Optimal power flow subject to security constraints solved with a particle swarm optimizer

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***Abstract*--This paper presents a novel approach to solve an optimal power flow problem with embedded security constraints (OPF-SC), represented by a mixture of continuous and discrete control variables, where the major aim is to minimize the total operating cost, taking into account both operating security constraints, and system capacity requirements. The particle swarm optimization (PSO) algorithm with reconstruction operators (PSO-RO) has been used as the optimization tool. Such operators guarantee searching the optimal solution within the feasible space, reducing the computation time and improving the quality of the solution. Results on systems from the specialized literature are adopted to validate the proposed approach.**

Index terms—Optimization methods, Power generation economics, Power system economics, Power system security, Unit commitment.

I. Introduction

THE power system optimal power flow (OPF) objective is to obtain a start-up and shut-down schedule of generating units to meet the required demand at minimum production cost, satisfying units' and system's operating constraints, by adjusting the power system control variables. Such ones include the active power supplied by each available generator, the tap position of transformers, and the reactive power generation of the VAR sources.

Furthermore, the power system must be capable to withstand the loss of some or several transmission lines, transformers or generators, guaranteeing its security; such events are often termed *probable* or *credible* contingencies. Different security criteria have been used to ensure sufficient security margins. One of them is the so named *N-1 criterion*, widely used nowadays. This criterion, in its simplest form, specifies that the system should be able to withstand the loss of any component (e.g., lines, transformers, generators) without jeopardizing the system's operation; such problems are known as optimal power flow with security constrains (OPF-SC). To determine the credible contingencies, a contingency ranking is used.

Through an optimal power flow formulation with the N-1 criterion, a method for computing the optimal pre- and post-contingency operating points is presented. Additionally, constraints in generating units' limits, minimum and maximum up- and down-time, slope-down and slope-up, and coupling constraints between the pre- and the post-contingency states have been taken into account.

The OPF-SC problem has been formulated as a non-linear, non-convex, and large-scale, mixed-integer, optimization problem. Several techniques have been used for solving such one, for instance: optimization methods involving derivative-based techniques such as those summarized in [1]-[2], and heuristic optimization techniques such as genetic algorithms [3]. Likewise, very important contributions on optimization applications in power systems have been presented in [4-13].

The particle swarm optimization (PSO) algorithm has been used to solve the OPF in [14]. Additionally, in [15, 16], a modified PSO, improving its convergence characteristics, is applied to the same problem. Three types of PSO algorithms are proposed in [17, 18], all of which have been applied to solve optimization problems related to reactive power and voltage control.

In this paper, a particle swarm optimizer with reconstruction operators (PSO-RO) for solving the OPF-SC is proposed. To handle the constraints of the problem, such reconstruction operators and an external penalty are adopted. The reconstruction operators allow that all particles representing a possible solution satisfy the units' operating constraints, while looking for the optimal solution only within the feasible space, reducing the computing time and improving the quality of the achieved solution.

This paper is organized as follows. Section II presents a summary of the PSO algorithm. The posed problem is exposed in Section III. Section IV and V describe the proposal for solving the OPF-SC problem and some results on power systems of the open literature, respectively.

II. Basics on the PSO algorithm

The PSO algorithm was introduced by Kennedy and Eberhart in 1995 as an alternative to genetic algorithms (GA) [19, 20]. The first version of PSO was intended to handle only nonlinear continuous optimization problems. However, its

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development elevated its capabilities for handling a wide class of complex optimization problems [21]. A PSO algorithm consists of a population continuously updating the knowledge of the given searching space. This population is formed by individuals denoted as particles; each one represents a possible solution. Unlike evolutionary algorithms, each particle moves in the searching space with a velocity, which is dynamically updated based on its previous velocity. The particle's location where the best fitness has been achieved, is denoted *pbest* in (1). *gbest* is that population where *pbest* is located. The velocity of the *i*-th element is updated as

$$\begin{aligned} V_{i,j}^{iter+1} &= wV_{i,j}^{iter} + C_1 * rand() * (pbest - X_i)... \\ &\quad + C_2 * Rand() * (gbest_i - X_i) \\ &for\ i = 1,2,....,NIND \quad j = 1,2,....NVAR \end{aligned} \tag{1}$$

where *iter* is the current iteration; C_1 and C_2 are two positive learning factors; X_i is the actual position of the i-th particle; *rand()* and *Rand()* are two randomly generated values within the range [0, 1]; *NIND,* is the number of particles*; NVAR,* is the number of variables; *w* is known as the inertia weight, and it plays the role of balancing the global and local search [22, 23].

The position of each particle is updated at each iteration; this is done by adding the velocity vector to the position vector, as described in (2).

$$X_{i,j}^{iter+1} = X_{i,j}^{iter} + V_{i,j}^{iter+1} \tag{2}$$

III. OPF-SC FORMULATION

The main objective of the OPF-SC problem is to minimize the total cost. This one includes the pre-contingency cost (superscript *0*) plus the cost of each credible contingencies (superscript *k*). The cost functions are constituted by two terms: one related with the generating costs, and a second one associated with a consumer benefit; this is defined as [2]

$$F^{obj} = \min_{P_{Gi}^o, P_{Gi}^k} C^o + \sum_{k=1}^{K} C^k \tag{3}$$

where:

K is the total number of credible contingencies.

C^o operating cost for the base case; this cost is evaluated by (4)

$$C^o = \sum_{t=1}^{T}\left\{\sum_{i=1}^{NG}\left(U_{i,t}^0 * f\left(P_{Gi,t}^o\right) + SUC_i\right) - \sum_{j=1}^{NLoad} B\left(P_{Loadj,t}\right)\right\} \tag{4}$$

T scheduled hours

$U_{i,t}^0$ i-th unit's pre-contingency state at time t, 1 *ON*, 0 *OFF.*

NG total number of available generators.

$f(P_{Gi,t}^o)$ i-th generator's function cost at time t.

$P_{Gi,t}^o$ active power supplied by the i-th generator at the pre-contingency state.

SUC_i i-th generator's start- up cost.

$NLoad$ total number of loads.

$B(P_{Loadj,t})$ is the consumer benefit curve for j-th load at time t.

$P_{Loadj,t}$ active power consumption at the j-th load.

C^k credible contingencies' cost; it is defined in (4.a).

$$C^k = \sum_{t=1}^{T}\left\{\sum_{i=1}^{NG}\left(U_{i,t}^k * f\left(P_{Gi,t}^k\right) + SUC_i\right) - \sum_{j=1}^{NLoad} B\left(P_{Loadj,t}\right)\right\}$$

(4.a))

The solution must satisfy constraints at the pre-contingency state, constraints at the post-contingency state, coupling constraints between such states, and security constraints.

A. Equality constraints at pre-contingency

The active and reactive power balance equations in the pre-contingency state are

$$0 = \sum_{i=1}^{NG} P_{Gi,t}^o - \sum_{j=1}^{NLoad}\left(P_{Loadj,t}\right) - P_{Loss_t}^{\ \ o} \qquad for\ t = 1,2,.....,T \tag{5.a}$$

$$0 = \sum_{i=1}^{NG} Q_{Gi,t}^o - \sum_{j=1}^{NLoad} Q_{Loadj,t} - Q_{Loss_t}^{\ \ o} \qquad for\ t = 1,2,.....,T \tag{5.b}$$

where $P_{Loss,t}$ represents the total active power losses, while $Q_{Loss,t}$ are the total reactive power ones, at time t.

B. Equality constraints at post-contingencies

The active and reactive power balance equations of each credible contingency become,

$$0 = \sum_{i=1}^{NG} P_{Gi,t}^k - \sum_{j=1}^{NLoad} P_{Loadj,t} - P_{Loss_t}^{\ \ k} \qquad for\ \ t = 1,2,.....,T \ \ k = 1,2,....,K \tag{6.a}$$

$$0 = \sum_{i=1}^{NG} Q_{Gi,t}^k - \sum_{j=1}^{NLoad} Q_{Loadj,t} - Q_{Loss_t}^{\ \ k} \qquad for\ \ t = 1,2,.....,T \ \ k = 1,2,....,K \tag{6.b}$$

where K is the total number of credible contingencies.

C. Inequality constraints at pre- and post-contingencies

The active power generated by each unit must satisfy the maximum and minimum operating limits, both for pre- and post-contingency states, (7.a)-(7.b).

$$P_{Gi\_MIN} \le P_{Gi,t}^o \le P_{Gi\_MAX} \tag{7.a}$$

$$P_{Gi\_MIN} \le P_{Gi,t}^k \le P_{Gi\_MAX} \tag{7.b}$$

where P_{Gi_MIN}, P_{Gi_MAX}, express the minimum and maximum thermal units' operating limits.

The active power flow through each branch of the network must satisfy the security limits, (8.a) y (8.b).

$$\left|FLOW_{ij}^{o}\right| \leq FLOW_{ij\_MAX}^{o} \quad \forall ij,\ i \neq j \qquad (8.a)$$

$$\left|FLOW_{ij}^{k}\right| \leq FLOW_{ij\_MAX}^{k} \quad \forall ij,\ i \neq j \qquad (8.b)$$

where $FLOW_{ij\_MAX}^{0}$, $FLOW_{ij\_MAX}^{k}$, represent the maximum active power that should flow through the branch connecting the buses *i-j*, during the pre-contingency and each post-contingency state, respectively.

D. Coupling constraints

These constraints are related with those ones that the elements should satisfy to get ahead from the pre-contingency to each post-contingency state.

The active power delivered by the thermal units at the pre-contingency state should be such that satisfies the generator's operative slopes, (9.a)-(9.b).

$$P_{Gi,t}^{o} \leq P_{Gi,t-1} + \Delta_{MAXi}^{+} \qquad (9.a)$$

$$P_{Gi,t}^{0} \geq P_{Gi,t-1} - \Delta_{MAXi}^{-} \qquad (9.b)$$

Δ_{MAXi}^{+}, Δ_{MAXi}^{-}, symbolize the i-th start-up and stop-down slopes, respectively; *t,* is the index time of the scheduled period.

Furthermore, it is necessary that the active power generated by the thermal units during each post-contingency state should be such that satisfies the generator's operative slopes, taking into account the system condition at the pre-contingency state, (10.a)-(10.b).

$$P_{Gi,t}^{k} \leq P_{Gi,t}^{o} + \Delta_{MAXi}^{+} \qquad (10.a)$$

$$P_{Gi,t}^{k} \geq P_{Gi,t}^{o} - \Delta_{MAXi}^{-} \qquad (10.b)$$

E. Security constraints

The contingency ranking is a measure to evaluate the relative severity of a contingency. Such ranking is formed through the active power performance index (PI) which is evaluated as follows [24].

$$PI = \sum_{i=1}^{all\ branches} \frac{W_l}{2}\left(\frac{\left|FLOW_{i,j}^{k}\right|}{FLOW_{i,j\_MAX}^{k}}\right)^2 \qquad (11)$$

where, W_l is a real non-negative weighting coefficient (in this paper it is assumed equal to unity). PI is a small number when all flows are less than the capability of the respective line, and it is a large value whenever there are overloaded lines. AC load flow is used to calculate the active power performance indices for every outage. The cases with the highest values' power performance indices are analyzed as credible contingencies.

Finally, thermal units must satisfy a minimum up- and down-time, (12.a)-(12.b).

$$T_i^{ON} \geq T_{MIN,i}^{ON} \qquad (12.a)$$

$$T_i^{OFF} \geq T_{MIN,i}^{OFF} \qquad (12.b)$$

where T_i^{ON}, T_i^{OFF}, symbolize the elapsed time since the i-th unit has been turned-on or turned-off, until the analyzed moment, respectively; $T_{MIN,i}^{ON}$, $T_{MIN,i}^{OFF}$, represent the minimum up-time and down-time that the i-th unit must satisfy.

IV. Proposed Solution

The proposed algorithm flowchart is depicted in Fig. 1. The major steps of the OPF-SC solution are summarized in the following.

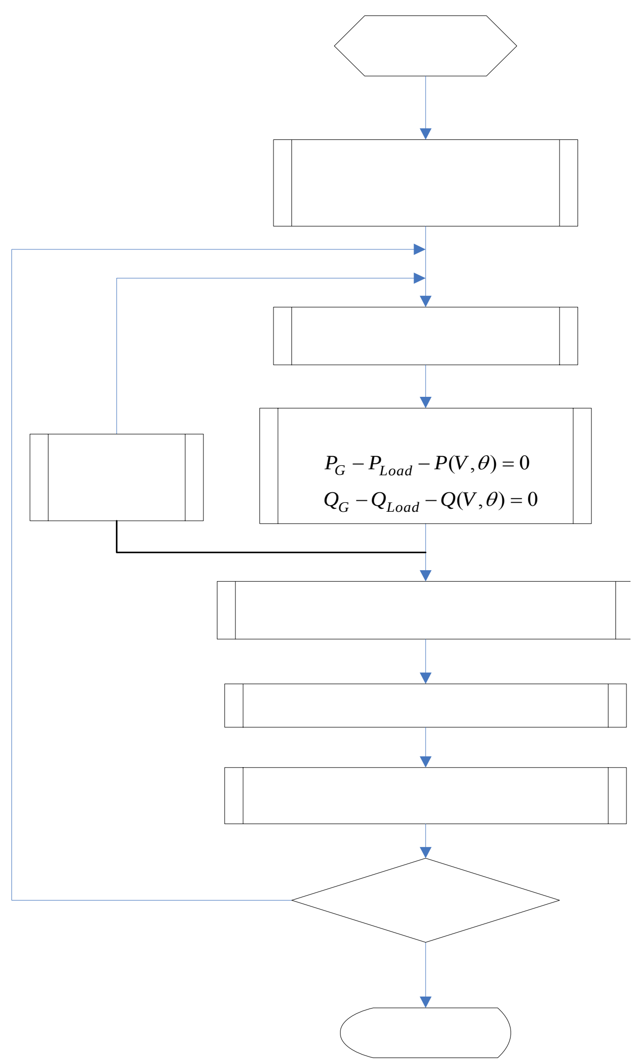

Fig. 1. Proposed algorithm flowchart

A. Initial population

In this paper, a particle is composed by continuous and discrete control variables. The continuous ones include the generators' active power output, and the discrete variables include the transformers-tap setting and var-injection values of the switched shunt capacitors/reactors.

The population is constituted by *K*+1 matrices, one for the base case and one for each of the *K* credible contingencies, (subpopulations) of size *T x(NG+ND) x NIND*; where *K* represents the total number of the counted contingencies; *T* is the number of scheduled hours; *NG* is the number of continuous variables (available thermal units); *ND* is the number of discrete variables (available transformers-tap and switched shunt elements); *NIND* is the number of particles.

Within a subpopulation, each particle is defined by a matrix of size *T* x *(NG+ND)*, where each of the *NG* first elements corresponds to the i-th active power generation at time *t*, while each of the *ND* elements corresponds to the k-th adjustable step size of the discrete control variable, Fig. 2.

	G_1		G_{NG}	ST_1		ST_{ND}
1	$P_{G1,1}$	…..	$P_{G1,NG}$	$ST_{1,1}$	….	$ST_{1,ND}$
2	$P_{G2,1}$	…..	$P_{2,NG}$	$ST_{2,1}$	….	$ST_{2,ND}$
3	$P_{G3,1}$	…..	$P_{3,NG}$	$ST_{3,1}$	….	$ST_{3,ND}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
T	$P_{GT,1}$	…..	$P_{GT,NG}$	$ST_{T,1}$	….	$ST_{T,ND}$
ST means discrete variables (shunts & taps)						

Fig. 2. Particle representation.

For each scenario (pre- and post-contingency states), the initial active power at time T_k are percentile and randomly allocated among the *NG* available thermal units, such that the load is satisfied, (13).

$$P_{Gi,t}^{n} = \frac{rand(T\ ,NG)}{\sum rand(T\ ,NG)} * \sum_{j=1}^{Nload} P_{LOADj,t} \quad ……(13)$$

$$for\ n = 1,2,....,K\ \ and\ \ t = 1,2...,T$$

where, rand(*T,NG*) represent a matrix of size *TxNG* randomly generated in the interval [0, 1], $\sum_{j=1}^{Nload} P_{LOADj,t}$ is a vector of size *T*, where each element corresponds to the total load at time *t*.

The transformer-tap setting and var-injection values of one switched shunt capacitor/reactor (discrete variable) are randomly generated between upper and lower limits (14.a), and taking the nearest value in the interval $[u_k^{\max}, u_k^{\min}$, step $M_i]$, with the steps M_i for each discrete variable is defined by (14.b).

$$ST_{j,t} = u_j^{\min} + rand()\left(u_j^{\max} - u_j^{\min}\right) \quad (14.a)$$

$$M_j = \frac{\left(u_j^{\max} - u_j^{\min}\right)}{Step_j} \quad (14.b)$$

for t = 1, 2,….., T and j = 1, 2,….., ND

where, $u_j^{\max}$, $u_j^{\min}$ express the j-th lower and upper discrete variable's operating limits, and $Step_j$ is the adjustable step size of the j-th discrete variable.

B. Reconstruction operators

In this paper, reconstruction operators are adopted for continuous variables [25]. Such operators satisfy the units' constraints. Taking into account the inequalities within a conventional formulation, through penalization for example, result in an execution where it is very feasible to choose wrong decisions, due to the great quantity of penalizing factors. Together, the PSO and the operators, accomplish such handling in a more transparent way limiting the use of penalization. That is, such mechanisms control that each continuous variables fulfills the slope restrictions, minimum times, and the generating limits for the pre- and post-contingencies states. For each state the following reconstruction operators have been applied.

1) Active power generation limits

For each particle in the population, and for each generation, the assigned active power to each thermal unit must satisfy lower and upper generation limits. For those units where the active power is below a predefined percentage of the minimum (in this paper chosen as λ=0.8), the *off* state is chosen for such unit (P_{Gi}= 0), (15).

$$P_{Gi}^{'} = \begin{cases} P_{Gi\_MAX} & if \quad P_{G,i} > P_{Gi\_MAX} \\ P_{Gi} & if \quad P_{G,i\_MIN} < P_{Gi} < P_{G,i\_MAX} \\ P_{Gi\_MIN} & if \quad \lambda * P_{Gi\_MIN} < P_{Gi} \le P_{Gi\_MIN} \\ 0 & otherwise \end{cases} \quad (15)$$

Thus, this operator is useful for avoiding the use of additional variables in order to determine the state of the units (*on*/*off)*, diminishing the total number of variables for optimizing.

2) Minimum down- and up-time

$$P_{Gi}^{''} = \begin{cases} 0 & if\ \ i-th\ unit\ is\ start\ and \quad T_i^{OFF} < T_{MIN,i}^{OFF} \\ P_{G,i\_MIN} & if\ i-th\ unit\ is\ OFF\ and \quad T_i^{ON} < T_{MIN,i}^{ON} \\ P_{Gi}^{'} & otherwise \end{cases} \quad ……(16)$$

3) Start/stop and up/down slopes

$$P_{Gi}^{'''} = \begin{cases} \min(P_{Gi}^{''}, \Delta_{ON}) & if \quad P_{G,t-1} = 0\ and\ P_{Gi}^{''} = 1 \\ \max(P_{Gi,t-1} - \Delta_{OFF}, 0) & if \quad P_{G,t-1} = 1\ \ and\ \ P_{Gi}^{''} = 0 \\ \min(P_{Gi\_MAX}, P_{Gi,t-1} + \Delta_{UP}) & if \quad P_{Gi}^{''} - P_{Gi,t-1} > 0 \\ \max(P_{\min}, P_{Gi,t-1} - \Delta_{DOWN}) & if \quad P_{Gi,t-1} - P_{Gi}^{''} > 0 \end{cases} \quad ……(17)$$

where

Δ_{ON}	the i-th unit start-slope limit.
Δ_{OFF}	the i-th unit stop-slope limit.
Δ_{UP}	the i-th unit up-slope limit.
Δ_{DOWN}	the i-th unit down-slope limit.

4) Constraints handling for load balancing

After the adjustment of the precedent steps *1-3*, the total active power assigned to the thermal units is not necessarily equal to the demanded load. Thus, a re-dispatch is required taking into account the available units. Such re-dispatch must handle the constraints *1-3*. In the following, the algorithm used to accomplish the re-dispatch is schematized.

STEP 1. Define a set $S_{on}=\{j|U_j=1$, *satisfying* $1 \le j \le NG\}$.

STEP 2.- Define a set $S_{off}=\{j|U_j=0$, *satisfying* $1 \le j \le NG\}$.

STEP 3.- Define a set $S_1=\{j|\, P_{Gj\_MIN} \le P_{Gj}^{'''} < P_{Gj\_MAX}$, *satisfying* $1 \le j \le NG\}$; $S_1 \subset S_{on}$

STEP 4.- Establish a set $S_2=\{j|\, P_{Gj\_MIN} < P_{Gj}^{'''} \le P_{Gj\_MAX}$, *satisfying* $1 \le j \le NG\}$; $S_2 \subset S_{on}$

STEP 5.-

Do while $\sum_{i=1}^{NG} P_{Gi}^{''''} < \sum_{j=1}^{NLoad} P_{Loadj}$

Choose randomly one element from S_1 *or from* S_{off} *in case* $S_1=\{\}$. *Call such one* P_{UP}, *then*

$$P_{Gi}^{''''} = \min\left(P_{Gi\_MAX}, P_{UP} + \left(\sum_{j=1}^{NLoad} P_{Loadj} - \sum_{i=1}^{NG} P_{Gi}^{''''}\right), P_{Gi} + \Delta\right)$$

Delete this element from the set it was taken

End

STEP 6.

Do while $\sum_{i=1}^{NG} P_{Gi}^{''''} > \sum_{j=1}^{NLoad} P_{Loadj}$

Choose randomly one element from S_2 *or from* S_{on} *in case* $S_2=\{\}$, *then*

$$P_{Gi}^{''''} = \max\left(P_{Gi\_MIN}, P_{Gi}^{''''} - \left(\sum_{i=1}^{NG} P_{Gi}^{''''} - \sum_{j=1}^{NLoad} P_{Loadj}\right), P_{Gi,k-1}^{''''} - \Delta, P_{Gi}^{''''} - \Delta\right)$$

Delete this element from the set it was taken

End

STEP 7.- End

S_1 (*STEP 3*) represents the set of units able to increase their generation, whereas S_2 (*STEP 4*) represents the set of units able to reduce their generation; both of them are subsets of S_{on}.

C. Losses

After that the reconstruction operators have been applied and the control variables' values are determined, for each particle, a load flow run is performed. Such one allows to evaluate the branches' active power flow and the total losses, which are assigned to that unit playing the role of the slack.

D. Fitness function

After the reconstruction operators have been applied to each particle, the transmission lines' limits are not necessarily satisfied.

Several techniques for handling constraints in evolutionary algorithms have been previously proposed in the specialized literature [26]. One of them considers the objective function penalization. Using this technique, the fitness function is formed by the objective function (3) plus penalty terms for particles that have violated some power flow constraints. Such fitness function can be expressed as (18)

$$F_i^{apt} = F_i^{obj} + \sum_{t=1}^{T}\left(Cte(iter)\sum_{j=1}^{N\_OFLW} OverFlow_{j,t}\right) \quad for \;\; i = 1,..,NIND \tag{18}$$

where, N_OFLW represents the total number of lines with over flow; $NIND$ is the number of individuals; $OverFlow_{j,t}$ is defined as in (19).

$$abs(FLOW_{i,j\_MAX} - FLOW_{i,j}) \tag{19}$$

$FLOW_{i,j\_MAX}$, is the maximum allowed flow across the line connecting buses *i-j*; *Cte(iter)* is a dynamically modified penalty value [30]: $Cte(iter) = 300\sqrt{iter}$; this one allows an increasing penalization along generations.

E. Updating

The new position (updating within the searching space) can be evaluated by (1) and (2).

In this paper, the weight *w* in (1), is defined by a linearly decreasing function (20), beginning from a relatively large value (w_{MAX} = 0.9) and diminishing toward a small one (w_{MIN} = 0.4), while the PSO-RO is evolving. This strategy aids to the global search at the beginning of the iterative process, and to the local search at the end of the iterative process [22].

$$w = w_{MAX} - \frac{w_{MAX} - w_{MIN}}{iter_{MAX}} * iter \tag{20}$$

where $iter_{MAX}$, expresses the maximum number of iterations; *iter* symbolizes the current iteration.

V. Results

The proposed methodology has been applied to two power systems. The first one is constituted by 39 buses, 46 branches, and 10 generators, Fig. 3 [27]. The second system possesses 26 buses, 46 branches 6 generators, 7 transformers and 9 shunt capacitors [28]. In both of them a period of one hour was considerated (T=1).

A. First Example.

The New England power system is depicted in Fig. 3. The base case data is available in [27], with a total load of 1000 MW. The transmission line ratings have been elected as 1000 MW and 1200 MW for the pre- and post-contingency states, respectively. For this example three cases have been taken into account.

CASE 1: The simple unit commitment problem (UC) is executed, where network constraints are not considered. The commitment schedule and the generating cost are shown in Table I.

CASE 2: Optimal power flow problem (OPF). In this case the impact of the transmission line's capacity limits is included. If the UC results are considered as possible solutions for this case, transmission flow violations will occur. Therefore a re-dispatch is necessary to satisfy the transmission line's capacity limits; this re-dispatch is displayed in Table I, where it is shown that some of the most expensive units must be dispatched to satisfy the transmission line's capacity limits.

CASE3: Optimal power flow with security constraints. The outage of line 2-19, based on results of the CASE 2, will cause overflows on line 2-11. The new dispatch for preventing post-contingency violations in the OPF-SC is exhibited in Table I.

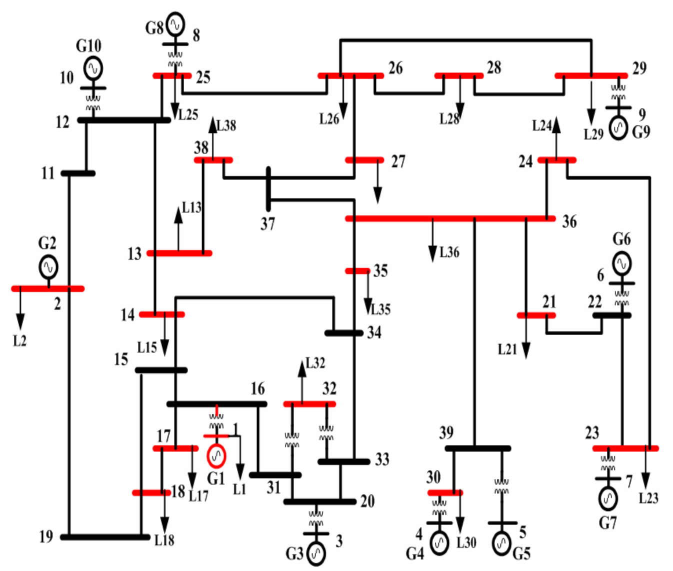

Fig. 3. New England power system

TABLE I
SIMULATION RESULTS

Unit	UC	OPF	OPF-SC pre-contingency	OPF-SC Line 2-19 out
$P_{G,1}$	455	213	213	234
$P_{G,2}$	455	455	455	393
$P_{G,3}$	0	0	0	0
$P_{G,4}$	0	0	0	130
$P_{G,5}$	0	162	162	162
$P_{G,6}$	80	80	80	80
$P_{G,7}$	0	0	0	0
$P_{G,8}$	10	55	55	0
$P_{G,9}$	0	35	35	0
$P_{G,10}$	0	0	0	0
Cost ($)	20869	25279	25279	22557

B. Second example

The power system contains six thermal units, 26 buses, 46 transmissions lines; the total load is 1263 MW, Fig. 4. The transmission lines' parameters and load data are taken from [28].

The generator's cost curve coefficients are shown in Table II. These one corresponds to the quadratic active power generation cost curve with a sinusoid component; this one was used to represent the valve-point loading effects (21) [29].

$$f_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i + \left| d_i \sin\left(e_i\left(P_{Gi\_MIN} - P_{Gi}\right)\right)\right| \quad \ldots\ldots(21)$$

TABLE II
GENERATING UNIT CAPACITY AND COEFFICIENTS

BUS	1	2	3	4	5	6
a [$/MW^2]	0.007	0.0095	0.009	0.009	0.008	0.0075
b [$/MW]	7	10	8.5	11	10.5	12
c [$]	240	200	220	200	220	190
d	100	80	80	50	80	50
e	0.0545	0.0825	0.071	0.093	0.0825	0.09
$P_{G\ MAX}$	500	200	300	150	200	120
$P_{G\ MIN}$	100	50	80	50	50	50
Slopes						
Start[MW]	500	200	300	150	200	120
Stop[MW]	500	200	300	150	200	120
Up	250	150	80	100	100	100
Down	250	150	80	100	100	100
Pinitial	0	0	0	0	0	0
Slopes Up & Down [MW/h]						

Likewise, the upper and lower generation limits and the slopes are shown in Table II.

The system has a total of 22 control variables, being: six active power generations, seven transformers-tap setting, and nine var-injection values. The adjustable step size is 0.01 p.u. for the transformers-tap setting; their adjustable range is [0.9, 1.1]. Related to the var-injection capacitors, the changing step size is taken as 0.5 Mvar; their adjustable range is [0.0, 5.0].

In Fig. 4 the transmission line's capacity limits (in MW) are exhibited. The lower value is the pre-contingency limit, while the value inside the parentheses is the post-contingency limit.

In order to validate the proposed PSO-RO, furthermore of the three same cases as the previous example, a simple Economic Dispatch (ED) have been analyzed. For the ED case, the sinusoid component in the cost functions is not considered. For the fourth case, the contingencies with highest value of the performance index (PI) have been elected as credible contingencies: lines 3-13 and 17-18.

After performing 100 independent runs, the simulations are summarized in Table III, which include the average, maximum, minimum, and standard deviation.

For the ED case, reference [29] reports some generating cost using both a GA and some PSO variants. The best result reported in [29] is $15450. As seen in Table IV, PSO-RO is able to obtain lower costs.

TABLE III
STATISTICS

Case	Average ($)	Min ($) (best)	Max ($) (worst)	Standard deviation
ED	15437.8	15436	15444	3.093
UC	15435	15428	15447	6.93
OPF	15561	15523	15601	18.35
OPF-SC	47535	47306	47863	112.6

The control variables' optimal setting are shown in Table IV.

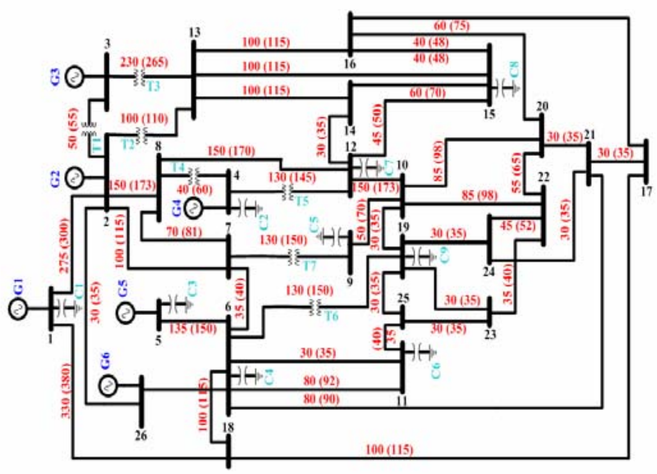

Fig. 4. 26- buses power system

TABLE IV
CONTROL VARIABLES' OPTIMAL SETTINGS

Unit	ED	UC	OPF	OPF-SC pre-cont.	OPF-SC Lines out 3 - 13	OPF-SC Lines out 17 - 18
$P_{G,1}$	446.4	445.8	446.7	448.2	493.3	449.4
$P_{G,2}$	173.1	200.0	166.3	199.6	200.0	200.0
$P_{G,3}$	261.9	300.0	255.8	197.7	117.7	209.7
$P_{G,4}$	136.5	130.7	150.0	148.4	150.0	119.1
$P_{G,5}$	171.1	200.0	166.6	164.6	199.4	200.0
$P_{G,6}$	86.1	0	90.3	120.0	120.0	110.2
Tap1	0.90	1.07	1.03	1.04	0.90	1.10
Tap2	0.97	1.00	1.10	0.95	0.91	1.09
Tap3	0.94	0.94	0.90	0.99	1.04	0.99
Tap4	0.98	0.97	0.98	0.90	1.07	0.90
Tap5	1.00	1.03	0.96	1.10	0.93	1.08
Tap6	0.96	0.98	0.95	1.10	1.10	0.90
Tap7	0.97	1.09	0.90	0.90	0.90	0.90
Shunt1	3.5	2.0	0.0	5.0	0.0	0.0
Shunt2	1.5	0.0	0.0	0.0	5.0	0.0
Shunt3	1	0.0	0.0	0.0	0.0	0.0
Shunt4	4	3.0	5.0	5.0	4.5	5.0
Shunt5	1.5	4.5	4.5	5.0	4.5	0.0
Shunt6	1	1.5	2.0	5.0	0.0	4.5
Shunt7	0	2.0	0.0	5.0	4.5	0.0
Shunt8	5	0.0	0.0	0.0	5.0	2.5
Shunt9	5	3.0	5.0	0.0	4.5	2.0
Cost ($)	15436	15428	15523	15662	15874	15770
P_{loos}	12.1	13.5	12.8	15.5	17.5	25.3
time [min]	5.0	5.0	7.0	16.0		

The main parameters of the PSO-RO are shown in Table V [17], [22].

TABLE V
PARAMETERS OF PSO-RO ALGORITHMS

C_1	C_2	w_{MAX}	w_{MIN}	*itermax*	*NIND*
2.1	2.1	0.9	0.4	50	100

The used code for these studies was implemented in Matlab, on a Personal Computer, Pentium IV, 3 GHz.

VI. CONCLUSIONS

In this paper we propose an approach for the solution of the OPF-SC problem through the use of a particle swarm optimizer algorithm with reconstruction operators. Conventionally, the PSO handles constraints by penalizing the objective function. In this paper, the reconstruction operators are used to handle the units' operative constraints, while particles that have violated some power flow constraint are penalized. The use of reconstruction operators allows to increase the suitable particles in the searching space. By varying the weight *w* throughout the course of the PSO-RO run, the algorithm performs a global search at the beginning of the iterative process, and a local (i.e., more focused) search towards the end, which improves the performance of our PSO. Additionally, these operators allow to increase the number of particles within the feasible region, giving rise to the algorithm exhibits greater capacity of searching. Thus, the dependency of the heuristic algorithms on the proper definition of the penalization terms is reduced.

The proposed methodology is able to take into account feasible and satisfactory solutions for both the base case and for a set of credible contingencies. In case that the power system is capable to get ahead from a pre-contingency state to post-contingencies states, generation and branches' operative constraints are satisfied.

The proposed approach has been applied to two power systems of the open literature with satisfactory results. As the examples illustrate, different cost functions have been used to test the robustness of the proposed method.

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