weights. The plot shows the percentage of positions solved for the median run in Table 2 during 200 generations. We considered a total of 4000 positions.

In Figure 2 we can see the percentage of positions solved for the best virtual player and the average positions solved by all virtual players for the median run in Table 3. At generation 0, the average positions solved by all virtual players was 22.39%, and 26.49% for the best virtual player. At generation 200, the average positions solved by all virtual players was 46.30%, and 46.35% for the best virtual player. Since we adopted Michalewicz's non-uniform mutation operator [20], the percentage of positions solved was equal for the average of all virtual players and the best virtual player at the end of the evolutionary process (generation 200).

In order to show the scalability of our method, we proceeded to repeat the previous experiments with the 33 weights shown in Table 1. It is worth mentioning that previously, in the paper [27], also we tuned successfully the weights of the material values and the mobility factor of the pieces.

Again, we carried out 31 runs using $P_r = 5, 10, \dots, 35, 40$ and found that $P_r = 10, 15, 20$ produced the highest number of positions properly solved. We carried out the following experiments:

1. 31 runs with $P_r = 0$ and 33 weights (without the historical mechanism). These runs are shown in Table 6.
2. 31 runs with $P_r = 10$ and 33 weights. These runs are shown in Table 7.
3. 31 runs with $P_r = 15$ and 33 weights. These runs are shown in Table 8.

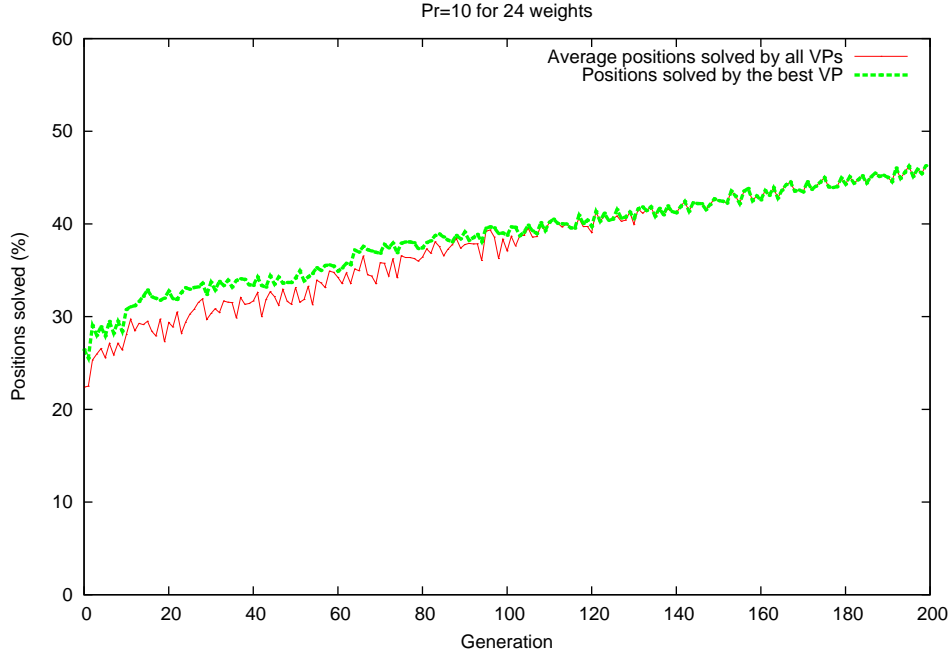


Figure 2: Evolutionary process for $P_r = 10$ with 24 weights. The plot shows the percentage of positions solved for the median run in Table 3 during 200 generations. We considered a total of 4000 positions.

4. 31 runs with $P_r = 20$ and 33 weights. These runs are shown in Table 9.

The description of these tables is the same as we described above. In these tables we can see that the positions solved for the median run at generation 200 were 40.81%, 53.10%, 50.12% and 55.16% for $P_r = 0$, $P_r = 10$, $P_r = 15$, $P_r = 20$, respectively. In Table 6 and Table 7, the weights obtained at generation 200 for the best virtual player was called $VP_{200,0}^{33}$ and $VP_{200,10}^{33}$, respectively.

In Figure 3 we can see the percentage of positions solved for the best virtual player and the average positions solved by all virtual players for the median run in Table 6. At generation 0, the average positions solved by all virtual players was 25.42%, and 30.51% for the best virtual player. At generation 200, the average positions solved by all virtual players was 40.81%, and 40.85% for the best virtual player.

In Figure 4 we can see the percentage of positions solved for the best virtual player and the average positions solved by all virtual players for the median run in Table 7. At generation 0, the average positions solved by all virtual players was 22.46%, and 25.38% for the best virtual player. At generation 200, the average positions solved by all virtual players was 53.10%, and 53.08% for the best virtual player. Since we adopted Michalewicz's non-uniform mutation operator [20], the percentage of positions solved was equal for the average of all virtual players and the best virtual player at the end of the evolutionary process (generation 200).

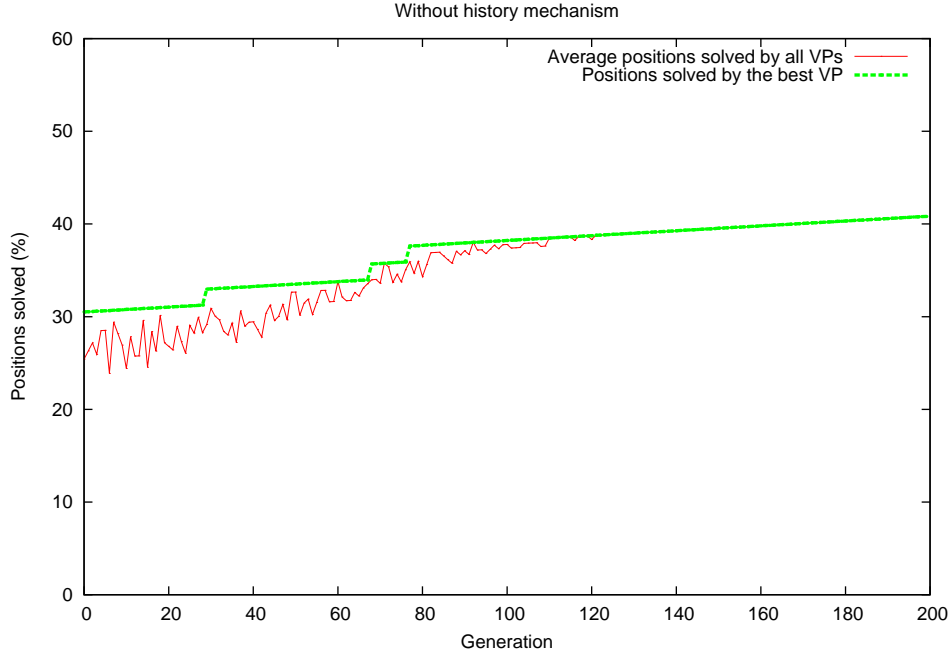


Figure 3: Evolutionary process without the historical mechanism for $P_r = 0$ with 33 weights. The plot shows the percentage of positions solved for the median run in Table 6 during 200 generations. We considered a total of 4000 positions.

At the end of the evolutionary process (generation 200), the percentage of positions solved when using the historical mechanism was larger (53.10%) than those achieved by the version of our algorithm that does not use the historical mechanism (33.50%). Also, the number of positions solved with the historical mechanism is larger than the value reported in [8].

In Table 1, we show the tuning weights for the virtual player $VP_{200,10}^{33}$. In this table we can see that the material values of the pieces are similar to the “theoretical” values known from chess theory [22].

In the experiments of this section, we used a search depth of one plie for our chess engine.

5.2. Experiment B

In this experiment we carried out 31 runs with the 33 weights shown in Table 1 for 1000, 2000 and 3000 positions from chess grandmaster games. The Table 10 shown in the first, second and third column, respectively, the training case, the median of the average positions solved by all virtual players and the standard deviation for the 31 runs at generation 200 with $P_r = 10$. The corresponding values for the training case 4000 were taken from the previous experiment. In this table we found that among smaller it is the size of the training case, minor will be the standard diversion and the of average positions solved by all virtual players.

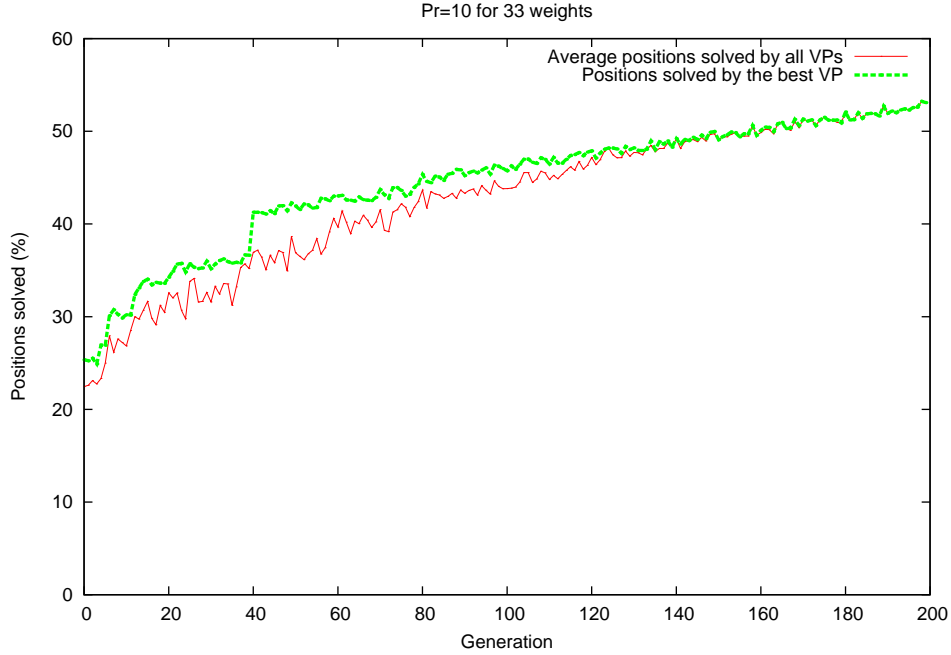


Figure 4: Evolutionary process for $P_r = 10$ with 33 weights. The plot shows the percentage of positions solved for the median run in Table 7 during 200 generations. We considered a total of 4000 positions.

In the experiments of this section, we used a search depth of one plie for our chess engine.

5.3. Experiment C

In order to show that the evolutionary process of our method produces virtual players with a higher rating, we carried out 200 games between $VP_{200,10}^{24}$ versus $VP_{200,0}^{24}$, $VP_{200,10}^{24}$ versus $VP_{0,0}^{24}$ and $VP_{200,0}^{24}$ versus $VP_{0,0}^{24}$ (each virtual player played 100 games with white pieces and 100 with black pieces). The virtual player $VP_{200,10}^{24}$ losses, draws, and wins 0, 97, and 103 games, respectively, versus the virtual player $VP_{200,0}^{24}$. The virtual player $VP_{200,10}^{24}$ losses, draws, and wins 0, 5, and 195 games, respectively, versus the virtual player $VP_{0,0}^{24}$. Finally, the virtual player $VP_{200,0}^{24}$ losses, draws, and wins 0, 14, and 186 games, respectively, versus the virtual player $VP_{0,0}^{24}$.

Based on these played games, we used the Bayeselo tool² to estimate the ratings of players using a minorization-maximization algorithm [17]. The obtained ratings are shown in Table 11. In this table, we can see that the rating for the virtual player $VP_{200,10}^{24}$ was 2341, the rating for the virtual player $VP_{200,0}^{24}$ was 2186, and the rating for the virtual player $VP_{0,0}^{24}$ was 1685, representing an increase of 656 rating points

²<http://remi.coulom.free.fr/Bayesian-Elo/>

between the evolved virtual player with the historical mechanism, and the virtual player without evolution.

We can have an idea of the playing strength of our virtual players using the classification of the United States Chess Federation (see appendix A). From Table 15, we can see that the strength of the virtual player $VP_{200,10}^{24}$ (2341 rating points) is at the level of an master in chess, the strength of the virtual player $VP_{200,0}^{24}$ (2186 rating points) is at the level of an expert in chess, and the strength of the virtual player $VP_{0,0}^{24}$ (1685 rating points) is at the level of a class B player.

Also, we carried out 200 games between $VP_{200,10}^{33}$ versus $VP_{200,0}^{33}$, $VP_{200,10}^{33}$ versus $VP_{0,0}^{33}$ and $VP_{200,0}^{33}$ versus $VP_{0,0}^{33}$ (again each virtual player played 100 games with white pieces and 100 with black pieces). The virtual player $VP_{200,10}^{33}$ losses, draws, and wins 0, 96, and 104 games, respectively, versus the virtual player $VP_{200,0}^{33}$. The virtual player $VP_{200,10}^{33}$ losses, draws, and wins 0, 1, and 199 games, respectively, versus the virtual player $VP_{0,0}^{33}$. Finally, the virtual player $VP_{200,0}^{33}$ losses, draws, and wins 0, 2, and 198 games, respectively, versus the virtual player $VP_{0,0}^{33}$. Again, we used the Bayeselo tool to estimate the ratings of players using a minorization-maximization algorithm [17]. The obtained ratings are shown in Table 12. In this table, we can see that the rating for the virtual player $VP_{200,10}^{33}$ was 2402, the rating for the virtual player $VP_{200,0}^{33}$ was 2245, and the rating for the virtual player $VP_{0,0}^{33}$ was 1509, representing an increase of 893 rating points between the evolved virtual player with the historical mechanism, and the virtual player without evolution.

Also, we carried out an additional experiment between the virtual players and Chessmaster₂₄₀₀, which is a version of the popular chess program *Chessmaster* (grand-master edition) which plays at 2400 rating points. Specifically, we carried out 200 games among Chessmaster₂₄₀₀ and each of the virtual players $VP_{200,10}^{33}$, $VP_{200,10}^{24}$, $VP_{200,0}^{33}$, and $VP_{200,0}^{24}$ (the number of games with white pieces and black pieces was the same for the virtual players and the program *Chessmaster*; for example, *Chessmaster* played 400 games with white pieces and 400 with black pieces). The results are shown in Figure 5. In this figure, we can see that Chessmaster₂₄₀₀'s losses, draws, and wins were 14, 169, and 17, respectively, versus the virtual player $VP_{200,10}^{33}$ (denoted as H1 in this Figure). Also, Chessmaster₂₄₀₀'s losses, draws, and wins were 10, 137, and 53, respectively, versus the virtual player $VP_{200,10}^{24}$ (denoted as H2 in this Figure), respectively, and so on.

Based on these played games, we used again the Bayeselo tool to estimate the ratings of the virtual players and Chessmaster₂₄₀₀. The obtained ratings are shown in Table 13. In this table we can see that the rating for the virtual players $VP_{200,10}^{33}$, $VP_{200,10}^{24}$, $VP_{200,0}^{33}$, and $VP_{200,0}^{24}$ were 2397, 2344, 2249, and 2194 rating points, respectively.

In the experiments of this section, we used a search depth of six plies for our chess engine, and in the opening phase we used the database *Olympiad.abk* included with the graphical user interface *Arena*³.

³<http://www.playwitharena.com/>

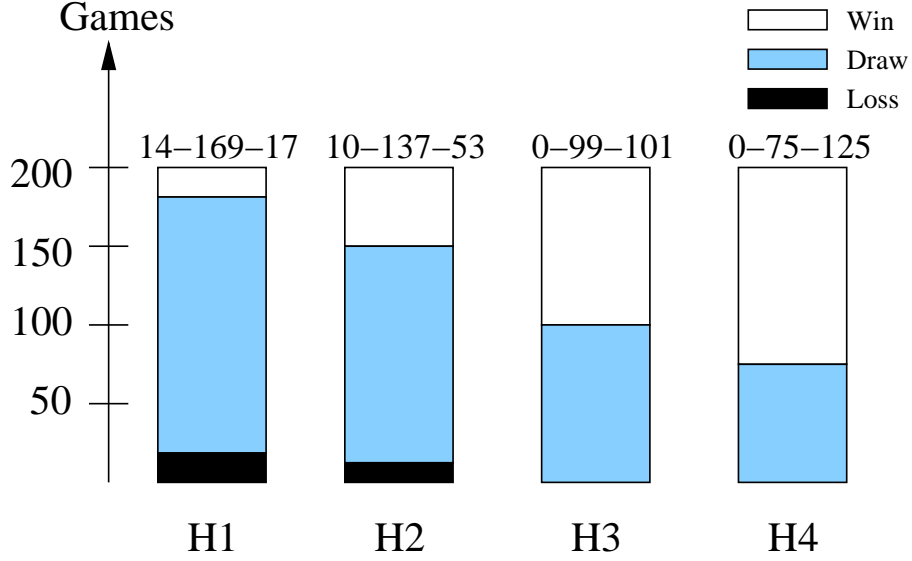


Figure 5: Histogram of losses, draws and wins for Chessmaster₂₄₀₀ against $VP_{200,10}^{33}$ (H1), $VP_{200,10}^{24}$ (H2), $VP_{200,0}^{33}$ (H3), and $VP_{200,0}^{24}$ (H4).

5.4. Experiment D

This experiment is divided in two stages. The first one was made in the section 5.1, and consisted of adjusting the weights of the 20 virtual players through chess grand-master games (this is known as supervised learning). In the second stage, we apply an evolutionary algorithm (also based on evolutionary programming [13]) to perform a tournament between virtual players (this is known as unsupervised learning or co-evolution).

Specifically, we carried out a tournament among $n = 20$ virtual players. Initially, the weights for this n virtual players were taken from the median run in Table 7. Afterwards, each virtual player is allowed to play $n/2$ games with randomly chosen opponents. The side (either black or white) is chosen at random. Games are executed until one of the virtual player receives checkmate or a draw condition arises. Depending on the outcome of the game, a virtual player obtains one point, half a point or zero points for a win, draw or loss, respectively. Draw conditions are given by the rule of 50 moves (after a pawn's move there are 50 moves to give a checkmate to the opponent), by the third repetition of the same position and by the lack of victory conditions (e.g., a king and a bishop versus a king).

After finishing the tournament, the selection mechanism chooses the $n/2$ virtual players having the highest number of points, and subsequently these virtual players are mutated to generate the remaining $n/2$ virtual players. Finally, the evolutionary algorithm continues running for 50 generations.

As we saw in the section 5.1, the best virtual player obtained in the first stage was called $VP_{200,10}^{33}$, and the best virtual player obtained in the second stage we called

VP_{50}^{33} .

Finally, we carried out 200 games between $VP_{200,10}^{33}$ versus VP_{50}^{33} , (again each virtual player played 100 games with white pieces and 100 with black pieces). The virtual player $VP_{200,10}^{33}$ losses, draws, and wins 53, 122, and 25 games, respectively, versus the virtual player VP_{50}^{33} . Again, we used the Bayeselo tool to estimate the ratings of the virtual players. The obtained ratings are shown in Table 14.

In this table, we can see that the rating for the virtual player $VP_{200,10}^{33}$ was 2404, and the rating for the virtual player VP_{50}^{33} was 2442, representing an increase of 38 rating points between the virtual player obtained with supervised learning, and the virtual player obtained with unsupervised learning.

In the experiments of this section, we used a search depth of six plies for our chess engine, and in the opening phase we used the database *Olympiad.abk*.

5.5. Validation

With the completion of the supervised learning, we used an additional 4000 positions for testing. Specifically, we let the virtual player $VP_{200,0}^{33}$ perform a 1-ply search on each of these positions, and the percentage of correctly solved positions was 39.7%. Also, we allow the virtual player $VP_{200,10}^{33}$ perform a 1-ply search on each of these positions, and the percentage of correctly solved positions was 52.1%. This indicates that the first 4000 positions used for training cover most of the types of positions that can arise.

6. Conclusions and Future Work

Most of the works that make use of evolutionary algorithms to tune the weights of a chess engine adopt co-evolution. In these methods, the virtual players hold tournaments among them, and the virtual players which obtain a larger number of victories acquire the right to pass to the next generation.

Our proposed approach uses supervised learning to perform the tuning of weights of a chess engine through a database of chess grandmaster games. The idea of the selection mechanism of our evolutionary algorithm is to favor virtual players that are able to “visualize” (or match) more movements from those registered in a database of chess grandmaster games. With our proposal, and after 200 generations, our virtual players can solve 40.81% of the positions of chess grandmaster games. Additionally, our evolutionary algorithm employs a historical mechanism that allows it to solve the 53.10% of the positions of chess grandmaster games. Note that this value is higher than the value of 32.4% reported in the work of David-Tabibi et al. [8], who also used supervised learning with a database of chess grandmaster games, unsupervised learned (through co-evolution) and a genetic algorithm. It is noteworthy that our work uses evolutionary programming, and David-Tabibi’s work uses a genetic algorithm. We believe that the number of positions solved by our method is satisfactory considering that this was achieved adopting only a depth of 1 ply in the search tree.

In the games held between our virtual players and *Chessmaster* (playing at a rating of 2400) we found that the best evolved virtual player without the historical mechanism played at 2249 rating points, and the best evolved virtual player with the historical

mechanism played at 2397 rating points. Thus, we conclude that the rating of our chess engine is competitive with that of the version of *Chessmaster* adopted in our study.

Additionally, we used unsupervised learning (through a tournament among virtual players) to improve the rating obtained with supervised learning. In this case we obtained an improved of 38 rating points (from 2404 to 2442 rating points).

We note that the standard deviation increases as we increase the size of the test cases. Also, our results indicate that the values of the chess pieces obtained by our proposed approach closely match the known values from chess theory.

As part of our future work, and with the idea of creating a chess program that is able to play better, we plan to add and tune more weights in our evaluation function. We also intend to add extensions to our program (e.g., passed pawn extensions, mate-threat, among others), and we plan to use bitbases or tablebases in the final phase of the game, aiming to increase the rating of our chess engine.

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Appendix A

The ELO rating system is a method that calculates the relative strength of players in games with two opponents, and was created by the mathematician Arpad Elo. In Table 15 we show the classification of the USCF (United States Chess Federation).

Table 1: Ranges of the weights adopted in our approach and final weights for the virtual player $VP_{200,10}^{33}$.

| <i>Number</i> | <i>Weight</i> | W_{low} | W_{high} | Weights for $VP_{200,10}^{33}$ |
|---------------|-----------------------------|-----------|------------|--------------------------------|
| 1 | PAWN_VALUE | 100 | 100 | 100.0 |
| 2 | KNIGHT_VALUE | 200 | 400 | 295.2 |
| 3 | BISHOP_VALUE | 200 | 400 | 315.4 |
| 4 | ROOK_VALUE | 400 | 600 | 489.7 |
| 5 | QUEEN_VALUE | 800 | 1000 | 921.3 |
| 6 | PAWN_DOUBLED_PENALTY_VALUE | -50 | 50 | -14.3 |
| 7 | PAWN_ISOLATED_PENALTY_VALUE | -50 | 50 | -23.2 |
| 8 | PAWN_BACKWARD_PENALTY_VALUE | -50 | 50 | -19.5 |
| 9 | PAWN_PASSED | -50 | 100 | 34.3 |
| 10 | PAWN_CENTRAL | -50 | 100 | 19.7 |
| 11 | KNIGHT_SUPPORTED | -50 | 100 | 16.2 |
| 12 | KNIGHT_OPERATIONS_BASE | -50 | 100 | 15.3 |
| 13 | KNIGHT_PERIPHERY_0 | -50 | 50 | -10.3 |
| 14 | KNIGHT_PERIPHERY_1 | -50 | 50 | 14.1 |
| 15 | KNIGHT_PERIPHERY_2 | -50 | 50 | 19.7 |
| 16 | KNIGHT_PERIPHERY_3 | -50 | 50 | 26.4 |
| 17 | ROOK_OPEN_COLUMN | -50 | 50 | 22.1 |
| 18 | ROOK_SEMIOPEN_COLUMN_BEHIND | -50 | 50 | -5.7 |
| 19 | ROOK_CLOSED_COLUMN | -50 | 50 | -11.3 |
| 20 | ROOK_SEVEN_RANK | -50 | 50 | 49.6 |
| 21 | KNIGHT_MOBILITY | 0 | 100 | 27.8 |
| 22 | BISHOP_MOBILITY | 0 | 100 | 34.1 |
| 23 | ROOK_MOBILITY | 0 | 100 | 37.2 |
| 24 | QUEEN_MOBILITY | 0 | 100 | 12.3 |
| 25 | KING_MOBILITY | 0 | 100 | 5.1 |
| 26 | ROOK_SEMIOPEN_COLUMN_AHEAD | -50 | 50 | 9.3 |
| 27 | KING_ATTACKING_MATERIAL | -100 | 0 | -67.8 |
| 28 | KING_DEFENDING_MATERIAL | 0 | 100 | 52.3 |
| 29 | KING_CASTLING | 0 | 100 | 51.2 |
| 30 | KING_PAWNS | 0 | 100 | 49.3 |
| 31 | BISHOP_AHEAD | -100 | 0 | 39.6 |
| 32 | BISHOP_PAWNS_MOBILITY | 0 | 100 | 21.3 |
| 33 | BISHOP_PAIR | 0 | 100 | 27.0 |

Table 2: 31 runs with $P_r = 0$ for 24 weights.

| Number of generations (G_{max}): | | 200 | | |
|--|----------------|---|----------------|-----------------------|
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 0 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 21.8 | 32.54 | 18.7 | 32.54 | 56:54 |
| 21.8 | 32.65 | 18.7 | 32.65 | 56:50 |
| 21.8 | 32.69 | 18.7 | 32.69 | 56:53 |
| 21.8 | 32.71 | 18.7 | 32.71 | 56:57 |
| 21.8 | 32.79 | 18.7 | 32.79 | 56:54 |
| 21.8 | 32.81 | 18.7 | 32.81 | 56:52 |
| 21.8 | 32.81 | 18.7 | 32.81 | 57: 6 |
| 21.8 | 32.94 | 18.7 | 32.94 | 57: 7 |
| 21.8 | 32.97 | 18.7 | 32.97 | 56:57 |
| 21.8 | 33.10 | 18.7 | 33.10 | 57: 0 |
| 21.8 | 33.15 | 18.7 | 33.15 | 57: 4 |
| 21.8 | 33.20 | 18.7 | 33.20 | 56:54 |
| 21.8 | 33.24 | 18.7 | 33.24 | 56:55 |
| 21.8 | 33.29 | 18.7 | 33.29 | 56:55 |
| 21.8 | 33.47 | 18.7 | 33.47 | 57: 1 |
| 21.85 | 33.61 | 18.70 | 33.61 | 56:50 (median) |
| 21.8 | 33.62 | 18.7 | 33.62 | 57: 2 |
| 21.8 | 33.63 | 18.7 | 33.63 | 56:50 |
| 21.8 | 33.64 | 18.7 | 33.64 | 57: 3 |
| 21.8 | 33.76 | 18.7 | 33.76 | 56:56 |
| 21.8 | 33.85 | 18.7 | 33.85 | 57: 1 |
| 21.8 | 33.87 | 18.7 | 33.87 | 56:52 |
| 21.8 | 33.95 | 18.7 | 33.95 | 56:50 |
| 21.8 | 33.99 | 18.7 | 33.99 | 57: 7 |
| 21.8 | 34.24 | 18.7 | 34.24 | 56:53 |
| 21.8 | 34.31 | 18.7 | 34.31 | 57: 5 |
| 21.8 | 34.36 | 18.7 | 34.36 | 56:58 |
| 21.8 | 34.46 | 18.7 | 34.46 | 56:56 |
| 21.8 | 34.50 | 18.7 | 34.50 | 57: 2 |
| 21.8 | 34.59 | 18.7 | 34.59 | 57: 7 |
| 21.8 | 34.60 | 18.7 | 34.60 | 57: 3 |
| Standard deviation of the fourth column: | | 0.65 | | |
| Average run time (min:sec): | | 56:58 | | |

Table 3: 31 runs with $P_r = 10$ for 24 weights.

| | | | | |
|--|----------------|---|----------------|-----------------------|
| Number of generations (G_{max}): | | 200 | | |
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 10 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 21.8 | 45.26 | 20.8 | 45.20 | 57: 9 |
| 18.2 | 45.29 | 16.9 | 45.23 | 57: 4 |
| 23.2 | 45.35 | 19.2 | 45.30 | 56:57 |
| 24.1 | 45.42 | 20.3 | 45.36 | 56:50 |
| 20.9 | 45.43 | 19.7 | 45.37 | 57: 2 |
| 27.0 | 45.43 | 24.3 | 45.38 | 57: 2 |
| 19.7 | 45.65 | 16.6 | 45.59 | 56:51 |
| 22.4 | 45.93 | 20.4 | 45.87 | 57: 6 |
| 25.5 | 45.94 | 21.8 | 45.89 | 56:53 |
| 28.6 | 46.07 | 24.2 | 46.01 | 56:54 |
| 20.9 | 46.07 | 18.5 | 46.02 | 57: 1 |
| 24.2 | 46.13 | 22.2 | 46.08 | 56:52 |
| 25.5 | 46.14 | 21.8 | 46.08 | 56:56 |
| 22.0 | 46.23 | 19.7 | 46.18 | 57: 4 |
| 19.3 | 46.32 | 17.6 | 46.27 | 56:50 |
| 26.49 | 46.35 | 22.39 | 46.30 | 56:51 (median) |
| 27.6 | 46.37 | 24.2 | 46.31 | 57: 5 |
| 26.2 | 46.41 | 21.5 | 46.36 | 57: 0 |
| 26.5 | 46.44 | 21.7 | 46.38 | 56:53 |
| 28.6 | 46.45 | 20.8 | 46.39 | 57: 0 |
| 31.2 | 46.59 | 24.4 | 46.53 | 57: 2 |
| 23.8 | 46.60 | 21.1 | 46.55 | 56:50 |
| 20.9 | 46.72 | 19.6 | 46.67 | 57: 8 |
| 26.1 | 46.94 | 20.3 | 46.89 | 57: 9 |
| 25.7 | 46.97 | 21.7 | 46.92 | 56:54 |
| 24.9 | 47.05 | 21.9 | 46.99 | 56:51 |
| 23.4 | 47.16 | 22.0 | 47.11 | 57: 4 |
| 24.7 | 47.22 | 22.0 | 47.17 | 57: 6 |
| 20.4 | 47.24 | 18.7 | 47.19 | 57: 6 |
| 22.3 | 47.38 | 18.9 | 47.33 | 57: 7 |
| 26.5 | 47.45 | 24.1 | 47.40 | 56:51 |
| Standard deviation of the fourth column: | | 0.66 | | |
| Average run time (min:sec): | | 56:59 | | |

Table 4: 31 runs with $P_r = 15$ for 24 weights.

| Number of generations (G_{max}): | | 200 | | |
|--|----------------|---|----------------|-----------------------|
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 15 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 16.8 | 41.19 | 14.9 | 41.21 | 57: 4 |
| 16.0 | 41.30 | 15.3 | 41.32 | 57: 3 |
| 21.0 | 41.30 | 17.2 | 41.32 | 56:54 |
| 17.6 | 41.31 | 16.0 | 41.32 | 56:50 |
| 18.5 | 41.34 | 16.4 | 41.36 | 57: 9 |
| 18.5 | 41.35 | 16.6 | 41.36 | 57: 9 |
| 18.5 | 41.40 | 17.4 | 41.41 | 57: 8 |
| 19.3 | 41.42 | 14.9 | 41.44 | 56:55 |
| 19.3 | 41.45 | 16.6 | 41.46 | 56:51 |
| 20.2 | 41.63 | 17.2 | 41.65 | 56:52 |
| 18.5 | 41.66 | 13.9 | 41.68 | 56:52 |
| 20.2 | 41.87 | 17.2 | 41.89 | 56:58 |
| 16.8 | 41.93 | 14.9 | 41.94 | 57: 8 |
| 18.5 | 41.94 | 16.4 | 41.96 | 56:53 |
| 19.3 | 41.96 | 16.2 | 41.98 | 57: 3 |
| 21.01 | 42.27 | 18.07 | 42.29 | 56:54 (median) |
| 23.5 | 42.29 | 19.1 | 42.31 | 56:52 |
| 22.7 | 42.32 | 19.7 | 42.34 | 57: 8 |
| 15.1 | 42.35 | 13.9 | 42.37 | 57: 6 |
| 20.2 | 42.39 | 18.3 | 42.41 | 57: 3 |
| 16.8 | 42.50 | 15.3 | 42.52 | 57: 6 |
| 21.0 | 42.76 | 17.6 | 42.77 | 56:58 |
| 22.7 | 42.89 | 17.0 | 42.90 | 56:52 |
| 24.4 | 43.00 | 21.0 | 43.02 | 57: 1 |
| 19.3 | 43.18 | 17.9 | 43.19 | 57: 2 |
| 19.3 | 43.20 | 17.0 | 43.22 | 56:50 |
| 19.3 | 43.27 | 16.6 | 43.29 | 56:53 |
| 17.6 | 43.29 | 16.4 | 43.31 | 57: 9 |
| 27.7 | 43.30 | 19.3 | 43.31 | 56:50 |
| 21.8 | 43.34 | 18.3 | 43.36 | 56:54 |
| 17.6 | 43.35 | 16.2 | 43.37 | 56:58 |
| Standard deviation of the fourth column: | | 0.76 | | |
| Average run time (min:sec): | | 56:59 | | |

Table 5: 31 runs with $P_r = 20$ for 24 weights.

| | | | | |
|--|----------------|---|----------------|-----------------------|
| Number of generations (G_{max}): | | 200 | | |
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 20 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 22.3 | 39.59 | 18.4 | 39.62 | 57: 4 |
| 23.1 | 39.66 | 20.2 | 39.68 | 56:59 |
| 23.1 | 39.87 | 20.7 | 39.89 | 57: 9 |
| 19.8 | 39.91 | 18.4 | 39.94 | 56:54 |
| 21.5 | 40.06 | 20.2 | 40.08 | 57: 1 |
| 23.1 | 40.08 | 21.5 | 40.11 | 57: 8 |
| 21.5 | 40.10 | 18.8 | 40.12 | 57: 6 |
| 21.5 | 40.22 | 19.6 | 40.25 | 57: 8 |
| 19.8 | 40.24 | 18.4 | 40.26 | 56:55 |
| 22.3 | 40.27 | 20.7 | 40.30 | 56:57 |
| 24.0 | 40.30 | 19.4 | 40.33 | 57: 4 |
| 22.3 | 40.45 | 19.8 | 40.47 | 56:55 |
| 20.7 | 40.51 | 18.4 | 40.54 | 56:51 |
| 18.2 | 40.58 | 16.7 | 40.60 | 57: 3 |
| 22.3 | 40.63 | 20.5 | 40.66 | 57: 2 |
| 23.97 | 40.66 | 21.07 | 40.69 | 56:59 (median) |
| 24.8 | 40.73 | 22.5 | 40.76 | 57: 9 |
| 24.8 | 40.80 | 20.9 | 40.82 | 56:52 |
| 23.1 | 40.81 | 19.4 | 40.83 | 56:59 |
| 27.3 | 40.99 | 20.9 | 41.01 | 57: 6 |
| 21.5 | 41.08 | 19.2 | 41.11 | 57: 8 |
| 24.0 | 41.32 | 19.0 | 41.34 | 56:50 |
| 24.8 | 41.47 | 21.9 | 41.49 | 57: 9 |
| 19.8 | 41.48 | 18.4 | 41.50 | 56:56 |
| 27.3 | 41.54 | 20.7 | 41.57 | 57: 7 |
| 24.0 | 41.65 | 21.3 | 41.67 | 56:52 |
| 20.7 | 41.67 | 19.2 | 41.69 | 56:51 |
| 19.8 | 41.68 | 19.2 | 41.71 | 57: 2 |
| 23.1 | 41.70 | 20.7 | 41.73 | 57: 2 |
| 21.5 | 41.72 | 20.7 | 41.74 | 57: 7 |
| 22.3 | 41.73 | 19.6 | 41.75 | 57: 8 |
| Standard deviation of the fourth column: | | 0.68 | | |
| Average run time (min:sec): | | 57: 1 | | |

Table 6: 31 runs with $P_r = 0$ for 33 weights.

| Number of generations (G_{max}): | | 200 | | |
|--|----------------|---|----------------|--------------------------|
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 0 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 25.4 | 39.82 | 24.2 | 39.78 | 63: 0 |
| 25.4 | 39.84 | 23.3 | 39.81 | 63: 1 |
| 30.5 | 39.87 | 24.6 | 39.83 | 63: 7 |
| 30.5 | 39.95 | 23.3 | 39.92 | 62:51 |
| 27.1 | 40.04 | 22.5 | 40.00 | 63: 2 |
| 27.1 | 40.04 | 24.2 | 40.01 | 62:54 |
| 28.8 | 40.06 | 22.0 | 40.03 | 62:57 |
| 30.5 | 40.16 | 24.2 | 40.12 | 62:57 |
| 28.8 | 40.30 | 21.6 | 40.26 | 63: 6 |
| 25.4 | 40.33 | 22.5 | 40.29 | 62:55 |
| 22.0 | 40.40 | 21.2 | 40.36 | 62:51 |
| 28.8 | 40.44 | 24.2 | 40.40 | 62:55 |
| 23.7 | 40.44 | 19.9 | 40.40 | 63: 6 |
| 28.8 | 40.59 | 25.0 | 40.56 | 63: 6 |
| 25.4 | 40.70 | 23.7 | 40.67 | 62:52 |
| 30.51 | 40.85 | 25.42 | 40.81 | 63: 3 (median 27) |
| 25.4 | 40.86 | 22.5 | 40.83 | 63: 0 |
| 27.1 | 40.91 | 23.7 | 40.87 | 63: 1 |
| 32.2 | 41.12 | 27.1 | 41.09 | 63: 7 |
| 27.1 | 41.14 | 24.2 | 41.11 | 63: 7 |
| 28.8 | 41.24 | 26.7 | 41.21 | 62:50 |
| 22.0 | 41.40 | 19.9 | 41.37 | 62:57 |
| 28.8 | 41.46 | 25.4 | 41.43 | 62:55 |
| 27.1 | 41.49 | 22.9 | 41.46 | 63: 0 |
| 25.4 | 41.57 | 21.6 | 41.54 | 63: 1 |
| 25.4 | 41.73 | 20.3 | 41.70 | 63: 5 |
| 23.7 | 41.75 | 21.2 | 41.72 | 62:55 |
| 32.2 | 41.75 | 25.8 | 41.72 | 63: 2 |
| 28.8 | 41.76 | 25.0 | 41.72 | 62:53 |
| 25.4 | 41.81 | 22.9 | 41.77 | 62:51 |
| 23.7 | 41.81 | 22.0 | 41.78 | 63: 6 |
| Standard deviation of the fourth column: | | 0.69 | | |
| Average run time (min:sec): | | 62:59 | | |

Table 7: 31 runs with $P_r = 10$ for 33 weights.

| | | | | |
|--|----------------|---|----------------|-------------------------|
| Number of generations (G_{max}): | | 200 | | |
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 10 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 25.5 | 52.10 | 22.6 | 52.12 | 63: 5 |
| 25.3 | 52.13 | 22.4 | 52.15 | 63: 9 |
| 25.6 | 52.14 | 22.7 | 52.16 | 62:58 |
| 24.6 | 52.17 | 21.8 | 52.19 | 62:50 |
| 25.1 | 52.20 | 22.2 | 52.22 | 63: 5 |
| 24.9 | 52.23 | 22.0 | 52.25 | 62:53 |
| 24.8 | 52.29 | 21.9 | 52.31 | 63: 2 |
| 25.3 | 52.37 | 22.4 | 52.39 | 62:52 |
| 24.4 | 52.43 | 21.6 | 52.45 | 62:54 |
| 25.1 | 52.78 | 22.3 | 52.80 | 62:55 |
| 25.2 | 52.78 | 22.3 | 52.80 | 62:58 |
| 25.3 | 52.79 | 22.4 | 52.81 | 63: 5 |
| 25.1 | 52.87 | 22.2 | 52.89 | 62:52 |
| 24.5 | 52.89 | 21.6 | 52.91 | 63: 0 |
| 25.2 | 53.07 | 22.3 | 53.08 | 62:57 |
| 25.38 | 53.08 | 22.46 | 53.10 | 63: 9 (median 3) |
| 25.2 | 53.08 | 22.3 | 53.10 | 63: 4 |
| 25.4 | 53.10 | 22.5 | 53.12 | 62:58 |
| 24.9 | 53.28 | 22.0 | 53.30 | 63: 4 |
| 24.5 | 53.31 | 21.7 | 53.33 | 62:54 |
| 25.3 | 53.45 | 22.4 | 53.47 | 62:54 |
| 25.4 | 53.48 | 22.5 | 53.50 | 62:58 |
| 25.0 | 53.53 | 22.2 | 53.55 | 62:57 |
| 25.2 | 53.54 | 22.3 | 53.56 | 63: 4 |
| 25.5 | 53.56 | 22.6 | 53.58 | 62:53 |
| 25.1 | 53.64 | 22.2 | 53.66 | 62:52 |
| 24.8 | 53.76 | 21.9 | 53.78 | 62:56 |
| 25.1 | 53.94 | 22.2 | 53.96 | 63: 6 |
| 25.1 | 53.94 | 22.2 | 53.96 | 63: 6 |
| 25.1 | 53.96 | 22.2 | 53.98 | 63: 7 |
| 24.8 | 54.17 | 22.0 | 54.19 | 62:50 |
| Standard deviation of the fourth column: | | 0.64 | | |
| Average run time (min:sec): | | 62:59 | | |

Table 8: 31 runs with $P_r = 15$ for 33 weights.

| | | | | |
|--|----------------|---|----------------|-----------------------|
| Number of generations (G_{max}): | | 200 | | |
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 15 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 18.0 | 49.15 | 16.0 | 49.07 | 62:58 |
| 22.0 | 49.16 | 18.0 | 49.08 | 63: 0 |
| 26.0 | 49.32 | 24.0 | 49.24 | 62:53 |
| 24.0 | 49.36 | 15.0 | 49.28 | 63: 0 |
| 23.0 | 49.38 | 0.0 | 49.30 | 62: 56 |
| 22.0 | 49.40 | 14.5 | 49.32 | 63: 1 |
| 18.0 | 49.56 | 17.0 | 49.48 | 63: 2 |
| 22.0 | 49.56 | 0.0 | 49.48 | 63: 10 |
| 30.0 | 49.68 | 23.5 | 49.60 | 62:54 |
| 26.0 | 49.69 | 17.5 | 49.61 | 62:51 |
| 24.0 | 49.79 | 20.0 | 49.71 | 62:57 |
| 22.0 | 49.93 | 16.0 | 49.85 | 63: 8 |
| 20.0 | 50.03 | 14.0 | 49.95 | 62:55 |
| 20.0 | 50.08 | 16.0 | 50.00 | 62:50 |
| 18.0 | 50.11 | 14.5 | 50.03 | 63: 6 |
| 26.00 | 50.20 | 21.00 | 50.12 | 63: 3 (median) |
| 24.0 | 50.25 | 17.5 | 50.17 | 62:54 |
| 24.0 | 50.37 | 19.5 | 50.29 | 62:55 |
| 18.0 | 50.43 | 14.0 | 50.35 | 63: 8 |
| 20.0 | 50.44 | 16.5 | 50.36 | 63: 7 |
| 22.0 | 50.56 | 18.0 | 50.48 | 63: 0 |
| 28.0 | 50.62 | 19.5 | 50.54 | 63: 9 |
| 24.0 | 50.64 | 18.0 | 50.56 | 62:54 |
| 22.0 | 50.64 | 19.0 | 50.56 | 62:57 |
| 18.0 | 50.78 | 15.5 | 50.70 | 62:54 |
| 22.0 | 50.80 | 19.5 | 50.72 | 63: 2 |
| 20.0 | 50.85 | 18.0 | 50.77 | 63: 0 |
| 22.0 | 50.91 | 16.0 | 50.83 | 63: 5 |
| 24.0 | 50.92 | 19.0 | 50.84 | 62:57 |
| 26.0 | 51.24 | 17.0 | 51.16 | 62:56 |
| 28.0 | 51.26 | 19.0 | 51.18 | 62:58 |
| Standard deviation of the fourth column: | | 0.63 | | |
| Average run time (min:sec): | | 63:05 | | |

Table 9: 31 runs with $P_r = 20$ for 33 weights.

| Number of generations (G_{max}): | | 200 | | |
|--|----------------|---|----------------|-----------------------|
| Population size: | | 20 | | |
| Number of training positions: | | 4000 | | |
| P_r : | | 20 | | |
| Positions solved by the best VP (%) | | Average positions solved by all VPs (%) | | Time (min:sec) |
| Generation 0 | Generation 200 | Generation 0 | Generation 200 | |
| 25.4 | 46.58 | 22.0 | 46.51 | 67:47 |
| 0.0 | 46.60 | 0.0 | 46.54 | 68:26 |
| 27.1 | 46.74 | 21.2 | 46.67 | 63:59 |
| 22.0 | 46.87 | 20.8 | 46.80 | 71:28 |
| 28.8 | 46.88 | 22.5 | 46.82 | 65:17 |
| 0.0 | 46.90 | 0.0 | 46.84 | 64:09 |
| 27.1 | 46.95 | 24.2 | 46.88 | 65:28 |
| 22.0 | 46.97 | 20.3 | 46.90 | 57:13 |
| 27.1 | 46.97 | 22.5 | 46.90 | 62:29 |
| 25.4 | 47.02 | 18.6 | 46.95 | 68: 5 |
| 20.3 | 47.04 | 19.1 | 46.97 | 67:36 |
| 23.7 | 47.12 | 22.0 | 47.05 | 63:29 |
| 23.7 | 47.39 | 21.2 | 47.32 | 59:15 |
| 27.1 | 47.44 | 24.2 | 47.37 | 67:35 |
| 23.7 | 47.45 | 22.5 | 47.39 | 68: 0 |
| 22.03 | 47.63 | 19.92 | 47.56 | 55:16 (median) |
| 25.4 | 47.77 | 22.5 | 47.70 | 58:14 |
| 25.4 | 47.91 | 22.5 | 47.85 | 60:42 |
| 23.7 | 47.94 | 23.3 | 47.87 | 64:11 |
| 22.0 | 48.01 | 18.2 | 47.94 | 59:12 |
| 25.4 | 48.01 | 23.3 | 47.94 | 60:10 |
| 27.1 | 48.02 | 22.9 | 47.96 | 59:58 |
| 22.0 | 48.14 | 17.4 | 48.07 | 59: 8 |
| 25.4 | 48.19 | 23.3 | 48.13 | 62:48 |
| 23.7 | 48.21 | 21.6 | 48.15 | 58:44 |
| 27.1 | 48.27 | 22.0 | 48.20 | 60:20 |
| 23.7 | 48.28 | 19.5 | 48.21 | 59:41 |
| 22.0 | 48.46 | 19.9 | 48.39 | 60:59 |
| 25.4 | 48.55 | 19.9 | 48.48 | 58:43 |
| 27.1 | 48.59 | 22.5 | 48.52 | 58:40 |
| 23.7 | 48.61 | 21.2 | 48.54 | 58:49 |
| Standard deviation of the fourth column: | | 0.65 | | |
| Average run time (min:sec): | | 62:58 | | |

Table 10: It shows the median of the average positions solved by all virtual players and the standard deviation for 31 runs at generation 200 and $Pr = 10$ with different training case.

| Training case | Average positions solved by all virtual players | Standard Deviation | Average evaluation time |
|---------------|---|--------------------|-------------------------|
| 1000 | 50.87% | 0.44 | 14 : 37 |
| 2000 | 51.07% | 0.47 | 31 : 12 |
| 3000 | 52.02% | 0.51 | 45 : 37 |
| 4000 | 53.10% | 0.56 | 62 : 59 |

Table 11: Ratings for $VP_{200,10}^{24}$, $VP_{200,0}^{24}$ and $VP_{0,0}^{24}$.

| Rank | Name | Elo | + | - | Games | Score (%) | Oppo. | Draws (%) |
|------|--------------------|------|----|----|-------|-----------|-------|-----------|
| 1 | $VP_{200,10}^{24}$ | 2341 | 26 | 25 | 400 | 87% | 1936 | 26% |
| 2 | $VP_{200,0}^{24}$ | 2186 | 24 | 24 | 400 | 60% | 2014 | 28% |
| 3 | $VP_{0,0}^{24}$ | 1685 | 43 | 53 | 400 | 2% | 2264 | 5% |

Table 12: Ratings for $VP_{200,10}^{33}$, $VP_{200,0}^{33}$ and $VP_{0,0}^{33}$.

| Rank | Name | Elo | + | - | Games | Score (%) | Oppo. | Draws (%) |
|------|--------------------|------|----|-----|-------|-----------|-------|-----------|
| 1 | $VP_{200,10}^{33}$ | 2402 | 27 | 26 | 400 | 88% | 1877 | 24% |
| 2 | $VP_{200,0}^{33}$ | 2245 | 26 | 27 | 400 | 62% | 1955 | 25% |
| 3 | $VP_{0,0}^{33}$ | 1509 | 78 | 121 | 400 | 0% | 2324 | 1% |

Table 13: Ratings for Chessmaster₂₄₀₀ and the virtual players in generation 200.

| Rank | Name | Elo | + | - | Games | Score (%) | Oppo. | Draws (%) |
|------|-----------------------------|------|----|----|-------|-----------|-------|-----------|
| 1 | Chessmaster ₂₄₀₀ | 2401 | 16 | 15 | 800 | 67% | 2296 | 60% |
| 2 | $VP_{200,10}^{33}$ | 2397 | 27 | 27 | 200 | 49% | 2401 | 85% |
| 3 | $VP_{200,10}^{24}$ | 2344 | 29 | 29 | 200 | 39% | 2401 | 69% |
| 4 | $VP_{200,0}^{33}$ | 2249 | 31 | 32 | 200 | 25% | 2401 | 50% |
| 5 | $VP_{200,0}^{24}$ | 2194 | 34 | 36 | 200 | 19% | 2401 | 38% |

Table 14: Ratings for $VP_{200,10}^{33}$ and VP_{50}^{33} .

| Rank | Name | Elo | + | - | Games | Score (%) | Oppo. | Draws (%) |
|------|--------------------|------|----|----|-------|-----------|-------|-----------|
| 1 | VP_{50}^{33} | 2442 | 19 | 18 | 200 | 57% | 2404 | 61% |
| 2 | $VP_{200,10}^{33}$ | 2404 | 18 | 19 | 200 | 43% | 2442 | 61% |

Table 15: ELO rating system

| Interval | Level |
|----------------|---------------|
| 2400 and above | Senior Master |
| 2200 – 2399 | Master |
| 2000 – 2199 | Expert |
| 1800 – 1999 | Class A |
| 1600 – 1799 | Class B |
| 1400 – 1599 | Class C |
| 1200 – 1399 | Class D |
| 1000 – 1199 | Class E |