PREFERENCE INCORPORATION IN MOEA/D USING AN OUTRANKING APPROACH WITH IMPRECISE MODEL PARAMETERS

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ABSTRACT

Multi-objective Optimization Evolutionary Algorithms (MOEAs) face numerous challenges when they are used to solve Many-objective Optimization Problems (MaOPs). Decomposition-based strategies, such as MOEA/D, divide an MaOP into multiple single-optimization sub-problems, achieving better diversity and a better approximation of the Pareto front, and dealing with some of the challenges of MaOPs. However, these approaches still require one to solve a multi-criteria selection problem that will allow a Decision-Maker (DM) to choose the final solution. Incorporating preferences may provide results that are closer to the region of interest of a DM. Most of the proposals to integrate preferences in decomposition-based MOEAs prefer progressive articulation over the "a priori" incorporation of preferences. Progressive articulation methods can hardly work without comparable and transitive preferences, and they can significantly increase the cognitive effort required of a DM. On the other hand, the "a priori" strategies do not demand transitive judgements from the DM but require a direct parameter elicitation that usually is subject to imprecision. Outranking approaches have properties that allow them to suitably handle non-transitive preferences, veto conditions, and incomparability, which are typical characteristics of many real DMs. This paper explores how to incorporate DM preferences into MOEA/D using the "a priori" incorporation of preferences, based on interval outranking relations, to handle imprecision when preference parameters are elicited. Several experiments make it possible to analyze the proposal's performance on benchmark problems and to compare the results with the classic MOEA/D without preference incorporation and with a recent, state-of-the-art preference-based decomposition algorithm. In many instances, our results are closer to the Region of Interest, particularly when the number of objectives increases.

Keywords: Preference incorporation; Outranking relations; Interval numbers; MOEA/D; MOEA/D-NUMS.

1 Introduction

Many real-world problems require one to solve multi-objective optimization problems (MOPs). Solving these problems requires the identification of a set of solutions in the so-called Pareto front that satisfy the compromise condition of not improving one objective without worsening some other objective. Pareto dominance-based multi-objective evolutionary algorithms (MOEAs) have been successfully used to solve problems with 2–3 objective functions; however, most of them cannot appropriately solve problems with four or more objectives (many objective optimization problems, or MaOPs).

The decomposition-based paradigm has outperformed Pareto dominance-based algorithms. MOEA/D [50] provided this new paradigm, which is more robust than Pareto-based MOEAs when handling higher dimensions in the objective space.

This algorithm decomposes a single, multiple-objective problem into many single objective optimization sub-problems by defining different weight vectors for different scalarizing functions. The sub-problems are solved simultaneously, and the different search processes influence each other. In each generation and sub-problem, genetic operators create offspring using two parents chosen from the sub-problem's neighborhood. Trivedi et al. [44] and Xu et al. [47] presented recent surveys of decomposition-based algorithms.

The process of solving an MOP is not complete until the best compromise is identified by the DM. This decision-making process requires the articulation of the DM´s preferences, which can be performed in three different ways: a priori, a posteriori, or interactively.

In traditional MOEAs, preferences are incorporated "a posteriori." This preference articulation approach assumes that a metaheuristic obtains an approximated Pareto front containing a representative subset of the Region of Interest (RoI), the zone more in agreement with the DM's preferences. The effectiveness of this assumption depends on the problem and the metaheuristic used. Once the approximation of the Pareto front has been obtained, the DM must solve a multi-criteria selection problem involving this set to choose the final solution. There are two methods:

i) One can make a heuristic selection. This method is based on assuming that the DM can consistently compare solutions on the approximated Pareto front and identify the best compromise. This could be a challenging task in problems with more than four objective functions due to the human mind's cognitive limitations in processing even a small amount of information simultaneously, as was stated by Miller in his famous paper [35].
ii) A formal multi-criteria decision method, which involves a model of the DM's preferences, can be used.

The "a priori" and progressive (interactive) articulation of preferences puts selective pressure toward solutions closer to the RoI, thus narrowing down the search space [7]. This could bring better approximations to the Pareto front and drastically reduce the number of solutions that are candidates for the best compromise, thus alleviating the cognitive effort required from the DM. Concerning MaOPs, these are relevant advantages compared to posterior preference incorporation.

As a consequence of the above discussion, there has been an increasing interest in combining MOEAs and multi-criteria decision-making (MCDM) techniques in recent years. It has been admitted that MOEAs and MCDM techniques can offer each other many advantages [7].

There are many proposals in the scientific literature that describe how to articulate preference information into the evolutionary search process to improve the selective pressure towards the RoI. According to Bechikh et al. [4], the most commonly used preference information structures in MOEAs are the following:

- *weights* (e.g., [5]),
- *the ranking of solutions* (e.g., [17]),
- *the ranking of objective functions* (e.g., [15]),
- *reference points or aspiration levels* (e.g., [42,49]),
- *reservation points* (e.g., [28,33])
- *trade-offs between objective functions* (e.g., [6]),
- *desirability thresholds* (e.g., [46]), and
- *outranking parameters* (e.g., [1,20,38]).

According to Li et al. [32], other ways to express the DM's preferences are the following:

- *the holistic pairwise comparisons of solutions* (e.g., [7,43]), and

- *the classification of solutions* (e.g., [13,14]).

Incorporating the DM's preferences into appropriate frameworks may provide results that are closer to the RoI than the results of MOEA/D, NSGA III, and IBEA [12,14,32]. Nevertheless, the experimental results in [23] showed that incorporating preferences into the evolutionary search does not always lead to a better approximation of the RoI compared to traditional MOEAs, especially when the number of objectives is small; as the number of objective functions increases, incorporating preferences becomes more important.

With the aim of taking advantage of their desirable conjoint properties, in recent years, several papers have proposed different ways to incorporate decision-maker preferences into decomposition-based algorithms. Pilat and Neruda [39] progressively incorporate preferences into MOEA/D through the coevolution of the weights. Ma et al. [34] proposed an "a priori" articulation of preferences in MOEA/D using the weight vectors in each sub-problem. De Souza et al. [16] suggested modifying the reference points in the scalarizing functions, and Li et al. [30] used T-MOEA/D in an interactive framework in which the DM articulates her/his preferences through a target region given by the preferred range of objective functions. Li et al. [31] proposed a Non-Uniform Mapping Scheme (NUMS) to map reference points to new positions close to the aspiration-level vector supplied by the DM. In R-MOEA/D, Yutao et al. [49] handled preferences interactively through scalarizing functions with reference points derived from a reference point specified by the DM. Li [29] proposes an interactive version of MOEA/D in which, after several generations, the DM should provide scores of a subset of solutions; from her/his response, an approximated value function is derived, and it is used to guide the search. In the decomposition-based algorithm IEMO/D, Tomczyk and Kadzinski [43] interactively use the information from the pairwise comparisons of solutions to build compatible L-norms, which guide the search process toward the RoI.

As was underlined in the above paragraph, most proposals to incorporate preferences in decomposition-based MOEAs prefer progressive articulation. Generally speaking, interactive methods are more popular than proposals with the "a priori" articulation of preferences due to the following three reasons:

First, the algorithm "learns" the DM's preferences and progressively increases its capacity to suggest more preferred solutions [43].

Second, within an interactive framework, the DM learns about her/his problem, discovers new solutions, considers the complex trade-offs among her/his objectives, and can revise and update her/his preferences; such a learning process helps the DM to choose more appropriate settings of the decision model parameters [14].

Third, the DM should be more confident with the final results since (s)he has been involved in the search process and has approved current solutions [14].

Most interactive methods require systematic comparisons among subsets of solutions or among solutions and reference points. This is a big concern when the number of objective functions increases up to four because the human mind's cognitive limitations impede comparable and transitive judgments. Without comparable and transitive preferences, interactive methods can hardly work [26]. This difficulty also strongly increases the cognitive effort required from the DM.

On the other hand, the "a priori" articulation of preferences is criticized because, to a certain extent, the appropriate parameters of preference models are influenced by complex trade-offs on the unknown Pareto front; their aprioristic setting is hence very difficult. Such a direct and "a priori" parameter elicitation is only possible with significant imprecision.

The "a priori" articulation of preferences is usually less cognitively demanding for the DM than interactive methods, and it does not require transitive judgments from the DM. The outranking approach has been successfully used in an "a priori" way (e.g., [1,19–21]). Non-compensatory and non-transitive preferences, incomparability, and veto situations are characteristics of many DMs facing real-world problems; these features are handled by ELECTRE multi-criteria decision methods and other outranking approaches (cf. [41]), which do not demand rational behavior from real DMs. However, except through complex sensitivity analyses, ELECTRE methods have no way of handling imprecision and ill-definition in preference model parameters (weights, vetoes, credibility threshold); particularly, the setting of the veto threshold is usually challenging for real DMs. This difficulty is even more relevant when the DM is an entity with ill-defined preferences (e.g., a heterogeneous group) or an inaccessible person (e.g., the CEO of a large enterprise or the head of a governmental organization).

To our knowledge, no outranking method has been combined with decomposition-based evolutionary algorithms; this is a research gap. A method for handling the imprecision and ill-definition of the outranking model's parameters is required to achieve a successful combination. Among several approaches to deal with imprecision, to our knowledge, only fuzzy sets have been used to propose extensions of ELECTRE and other outranking methods (e.g., [9,10]). Perhaps due to their mathematical sophistication or to a certain arbitrariness of some operators, the fuzzy extensions of ELECTRE methods have not gained the approval of the multi-criteria decision aid community.

Recently, Fernández et al. [22] proposed an interval-based outranking method, which shares desirable properties with ELECTRE methods for handling non-compensatory and non-transitive preferences in incomparability and veto situations. Such a proposal can address imprecisions in model parameter values, which is a clear advantage when either "a priori" parameter setting is performed by a single DM or the DM is a collective entity with ill-defined preferences. Collective preferences are usually conflicting, and some undesirable effects, such as manipulation and dictatorship, are possible. The usefulness of the interval-based approach rests on the idea that, usually, the DM (or the group in charge of the decision-making process), feels more comfortable setting the model parameter values as interval numbers rather than as precise numbers. The relevance of negotiation and consensus-reaching processes to identifying the region of interest for groups of DMs has been underlined by [3]. In this sense, the aggregation of diverse parameter settings as interval numbers could be a reasonable model of collective preferences that could be used to identify solutions that are close to the so-called "social RoI." From a different perspective, in [2] and [24], consensus search in group evolutionary multi-objective optimization has been addressed by using the interval outranking approach. In such papers, consensus is identified through the optimization of a measure of group satisfaction/dissatisfaction. Additionally, the interval outranking model is simpler than fuzzy extensions of ELECTRE methods. Fernández et al. in [23] proposed an indirect elicitation method that can infer the whole parameter set of the interval outranking approach if the DM is reluctant to perform direct parameter setting. To use this method, the DM should provide a set of pairwise comparisons of real or fictitious solutions in terms of strict preference, weak preference, k-preference, indifference, or incomparability.

This paper explores how to incorporate decision-maker preferences in the MOEA/D update phase by using the interval outranking approach from [22]. The basic MOEA/D algorithmic structure is kept, but in the update phase, the Tchebychev norm is complemented by a binary preference relation, which derives from the interval outranking information. To the best of our knowledge, there is no published paper incorporating preferences in this way; additionally, the use of an outranking approach confers desirable properties to address non-compensatory preferences and veto effects. Also, the use of intervals allows one to handle imprecision and ill-definition in the model parameters and poorly defined preferences. In extensive experiments, six different ways to incorporate the interval outranking-based preferences are analyzed and compared with the original MOEA/D and with the state-of-the-art preference-based MOEA/D-NUMS, proposed by Li et al. in [31], for nine DTLZ test problems that cover a wide range of objective space dimensions.

The present paper comprises the following parts: Section 2 gives background information, including a description of the interval outranking approach by Fernández et al. ([22]) and the basic implementation of MOEA/D. Section 3 describes in detail MOEA/D/O, which is the main proposal of this paper, and the different ways used to incorporate the DM's preferences. The experimental design is described in Section 4. The results and discussion are presented in Section 5. Finally, some general conclusions are given in Section 6.

2. Background

2.1 A brief description of MOEA/D

A multi-objective optimization problem (MOP) is a problem with more than one conflicting objective that must be optimized simultaneously. Equation (1) formally defines an MOP with m objectives:

$$\max \mathrm{F}(x) = (f_1(x), f_2(x), \ldots, f_m(x)), \quad \text{subject to: } x \in \Omega. \tag{1}$$

The MOEA/D framework proposed by [50] is one of the most popular decomposition-based EMO algorithms. It solves the MOP by decomposing it into several scalar optimization problems with the help of uniformly distributed weight vectors $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_N\}$. For each weight vector, this work generates one scalar optimization problem using the Tchebychev metric. Starting from an initial set of solutions $x = \{x_1, x_2, \ldots, x_N\}$, the underlying evolution process updates the neighbor solutions x_j associated with a weight vector $\varpi_i = \{\varpi^1{}_i, \varpi^2{}_i, \ldots, \varpi^m{}_i\}$ and the set $z = \{z_1, z_2, \ldots, z_m\}$ of best objectives values. The update of x_j is performed with those new solutions y that satisfy the Tchebychev condition $g^{te}(y \mid \varpi_i, z) \leq g^{te}(x_j \mid \varpi_i, z)$, where z is the set of best objective values and $g^{te}(y \mid \varpi_i, z) = \max\{\varpi^k{}_i|f_k(x) - z_k|\}$, for $1 \leq k \leq m$. During the process, each iteration generates two new solutions per weight vector using genetic operators. Zhang and Li [50] offer a detailed description of this approach.

2.2 Some basic information on interval numbers

An interval number is a range of values that lie between two limits. Interval numbers are an easy way to model the imprecision derived from inaccurate measurements or the variability of the DM's judgments and beliefs [1,27]. Interval numbers are an extension of real numbers and a subset of the real line $\mathbb{R}$ (cf. [36]).

Interval numbers will be denoted by bold italic letters, e.g., $\boldsymbol{E} = [\underline{E}, \overline{E}]$, where $\underline{E}$ and $\overline{E}$ correspond to the lower and upper limits. Out of the several operations that can be used to handle interval numbers, we only describe the addition arithmetic operation and the order relations defined below.

Let $\boldsymbol{D}$ and $\boldsymbol{E}$ be two interval numbers. The addition operation is defined as

$$\boldsymbol{D} + \boldsymbol{E} = [\underline{D} + \underline{E}, \overline{D} + \overline{E}].$$

The relations $\geq$ and $>$ on interval numbers are defined using the possibility function $P(\boldsymbol{E} \geq \boldsymbol{D})$. This function is defined by Equation (2) [48]:

$$P(\boldsymbol{E} \geq \boldsymbol{D}) = \begin{cases} 1 & \text{if } p_{ED} > 1, \\ p_{ED} & \text{if } 0 \leq p_{ED} \leq 1, \\ 0 & \text{if } p_{ED} \leq 0, \end{cases} \tag{2}$$

where $p_{ED} = \dfrac{\overline{E} - \underline{D}}{(\overline{E} - \underline{E}) + (\overline{D} - \underline{D})}$.

For the particular case when $\boldsymbol{D}$ and $\boldsymbol{E}$ are real numbers E and D (degenerate intervals), $P(\boldsymbol{E} \geq \boldsymbol{D}) = 1$ if and only if E ≥ D; otherwise, $P(\boldsymbol{E} \geq \boldsymbol{D}) = 0$.

A realization is a real number e that lies within an interval $\boldsymbol{E}$ (cf. [25]). Fernández et al. [22] interpret the possibility function value $P(\boldsymbol{E} \geq \boldsymbol{D}) = \alpha$ as the degree of credibility of the following statement: once two realizations are obtained from $\boldsymbol{E}$ and $\boldsymbol{D}$, the realization d will be smaller than or equal to the realization e.

The relations $\boldsymbol{E} \geq \boldsymbol{D}$ and $\boldsymbol{E} > \boldsymbol{D}$ are defined by $P(\boldsymbol{E} \geq \boldsymbol{D}) \geq 0.5$ and $P(\boldsymbol{E} \geq \boldsymbol{D}) > 0.5$, respectively. Of course, these relations can also compare real numbers.

The paper by Fernández et al. [22] provides a detailed description of interval numbers and the properties of the possibility function.

2.3 Interval outranking approach

Intervals can model imprecision and uncertainty in the judgments and beliefs of the DM. Taking advantage of this feature, Fernández et al. [22] proposed an interval outranking approach (IOA) that is able to simultaneously handle multi-criteria non-compensatory preferences and poor information in model parameters and criterion scores. The IOA can help to approximate the region of interest (RoI) of a particular MOP. The IOA is an extension of ELECTRE methods, and the remainder of this section formalizes it.

Given two solutions x and y of an MOP, the credibility index $\sigma(x, y, \boldsymbol{\lambda})$ of the assertion "x is at least as good as y" models the DM's preferences. Equation (3) defines the value of $\sigma(x, y, \boldsymbol{\lambda})$ using the concordance and discordance that exist among the assertion and criteria values:

$$\sigma(x,y) = \max\{\sigma_\gamma \mid \gamma \in \Omega\}, \tag{3}$$

where σ_γ is given by Equation (4):

$$\sigma_\gamma = \min\{\gamma, P(\boldsymbol{c}(x, y, \gamma) \geq \boldsymbol{\lambda}), 1 - \max\{d_j(x, y) \mid f_j \in \mathrm{D}(x\mathrm{S}_\gamma y)\}\}. \tag{4}$$

In Equation (4),

$\boldsymbol{\lambda}$ is an interval number that reflects a majority threshold;

$\Omega = \{\delta_j > 0, j = 1, \ldots, m\}$ (m is the number of criteria); and

$\delta_j(x, y)$ is the credibility degree of the statement "x outranks y with respect to criterion j."

Table 1 defines each element present in Equation (4). The DM's value system, denoted DM= $(\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \boldsymbol{\beta})$, consists of the weight vector $\boldsymbol{w}$, the veto threshold vector $\boldsymbol{v}$, the majority threshold $\boldsymbol{\lambda}$, the marginal credibility threshold γ, and the overall credibility threshold $\boldsymbol{\beta}$. A solution x has an image $\{f_1(x), f_2(x), \ldots, f_m(x)\}$. The single assumption behind the interval outranking approach is the DM's ability to set criteria scores, weights, veto thresholds, the majority threshold, and the credibility threshold as interval numbers. There are no requirements of "rational" behavior from the DM.

Table 1. Definition of parameters used for the computation of σ_γ.

Elements	**Description**
$f_j(x)$ or f_j	*j-th* criterion value of a solution
$\delta_j(x,y)$ or δ_j	Credibility index of the assertion "x is at least as good as y in criterion f_j." This index is computed as follows: $\delta_j(x, y) = P(f_j(x) \geq f_j(y))$.
xS_jy	Expression denoting "*x outranks y* regarding criterion f_j" if f_j is γ–concordant, i.e., if it holds that $\delta_j(x, y) \geq \gamma$
$\mathrm{C}(x\mathrm{S}_\gamma y)$	Concordance coalition formed by each criterion f_j that satisfies xS_jy

D($xS_\gamma y$)	Discordance coalition formed by each criterion f_j that does not satisfy xS_jy
$\boldsymbol{c}(x,y,\gamma)$	Concordance index defined according to γ, assuming that $\sum_{j=0}^{N} w_j^- \leq 1$ and $\sum_{j=0}^{N} w_j^+ \geq 1$, for the weights $\boldsymbol{w_j}$ of a DM. This parameter has an interval value $\boldsymbol{c}(x, y, \gamma) = [c^-(x, y), c^+(x, y)]$. The value of $c^-(x, y)$ is $c^-(x,y) = \sum_{f_j \in C(xS_\gamma y)} w_j^-$, if it is true that $\sum_{f_j \in C(xS_\gamma y)} w_j^- + \sum_{f_j \in D(xS_\gamma y)} w_j^+ \geq 1$; otherwise, its value is $c^-(x,y) = 1 - \sum_{f_j \in D(xS_\gamma y)} w_j^+$. The value of $c^+(x, y)$ is $c^+(x,y) = \sum_{f_j \in C(xS_\gamma y)} w_j^+$, if it is true that $\sum_{f_j \in C(xS_\gamma y)} w_j^+ + \sum_{f_j \in D(xS_\gamma y)} w_j^- \leq 1$; otherwise it is $c^+(x,y) = 1 - \sum_{f_j \in D(xS_\gamma y)} w_j^-$.
$d_j(x, y)$	Credibility index of the assertion "*xSy* is vetoed by the criterion f_j." It is computed as $P(f_j(y) \geq f_j(x) + \boldsymbol{v}_j)$, where $\boldsymbol{v}_j$ is the veto threshold related to the j-th criterion.

2.4 Some binary preference relations derived from the IOA

From the interpretation of σ(*x*, *y*) as a degree of credibility of the outranking (the credibility of the assertion "*x* is at least as good as *y*"), different binary preference relations arise from the DM's value system DM = ($\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}$, β) and the IOA defined in the previous section. Table 2 present a summary of these relations. Such relations have been utilized in the definition of strategies to guide the search process in decomposition algorithms, as described in Section 3.

Table 2. Binary preference relations.

Relation	**Definition**	**Description**
R_1	$\sigma(y, x) > \sigma(x, y)$	Indicates a certain preference in favor of *y*, although its credibility may be low
R_2	$\sigma(y, x) \geq \boldsymbol{\beta}$	*y* is at least as good as *x* with a credibility threshold $\boldsymbol{\beta} > 0.5$
R_3	$\sigma(y, x) \geq \boldsymbol{\beta}$ and $\sigma(x, y) < \boldsymbol{\beta}$	Indicates an asymmetric preference in favor of *y* with a credibility threshold $\boldsymbol{\beta} > 0.5$
R_4	$\sigma(y, x) > \sigma(x, y)$ and $\sigma(y, x) > 0.5$	Indicates a certain preference in favor of *y* with a credibility threshold of **0.5**
R_5	$\sigma(y, x) \geq \boldsymbol{\beta}$ and $\sigma(x, y) < 0.5$	Indicates a strict preference in favor of *y*

In the rest of the paper, $x\ R_k^{DM}\ y$ denotes the preference relation R_k between solutions *x* and *y* under the decision-maker value system DM = ($\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}$, β). As a consequence of veto effects and Condorcet's paradox[1], these relations are not transitive; that is, $x\ R_k^{DM}\ y$ and $y\ R_k^{DM}\ z$ do not imply $x\ R_k^{DM}\ z$. Incomparability (*not*($x\ R_k^{DM}\ y$) and *not*($y\ R_k^{DM}\ x$)) is also possible. Except for R_2, the remaining preference relations are asymmetric.

3. MOEA/D/O: an MOEA/D approach combined with the interval outranking approach

[1] Condorcet's paradox is a paradox of intransitive preferences arising from the aggregation of individual preferences under a majority rule.

This work proposes a novel interval outranking MOEA/D algorithm called MOEA/D/O. The algorithm combines interval and outranking models that handle imprecision and ill-definition in DMs' preferences and guide an evolutionary strategy's search process. Specifically, MOEA/D/O does the following:

a) It modifies the Tchebychev scalarizing function in MOEA/D to integrate outranking relations on the evolutionary search; and

b) it uses interval numbers to define the DM's value system DM = ($\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \boldsymbol{\beta}$) to handle imprecision in the process of the "a priori" elicitation of these preference parameters.

Algorithm 1 shows the pseudocode that summarizes the proposed MOEA/D/O. Among the algorithm's new parameters are the DM's value system and the outranking relation. Line 6 represents the changes in the new algorithm compared to the original MOEA/D: for each $j \in B(i)$ and $y \in \{y_1, y_2\}$, if $g'^{te}(y \mid \varpi_i, z) \leq g'^{te}(x_j \mid \varpi_i, z)$ and $x\ R_k^{DM}\ y$, set $x_j = y$ and $FV_j = F(y)$. Now the solution update process must consider both the Tchebychev condition and the specified preference relation.

Algorithm 1. **MOEA/D/O**

Input:

number of objectives m
number of scalar subproblems N
set of weight vectors $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_N\}$
weight vectors' neighborhood size T
decision maker value system DM= ($\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \boldsymbol{\beta}$)
preference relation $\boldsymbol{R_k}$
maximum number of evaluations *maxEvaluations*

Output:

last generation of solution associated to the weight vectors $x = \{x_1, x_2, \ldots, x_N\}$

1. $(x, z, FV, B(i), eval) \leftarrow$ Initialization(λ, N, m, T)
2. **repeat**
3.   **for** $i \leftarrow 1$ **to** N **do**
4. $\{y_1, y_2\} \leftarrow$ Reproduction(x,B(i),T);
5.   UpdateZ(z, $\{y_1, y_2\}$);
6.   **UpdateNeighborhood($\boldsymbol{x}, \boldsymbol{B(i)}, \boldsymbol{\lambda_i}, \boldsymbol{FV}, \{\boldsymbol{y_1}, \boldsymbol{y_2}\}, \boldsymbol{R}$);**
7.   **end**
8. $eval \leftarrow eval+1$;
9. **until** $eval < maxEvaluations$

According to the discussion in subsection 2.4, this work considers six variants of scalarizing functions used to incorporate preferences. Each scalarizing function is related to one of the binary preference relations defined in Table 2; the last variant is a result of the disjunction of the first five variants. Table 3 summarizes these variants, denoted as $\{VAR_1, VAR_2, VAR_3, VAR_4, VAR_5, VAR_6\}$. The variants require the computation of $\sigma(y, x_j)$ and $\sigma(x_j, y)$, as defined in subsection 2.3.

Table 3. MOEA/D/O variants of the scalarizing functions.

Variant	Scalarizing Functions
MOEAD	$g^{te}(y \mid \lambda_j, z) \leq g^{te}(x_j \mid \lambda_j, z)$

VAR_1	$g^{te}(y \mid \lambda_j,z) \leq g^{te}(x_j \mid \lambda_j,z)$ && y R_1 x
VAR_2	$g^{te}(y \mid \lambda_j,z) \leq g^{te}(x_j \mid \lambda_j,z)$ && y R_2 x
VAR_3	$g^{te}(y \mid \lambda_j,z) \leq g^{te}(x_j \mid \lambda_j,z)$ && y R_3 x
VAR_4	$g^{te}(y \mid \lambda_j,z) \leq g^{te}(x_j \mid \lambda_j,z)$ && y R_4 x
VAR_5	$g^{te}(y \mid \lambda_j,z) \leq g^{te}(x_j \mid \lambda_j,z)$ && y R_5 x
VAR_6	$g^{te}(y \mid \lambda_j,z) \leq g^{te}(x_j \mid \lambda_j,z)$ && (y R_1 x || y R_2 x || y R_3 x || y R_4 x || y R_5 x)

4. Experimental design

4.1 Benchmark problems

DTLZ1–9 are the benchmark problems used to assess the performance of MOEA/D/O. The number of objectives is $m = \{3, 5, 10\}$. Table 4 summarizes the standard problems' characteristics, including the number of decision variables, which combine the position and distance variables.

Table 4. Characteristics of the standard problems used in the experiments.

Problem	**Number of Criteria *M***	**Number of Decision Variables *n***	**Position *k***	**Distance *L***
DTLZ1	{3,5,10}	$m + 5 - 1$	$m - 1$	$n - k$
DTLZ2	{3,5,10}	$m + 10 - 1$	$m - 1$	$n - k$
DTLZ3	{3,5,10}	$m + 10 - 1$	$m - 1$	$n - k$
DTLZ4	{3,5,10}	$m + 10 - 1$	$m - 1$	$n - k$
DTLZ5	{3,5,10}	$m + 10 - 1$	$m - 1$	$n - k$
DTLZ6	{3,5,10}	$m + 10 - 1$	$m - 1$	$n - k$
DTLZ7	{3,5,10}	$m + 20 - 1$	$m - 1$	$n - k$
DTLZ8	{3,5,10}	$10m$	$m - 1$	$n - k$
DTLZ9	{3,5,10}	$10m$	$m - 1$	$n - k$

4.2 An approximation of the Region of Interest compatible with the interval outranking model

In this subsection, we present a model of the Region of Interest, which is compatible with the interval outranking approach (i.e., it is compatible with the preferences of a DM who accepts the interval outranking as her/his preference model).

The RoI is the most preferred zone of the Pareto front. Its definition is closely related to the best compromise solution. Let us denote by P_r the DM's strict preference relation. According to Fernandez et al. [19,20] and [1], the best compromise x^* must fulfill the following conditions:

- x^* is a Pareto optimal solution;
- there is no feasible solution y such that y P_r x^*; and
- there are sufficient arguments to justify that x^* is better than many other solutions in the Pareto front.

For the standard problems DTLZ, we form the RoI by combining the best compromise with other solutions that could be indifferent to the best compromise. For this purpose, we should consider the DM's value system DM = ($\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \beta$), the strict preference relation defined by R_5, the net outranking flow $\phi(.)$, and the credibility index $\sigma(y, x^*)$.

To identify that "there is no y preferred to x^*," we use the strict preference relation R_5. For the arguments in favor of x^*, we use the outranking net flow score $\varphi(x^*) = \varphi^+(x^*) - \varphi^-(x^*) = \sum_{y \in PF} \sigma(x^*, y) - \sum_{y \in PF} \sigma(y, x^*)$. Then, the best compromise BC for a given optimization problem will be the solutions that satisfy Equation (5):

$$BC^{DM} = \left\{ x \in PF \,\middle|\, \{y \in PF | y\, R_5^{DM} x\} = \emptyset, \varphi(x) = \max_{y \in PF}\{\varphi^{DM}(y)\} \right\}. \quad (5)$$

Equation (5) is an expression of the above conditions that can be used to find the best compromise. In Equation (5), the strict preference P_r is taken as the binary preference R_5 defined in Table 2, which is the stronger preference in this table. The statement "there are sufficient arguments to justify that x^* is better than many other solutions in the Pareto front" is validated by using the outranking net flow score, a measure that is very suitable for ranking decision sets on which fuzzy preference relations are defined.

Given the large size of the PF, a sample S from the PF will be used in Equation (5). The values of $\varphi^+(x^*)$ and $\varphi^-(x^*)$ are computed using $\sum_{y \in S} \sigma(x^*, y)$ and $\sum_{y \in S} \sigma(y, x^*)$, respectively. Let us note that this work's sample size was 100,000 solutions sampled from the Pareto front.

Once the best compromise solutions have been identified, those y in the wide sample S such that $\sigma(y, x^*) \geq \beta$ (to ensure sufficiently high credibility of the assertion "y is at least as good as x^*") should be part of the RoI. Finally, the RoI is approximated by the union of the solutions in BC^{DM} and those solutions $y \in S$ that satisfy the condition $\sigma(y, x^*) \geq \boldsymbol{\beta}$, with x^* belonging to BC^{DM}.

4.3 Random simulation of preferences

With the purpose of achieving robust results, we randomly simulate different decision-maker preferences. The simulation of a DM uses random weights $\boldsymbol{w}$ and vetoes $\boldsymbol{v}$ for the value system DM = $(\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \boldsymbol{\beta})$ and the fixed values $\boldsymbol{\lambda} = [0.51, 0.75]$ and $\boldsymbol{\beta} = [0.51, 0.67]$ for the majority and credibility thresholds, respectively.[2]

The method that generates $\boldsymbol{w} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_m\}$ utilizes the strategy proposed by [8]. First, it generates a vector d of $m - 1$ non-interval random numbers in the range (0, 1), following a uniform distribution. After that, the vector d is sorted in increasing order, and the vector $w' = \{w'_1, w'_2, \ldots, w'_m\}$ is computed as $w'_i = d_i - d_{i-1}$, for $1 < i < m$; $w'_1 = d_1 - 0$, and $w'_m = 1 - d_{m-1}$. If any w'_i does not satisfy the condition $w'_i > 0.5 \cdot (1 - w'_i)$, the vectors d and w'_i are discarded, and the method starts again. If w' is accepted, it is transformed into the interval vector $\boldsymbol{w} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_m\}$ by computing $\boldsymbol{w}_i \leftarrow [w_i - u \cdot w_i, w_i + u \cdot w_i]$ using a value u randomly generated in the range (0.1, 0.3).

Finally, to complete the generation of the preferences of the artificial DM, Algorithm 2 provides a random interval vetoes vector of size m. The method normalizes the extreme values of the objectives in the range [0, 1]. The algorithm's input is the previously generated vector of interval weights $\boldsymbol{w} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_m\}$. The method creates veto interval values one at a time, using a random middle-value v_h and also increasing/decreasing it at random to define the lower and upper bounds (see Line 5). The method makes use of the correlation between interval weights and vetoes among objectives (more important criteria have greater veto power). The generated vector of vetoes has normalized values; those values must be mapped to their real domain using the extreme original values of the objectives.

Algorithm 2. GenerateVetoes($\boldsymbol{w}$)

Input:

interval weight vector $\boldsymbol{w} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_m\}$

[2] Being a structure composed of interval numbers, $(\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \boldsymbol{\beta})$ can be either imprecise preference parameters from a single DM or poorly defined preference parameters from a group of DMs.

Output:

Interval vetoes vector $\boldsymbol{v}=\{\boldsymbol{v}_1, \boldsymbol{v}_2,\ldots, \boldsymbol{v}_m\}$

```
for i←1 to m do
    repeat
        v_h ← 1/2 + U(−1,1)/8;
        u ← U(0,1)/10;
        v_i ← [v_h − u·v_h, v_h + u·v_h];
        valid ← true;
        for j←1 to i − 1 do
            if P(w_j ≥ w_i) > 0.5 and P(v_j ≥ v_i) > 0.5 then
                valid ← false;
                break;
            end
        end
    until valid = true
end
return v
```

4.4 Experimental conditions

Table 5 shows the working conditions for the experiment used to validate the performance of MOEA/D/O (the weight vectors' neighborhood size T was set to N/10). The experiment compared the performances of the variants of MOEA/D/O on the standard problems. The intended goal was to identify the strategy that best approximates the region of interest for a set of randomly generated DM preferences. The particular genetic operators used were a) the random selection of parents, b) SBX crossover, and c) polynomial mutation.

Table 5. Experimental conditions for the validation of MOEA/D/O variants.

Algorithms	MOEAD, MOEA/D-NUMS, VAR_1, VAR_2, VAR_3, VAR_4, VAR_5, VAR_6
Standard Problems	DTLZ1 a DTLZ9
Number of objectives	3,5,10
Number of DMs	10
Population Size *N*	100
Evaluations Per Run	100000
Runs Per Variant	30

4.5 Performance indicators

The comparison of MOEA/D and the variants VAR1, VAR2, VAR3, VAR4, VAR5, and VAR6 considers four performance indicators. These indicators use the region of interest RoI_j of each simulated DM (calculated as in sub-section 4.2) and the set of solutions x^{**} reported by MOEA/D/O. They can be defined as follows:

a) *Min-Eucl*(RoI_j, x^{**}). Given the state of the last set of solutions x of MOEA/D/O, denoted x^{**}, and the region of interest of the j-th DM, this indicator is the minimum Euclidean distance between any of the solutions $s \in x^{**}$ and RoI_j.

b) *Avg-Eucl*(RoI_j, x^{**}). Using the above notation, this indicator is the average Euclidean distance among the solutions $s \in$

x^{**} and RoI_j.

c) $Min\text{-}Tcheb(RoI_j, x^{**})$. This indicator is the minimum Tchebychev distance between any of the solutions $s \in x^{**}$ and RoI_j.

d) $Avg\text{-}Tcheb(RoI_j, x^{**})$. This indicator is the average Tchebychev distance among the solutions $s \in x^{**}$ and RoI_j.

These four indicators measure the similarity between the solutions in x^{**} and those in the RoI. In general, the Euclidean indicators would be applicable in decision problems whose DM has a compensatory perspective. In contrast, the Tchebychev-based indicators become relevant if the DM's perspective is non-compensatory (that is, gains in some objectives do not justify losses in others). Furthermore, the minimum distances measure the performance of the algorithm considering the best solution alone, and the averages measure the overall trend of the solutions. The minimum and average distances are complementary indicators of the same type of distance, although the minimum distances are more important for identifying the best solution.

4.6 Statistical validation

The statistical comparison of the distinct variants of MOEA/D/O was carried out using the indicators described in subsection 4.5. For each couple (*Problem*, *Indicator*), where *Problem* = {DTLZ1, DTLZ2, …, DTLZ9} and *Indicator* = {*Min-Euclid*, *Avg-Euclid*, *Min-Tcheb*, *Avg-Tcheb*}, a group of 300 data points was generated by each variant. Each data point is associated with the specific indicator value measured on a single run, considering a particular DM j. The number 300 corresponds to 30 runs per decision-maker.

Using the previously gathered data, Friedman's non-parametric statistical test, and Nemenyi's post-hoc analysis, the following null hypothesis H0 was validated: "the mean of the results between each pair of groups is the same," where a group is associated with a variant of a specific standard problem for each indicator. Appendix 1 shows the general results derived from the experiment; additionally, the next section presents a discussion of their relevance.

5 Summary of results

This section organizes the results into two subsections. Subsection 5.1 compares the proposed approaches with an implementation of the classic MOEA/D without preference incorporation (see Section 2). Subsection 5.2 compares the proposed approaches with an implementation of the strategy MOEA/D-NUMS, which is an MOEA/D variant that integrates preferences into the search process based on the Non-Uniform Mapping Scheme developed in [31].

5.1 Comparison with MOEA/D, an approach without preferences

This subsection summarizes the results obtained from the comparison with MOEA/D. The results are shown in Table A.1 in Appendix 1. In this table, for each DTLZ problem, distance indicator, and number of objective functions, the different variants of preference incorporation and MOEA/D are ranked from best to worst. These results are summarized in this section.

5.1.1 With three-objective functions

Some results related to the Euclidean distance to the RoI are provided in Tables 6(a) and 6(b); these results, and those in Table A.1 in Appendix 1, show that no approach outperforms all the others for all nine DTLZ problems. To aggregate the rank orders from all the problems, we use the Borda count. Table 6(b) shows the Borda score and its suggested ranking. Although they show poor performance on DTLZ7, VAR3 and VAR5 seem to be the most promising ways of incorporating preferences. MOEA/D performs very well on DTLZ7, but it is outperformed by most variants in most problems, as evidenced by Table 6(a) and the Borda scores. Effect size analysis was also performed using the statistical test of Vargha and Delaney [45]. This measure determines the strength of the correlation between two variables [37], obtaining the expected probability that one algorithm will outperform another in a random execution [18]. The effect size information on the Min-Euclidean distance, in Tables 6(e) and 6(f), shows results consistent with the Borda analysis. Concerning the Tchebychev distance, the results are completely different, as shown by Tables 6(c) and 6(d); in most problems, MOEA/D strongly outperforms the approaches with preferences. Table 7 compares VAR3 and VAR5 using the Euclidean distance.

Table 6. Results with three objectives.

a) Euclidean Best and Worst Approaches

PROBLEM	EUCLIDEAN			
	Best variants		Worst variants	
	MIN	AVE	MIN	AVE
DTLZ1	3	3	M	1,2,4,5,6,M
DTLZ2	5	1,3,4,5	M	M
DTLZ3	2,M	5	1,3,4,5	M
DTLZ4	3,5	3,5	M	M
DTLZ5	1,2,3,4,5,6,M	1,2,3,4,5,6,M	-----	-----
DTLZ6	1,2,3,4,5,6	1,2,3,4,5,6	M	M
DTLZ7	1,4,M	M	2,3,5	2,3,5
DTLZ8	1,2,3,4,5,6,M	1,2,3,4,5,6,M	-----	-----
DTLZ9	3,5	3,5	1,2,4,6,M	1,2,4,6,M

x and M denote Var*x* and MOEA/D, respectively

b) Euclidean Borda score and aggregated ranking

MIN-EUCLIDEAN		
VAR3	**29.0**	**1**
VAR5	**31.0**	**2**
VAR6	**36.5**	**3**
VAR1	37.0	5
VAR2	37.0	5
VAR4	37.0	5
MOEAD	44.5	7

AVG-EUCLIDEAN		
VAR3	**28.0**	**1**
VAR5	**28.5**	**2**
VAR1	**34.5**	**3.5**
VAR4	34.5	3.5
VAR6	39.0	5
VAR2	41.0	6
MOEAD	46.5	7

c) Tchebychev Best and Worst Approaches

PROBLEM	**TCHEBYCHEV**			
	Best variants		**Worst variants**	
	MIN	**AVE**	**MIN**	**AVE**
DTLZ1	M	3,M	3,4,5	1,2,4,5,6
DTLZ2	M	1,2,3,4,5,6	3,4,5	M
DTLZ3	M	5	1,3,4,5,6	M
DTLZ4	M	1,4	2,3,5,6	2,3,5,6
DTLZ5	M	M	1,2,3,4,5,6	1,2,3,4,5
DTLZ6	M	M	1,2,3,4,5,6	1,2,3,4,5,6
DTLZ7	1,4	1,4	3,5	3,5
DTLZ8	1,2,3,4,5,6,M	1,2,3,4,5,6,M	-----	-----
DTLZ9	2,6,M	2,6,M	1,3,4,5	1,3,4,5

x and M denote Var*x* and MOEA/D, respectively

d) Tchebychev Borda score and aggregated ranking

MIN-TCHEBYCHEV		
MOEAD	**15.0**	**1**
VAR2	**32.5**	**2**
VAR1	**35.0**	**3.5**
VAR6	35.0	3.5
VAR4	39.5	5
VAR3	47.5	6.5
VAR5	47.5	6.5

AVG- TCHEBYCHEV		
MOEAD	**29.5**	**1**
VAR1	**34.5**	**2.5**
VAR4	**34.5**	**2.5**
VAR6	35.0	4
VAR2	38.0	5
VAR3	40.0	6
VAR5	40.5	7

e) Effect Size Score

EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE						
Instance	V1-M	V2-M	V3-M	V4-M	V5-M	V6-M
DTLZ1	0.67	0.73	0.73	0.71	0.71	0.75
DTLZ2	0.99	0.98	0.99	0.99	0.99	0.97
DTLZ3	0.42	0.53	0.36	0.42	0.36	0.50
DTLZ4	0.73	0.75	0.84	0.72	0.84	0.72
DTLZ5	0.50	0.50	0.50	0.50	0.50	0.50
DTLZ6	0.86	0.86	0.86	0.86	0.86	0.87
DTLZ7	0.46	0.30	0.25	0.46	0.25	0.33
DTLZ8	0.50	0.50	0.50	0.50	0.50	0.50
DTLZ9	0.52	0.52	0.60	0.59	0.60	0.50

V*x* and M denote Var*x* and MOEA/D, respectively

f) Average Effect Size Score

AVERAGE EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE							
	M	V1	V2	V3	V4	V5	V6
M		0.37	0.37	0.37	0.36	0.38	0.3
V1	0.63		0.50	0.48	0.49	0.48	0.5
V2	0.63	0.50		0.47	0.49	0.47	0.5
V3	0.63	0.52	0.53		0.51	0.50	0.5
V4	0.64	0.51	0.51	0.49		0.49	0.5
V5	0.62	0.52	0.53	0.50	0.51		0.5
V6	0.63	0.49	0.49	0.46	0.48	0.46	

V*x* and M denote Var*x* and MOEA/D, respectively

Table 7. VAR3 vs. VAR5 (three-objective functions).

MIN-EUCLIDEAN			AVE-EUCLIDEAN		
APPROACH	Outperforms VAR5 in problem	Outperformed by VAR5 in problem	**APPROACH**	Outperforms VAR5 in problem	Outperformed by VAR5 in problem
VAR3	1	2	VAR3	1	3

5.1.2 With five-objective functions

The results in terms of the Euclidean distance to the RoI are summarized in Tables 8(a) and 8(b). Again, there is no approach that outperforms all the others for all nine DTLZ problems. The Borda method is used to aggregate the conflicting rank orders from all the problems, and its score is provided in Table 8(b). The performances of VAR3 and VAR5 degrade compared to their performances with three objective functions. VAR1, VAR2, and VAR6 seem to be the most promising approaches. MOEA/D is the best method for DTLZ8, but it is clearly outperformed by the other approaches for most of the remaining problems. The effect size information on the Min-Euclidean distance, in Tables 8(e) and 8(f), also provides similar results to those presented in Tables 8(a) and 8(b).

The results in terms of the Tchebychev distance are summarized in Tables 8(c) and 8(d). Compared with the results using three objectives, MOEA/D keeps a good performance with respect to the minimum distance to the RoI, although it is outperformed by VAR6 in four problems; this is also supported by the Borda score. Nevertheless, with respect to this indicator, MOEA/D is the best approach for the most difficult DTLZ problems. VAR2 performs very well in terms of the average Tchebychev distance. MOEA/D is the best method for solving DTLZ6 but is clearly outperformed by VAR2 and VAR6 for most problems.

Table 8. Results with five objectives.

a) Euclidean Best and Worst Approaches

PROBLEM	EUCLIDEAN			
	Best variants		Worst variants	
	MIN	AVE	MIN	AVE
DTLZ1	1,2,4,5	1,2,3,4,5,6	3,5,M	M
DTLZ2	2,3	2	M	M
DTLZ3	6	1,2,4,6	3,5	3,5,M
DTLZ4	5,3	3,5	M	M
DTLZ5	1,2,3,4.5,6,M	2	-----	M
DTLZ6	1	1,6	M	M
DTLZ7	1	1,3,4,5	M	M
DTLZ8	M	M	3,4,5	1,2,3,4,5
DTLZ9	2	2	M	M

x and M denote Var*x* and MOEA/D, respectively

b) Euclidean Borda score and aggregated ranking

MIN-EUCLIDEAN		
VAR1	27.5	1
VAR6	29.0	2
VAR2	32.0	3
VAR4	36.0	4
VAR3	37.5	5
VAR 5	40.5	6
MOEAD	49.5	7

AVG-EUCLIDEAN		
VAR2	29.5	1
VAR4	32.0	2
VAR3	33.5	4
VAR5	33.5	4
VAR6	33.5	4
VAR 1	34.0	6
MOEAD	56.0	7

c) Tchebychev Best and Worst Approaches

PROBLEM	TCHEBYCHEV			
	Best variants		Worst variants	
	MIN	AVE	MIN	AVE
DTLZ1	6	1,2,3,4,5,6	3.5.M	M
DTLZ2	M	2	1,2,3,4,5,6	1,4,5
DTLZ3	6	1,2,4,6	3,5	3,5,M
DTLZ4	M	1,2,3,4,5,6	1,3,4,5	M
DTLZ5	M	1,2,3,4,5,6	1,2,3,4,5,6	M
DTLZ6	M	M	2,3,4,5	2,3,4,5
DTLZ7	2,6,M	2,4,6	3,5	M
DTLZ8	1,2,3,4,5,6	2,3,4,5	M	M
DTLZ9	6	2,6	M	M

x and M denote Var*x* and MOEA/D, respectively

d) Tchebychev Borda score and aggregated ranking

MIN-TCHEBYCHEV		
VAR6	**22.0**	**1**
MOEAD	**29.5**	**2**
VAR2	**31.5**	**3**
VAR1	36.0	4
VAR4	39.0	5
VAR3	47.0	6.5
VAR5	47.0	6.5

AVG-TCHEBYCHEV		
VAR2	**25.5**	**1**
VAR6	**30.5**	**2**
VAR4	**33.5**	**3**
VAR1	36.5	4
VAR3	37.0	5.5
VAR5	37.0	5.5
MOEAD	52.0	7

e) Effect Size Score

EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE						
Instance	V1-M	V2-M	V3-M	V4-M	V5-M	V6-M
DTLZ1	0.67	0.65	0.50	0.65	0.54	0.70
DTLZ2	0.95	0.97	0.97	0.96	0.97	0.95
DTLZ3	0.50	0.48	0.38	0.47	0.39	0.56
DTLZ4	0.81	0.74	0.85	0.81	0.84	0.68
DTLZ5	0.50	0.50	0.50	0.50	0.50	0.50
DTLZ6	0.71	0.64	0.62	0.65	0.65	0.69
DTLZ7	0.91	0.81	0.88	0.88	0.87	0.77
DTLZ8	0.23	0.17	0.15	0.15	0.13	0.27
DTLZ9	0.82	0.89	0.83	0.81	0.27	0.86

V*x* and M denote Var*x* and MOEA/D, respectively

f) Average Effect Size Score

AVERAGE EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE							
	M	V1	V2	V3	V4	V5	V6
M		0.32	0.35	0.37	0.35	0.36	0.34
V1	0.68		0.54	0.54	0.52	0.54	0.52
V2	0.65	0.46		0.51	0.48	0.50	0.49
V3	0.63	0.46	0.49		0.48	0.49	0.48
V4	0.65	0.48	0.52	0.52		0.52	0.50
V5	0.64	0.46	0.50	0.51	0.48		0.48
V6	0.66	0.48	0.51	0.52	0.50	0.52	

V*x* and M denote Var*x* and MOEA/D, respectively

Table 9(a) shows pairwise comparisons among VAR1, VAR2, and VAR6. VAR1 outperforms VAR2 and VAR6 for the most difficult DTLZ problems. Table 9(b) provides a comparison between VAR6 and VAR2 based on Tchevychev distances. Table 9(c) shows the comparison of the best approaches with MOEA/D in terms of minimum distances.

Table 9. Relevant comparisons with respect to distances for five objectives.

a) Comparisons of VAR1, VAR6, and VAR2 (5 objectives)

MIN-EUCLIDEAN				AVE-EUCLIDEAN			
APPROACH	Outperforms VAR1 in problems	Outperforms VAR6 in problems	Outperforms VAR2 in problems	**APPROACH**	Outperforms VAR1 in problems	Outperforms VAR6 in problems	Outperforms VAR2 in problems
VAR1		4,6,7	4,6,7,8	VAR1		4,7	4,6,7
VAR6	3,8,9		3,6,8	VAR6	8,9		6,8
VAR2	2,9	2,9		VAR2	2,5,9	2,5,9	

b) Comparison between VAR2 and VAR6 regarding Tchebychev distance

MIN-TCHEBYCHEV			AVE-TCHEBYCHEV		
APPROACH	Outperforms VAR6 in problems	Outperformed by VAR6 in problems	**APPROACH**	Outperforms VAR6 in problems	Outperformed by VAR6 in problems
VAR2	---------	1,3,4,6,9	VAR2	2,8	6

c) Best approaches vs. MOEA/D regarding minimum distances

MIN-EUCLIDEAN			MIN-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D in problems	Outperformed by MOEA/D in problems	**APPROACH**	Outperforms MOEA/D in problems	Outperformed by MOEA/D in problems
VAR1	1,2,4,6,7,9	8	VAR6	1,3,8,9	2,4,5,6
VAR6	1,2,3,4,6,7,9	8	VAR2	1,8,9	2,4,5,6
VAR2	1,2,4,6,7,9	8	VAR1	1,8,9	2,4,5,6,7

5.1.3 With ten-objective functions

The results in terms of the Euclidean distance to the RoI are given in Tables 10(a) and 10(b). Confirming the previous results with five objectives, again VAR1, VAR2, and VAR6 seem to be the most promising approaches. MOEA/D is the best method for DTLZ6, but it is clearly outperformed by the other approaches for most of the remaining problems. Tables 10(c) and 10(d) summarize our results in terms of the Tchebychev distance. VAR2 and VAR6 are the most promising approaches. MOEA/D is the best method for DTLZ6 but is clearly outperformed by the other approaches for most DTLZ problems, mainly with respect to the average distance. In addition, Tables 10(e) and 10(f) show the effect size information on the Min-Euclidean distance, which corroborates the previously obtained results.

Table 10. Results with ten objectives.

a) Euclidean Best and Worst Approaches

PROBLEM	EUCLIDEAN			
	Best variants		Worst variants	
	MIN	AVE	MIN	AVE
DTLZ1	2,6	2,6	3,5	1,3,4,5
DTLZ2	2,6	2	1,3,4,5	M
DTLZ3	2,6,M	2,6	3,4,5	1,3,4,5
DTLZ4	3,5	5,3	6,M	6,M
DTLZ5	M,6,1,4,3,2,5	2	-------	1,4,6,M
DTLZ6	M	M	2,3,4,5	3,5
DTLZ7	1,5,3	5,1,3,4	M	M
DTLZ8	4	4	M	M
DTLZ9	4,6,1,2	4,6,1,2	5,M	5,M

x and M denote Var*x* and MOEA/D, respectively

b) Euclidean Borda score and aggregated ranking

MIN-EUCLIDEAN		
VAR1	**30.5**	**1.5**
VAR6	**30.5**	**1.5**
VAR2	**31.0**	**3**
VAR3	38.5	4.5
VAR4	38.5	4.5
MOEAD	40.5	6
VAR5	42.5	7

AVG-EUCLIDEAN		
VAR2	**28.0**	**1**
VAR6	**33.0**	**2**
VAR1	**34.0**	**3**
VAR4	35.0	4
VAR3	36.5	5
VAR5	39.0	6
MOEAD	46.5	7

c) Tchebychev Best and Worst Approaches

PROBLEM	TCHEBYCHEV			
	Best variants		Worst variants	
	MIN	AVE	MIN	AVE
DTLZ1	2,6	2,6	3,5	1,3,4,5,M
DTLZ2	M	M,2,6	1,3,4,5	1,3,4,5
DTLZ3	2,6,M	2,6	3,4,5	1,3,4,5,M
DTLZ4	5	5	M	M
DTLZ5	M,1,4,3,2,5.6	2	---------	1,4,6,M
DTLZ6	M	M	2,3,4,5	2,3,5
DTLZ7	1,5,3,4	5,1,3,4	M	M
DTLZ8	1,2,3,4,5,6	1,3,4,5	M	M
DTLZ9	2	2	3,5,M	5,M

x and M denote Var*x* and MOEAD, respectively

d) Tchebychev Borda score and aggregated ranking

MIN-TCHEB		
VAR6	28.5	1
VAR2	30.0	2
VAR1	32.5	3
MOEAD	38.5	4
VAR4	40.5	5.5
VAR5	40.5	5.5
VAR3	41.5	7

AVG-TCHEBYCHEV		
VAR2	29.5	1
VAR6	32.0	2
VAR1	34.0	3
VAR3	36.0	4
VAR5	37.0	5
VAR4	37.5	6
MOEAD	46.0	7

e) Effect Size Score

EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE						
Instance	V1-M	V2-M	V3-M	V4-M	V5-M	V6-M
DTLZ1	0.46	0.68	0.34	0.41	0.32	0.68
DTLZ2	0.42	0.51	0.44	0.43	0.44	0.51
DTLZ3	0.38	0.48	0.29	0.34	0.32	0.51
DTLZ4	0.66	0.65	0.77	0.69	0.77	0.57
DTLZ5	0.50	0.49	0.49	0.49	0.48	0.50
DTLZ6	0.12	0.03	0.03	0.03	0.03	0.10
DTLZ7	0.84	0.85	0.80	0.82	0.84	0.82
DTLZ8	0.62	0.59	0.61	0.62	0.59	0.58
DTLZ9	0.74	0.73	0.62	0.76	0.51	0.76

V*x* and M denote Var*x* and MOEA/D, respectively

f) Average Effect Size Score

AVERAGE EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE							
	M	V1	V2	V3	V4	V5	V6
M		0.47	0.44	0.51	0.49	0.52	0.44
V1	0.53		0.50	0.56	0.53	0.56	0.48
V2	0.56	0.50		0.56	0.53	0.56	0.48
V3	0.49	0.44	0.44		0.47	0.51	0.42
V4	0.51	0.47	0.47	0.53		0.54	0.45
V5	0.48	0.44	0.44	0.49	0.46		0.42
V6	0.56	0.52	0.52	0.58	0.55	0.58	

V*x* and M denote Var*x* and MOEA/D, respectively

Finally, Table 11(a) shows the comparison of the best approaches and MOEA/D in terms of the minimum distances, and Table 11(b) provides a comparison between VAR2 and VAR6 in terms of the Tchebychev distance.

Table 11. Relevant comparisons with respect to distances for ten objectives.

a) Best approaches vs. MOEA/D with respect to minimum distances

MIN-EUCLIDEAN			MIN-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D in problems	Outperformed by MOEA/D in problems	**APPROACH**	Outperforms MOEA/D in problems	Outperformed by MOEA/D in problems
VAR1	4,7,8,9	2,3,6	VAR6	1,4,7,8,9	2,6
VAR6	1,2,7,8,9	6	VAR2	1,4,7,8,9	2,6
VAR2	1,2,4,7,8,9	6	VAR1	4,7,8,9	2,3,6

b) VAR2 vs. VAR6 regarding Tchebychev distance

MIN-TCHEBYCHEV			AVE-TCHEBYCHEV		
APPROACH	Outperforms VAR6 in problem	Outperformed by VAR6 in problem	**APPROACH**	Outperforms VAR6 in problems	Outperformed by VAR6 in problem
VAR2	9	6	VAR2	5,9	6

We should make the following remarks:

a) MOEA/D exhibits the best performance on DTLZ6 when the ten simulated DMs are taken into account; nevertheless, it should be stated that, for one of the simulated settings ($\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{\lambda}, \beta$), VAR1 outperforms MOEA/D in terms of the four distance indicators.

b) Concerning the most difficult test problems (DTLZ4 and DTLZ7)[3], MOEA/D performs the worst according to the four distance indicators.

5.2 Comparison against MOEA/D-NUMS, an approach that integrates preferences

This section compares the developed strategies against a recent state-of-the-art approach based on the non-uniform mapping scheme (NUMS) proposed in [31]. The scheme was implemented in the MOEA/D framework, and it offers a well-generalized approach to biasing the distribution of the reference points used by MOEA/D towards the RoI for each particular set of DM preferences. Hence, this scheme represents a good point of reference for comparison.

MOEA/D-NUMS was implemented following the configuration of parameters and operators described in [31]. Due to the algorithms' sensitivity when using aspiration levels, particularly in the presence of invalid values (cf. [32]), a fine-tuning process was used. Based on the previously generated DM's value system, the tuning process involved two steps.

[3] DTLZ4 is multi-frontal and Pareto many-to-one. Its objectives are non-separable, and the geometry of the Pareto front is concave and biased. DTLZ7 is singularly challenging because its Pareto front is disconnected and has mixed concave/convex regions, and the fitness landscape is one-to-one.

The first step randomly samples the decision space to create a very wide set of solutions, and the aspiration level was defined as the solution in this set with the maximum net flow score (as defined in subsection 4.2). With this aspiration level, the results mainly showed that MOEA/D-NUMS was outperformed by the outranking-based variants of articulating preferences. Such a relatively poor NUMS performance was due to a too-conservative aspiration level. As a result, the tuning process continued with the second step.

In the second step, we changed the objective values of the previously obtained aspiration levels by reducing them according to the relevance order given by their weights. Thus, the aspiration levels of the most important objectives suffered a stronger reduction than those of the less important objectives. This second strategy revealed better competitiveness in comparison to MOEA/D/O. The results are shown in Table A.2 in Appendix 2. In this table, for each DTLZ problem, distance indicator, and number of objective functions, the different variants of preference incorporation and MOEA/D-NUMS are ranked from best to worst. These results are summarized in this section.

5.2.1 With three-objective functions

Table 12 provides some results related to the distance to the RoI in instances with three objective functions. As shown in this table and Table A.2 in Appendix 2, there is no approach that outperforms all the others for all nine DTLZ problems. In order to aggregate the rank orders from all the problems, we use the Borda count. Again, based on the Borda score shown, the ranking suggests the most promising variants to use to integrate preferences, which are shown in Table 12 according to each indicator and compared with MOEA/D-NUMS. The same conclusion is obtained with the effect size measure, as shown in Table 12.

Table 12. Summary of results from comparison with MOEA/D-NUMS for three objectives.

a) Borda score and aggregated ranking

MIN-EUCLIDEAN		
VAR3	**28.5**	**1**
VAR5	30.5	2
VAR2	33.5	3
VAR1	34.0	4.5
VAR4	34.0	4.5
VAR6	34.5	6
NUMS	57.0	7

AVG-EUCLIDEAN		
VAR3	**31.0**	**1.5**
VAR5	**31.0**	**1.5**
VAR1	33.5	3.5
VAR4	33.5	3.5
VAR6	38.0	5
VAR2	40.0	6
NUMS	45.0	7

MIN-TCHEBYCHEV		
VAR2	**26.0**	**1**
VAR6	29.0	2
VAR1	31.0	3
VAR4	32.5	4
VAR3	41.0	5.5
VAR5	41.0	5.5
NUMS	51.5	7

AVG-TCHEBYCHEV		
VAR1	**33.5**	**1.5**
VAR4	**33.5**	**1.5**
VAR6	35.0	3
VAR2	37.0	5
VAR3	37.0	5
VAR5	37.0	5
NUMS	39.0	7

b) Effect Size Score

EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE						
Instance	V1-MN	V2-MN	V3-MN	V4-MN	V5-MN	V6-MN
DTLZ1	0.87	0.89	0.84	0.85	0.86	0.91
DTLZ2	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ3	0.65	0.73	0.62	0.65	0.61	0.70
DTLZ4	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ5	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ6	0.95	1.00	1.00	0.98	1.00	1.00
DTLZ7	0.99	0.99	0.99	0.99	0.99	0.99
DTLZ8	0.00	0.00	0.00	0.00	0.00	0.00
DTLZ9	0.77	0.80	0.76	0.77	0.77	0.80

V*x* and MN denote Var*x* and NUMS, respectively

c) Average Effect Size Score

AVERAGE EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE							
	MN	V1	V2	V3	V4	V5	V6
MN		0.20	0.18	0.20	0.19	0.20	0.18
V1	0.80		0.49	0.49	0.49	0.49	0.50
V2	0.82	0.51		0.49	0.49	0.49	0.51
V3	0.80	0.51	0.51		0.51	0.50	0.52
V4	0.81	0.51	0.51	0.49		0.49	0.52
V5	0.80	0.51	0.51	0.50	0.51		0.52
V6	0.82	0.50	0.49	0.48	0.48	0.48	

V*x* and MN denote Var*x* and NUMS, respectively

d) Best approaches vs. MOEA/D-NUMS with three objective functions for minimum distances

MIN-EUCLIDEAN			MIN-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems	**APPROACH**	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems
VAR3	1,2,3,4,5,6,7,9	8	VAR2	1,2,3,4,5,7,8,9	6
VAR5	1,2,3,4,5,6,7,9	8	VAR6	1,2,4,5,7,8,9	6

e) Best approaches vs. MOEA/D-NUMS with three objective functions for average distances

AVE-EUCLIDEAN			AVE-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems	**APPROACH**	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems
VAR3	2,4,5,6,7,9	1,3,8	VAR2	4,5,7,8,9	1,2,3,6
VAR5	2,4,5,6,7,9	1,3,8	VAR6	4,5,7,8,9	1,2,3,6

5.2.2 With five-objective functions

Table 13 provides some results related to the distance to the RoI in instances with five objective functions. The Borda count aggregates the rank orders from all the problems. The resulting ranking suggests the most promising variants to use to integrate preferences, which are shown in Table 13 according to each indicator and compared with MOEA/D-NUMS. This conclusion is corroborated by the effect size measure shown in the same table.

Table 13. Summary of results from comparison with MOEA/D-NUMS for five objectives.

a) Borda score and aggregated ranking

MIN-EUCLIDEAN		
VAR6	**28.5**	**1**
VAR1	29.5	2
VAR2	32.5	3
VAR3	37.0	4
VAR4	38.0	5
VAR5	40.0	6
NUMS	46.5	7

AVG-EUCLIDEAN		
VAR2	**30.5**	**1**
VAR6	35.0	2
VAR3	35.5	3.5
VAR5	35.5	3.5
VAR1	37.5	5.5
VAR4	37.5	5.5
NUMS	40.5	7

MIN-TCHEBYCHEV		
VAR6	**22.5**	**1**
NUMS	32.0	2
VAR2	32.5	3
VAR1	34.5	4
VAR4	39.5	5
VAR3	45.5	6.5
VAR5	45.5	6.5

AVG-TCHEBYCHEV		
VAR6	**31.0**	**1.5**
NUMS	**31.0**	**1.5**
VAR2	31.5	3
VAR4	37.0	4
VAR3	39.5	5
VAR1	40.0	6
VAR5	42.0	7

b) Effect Size Score

EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE						
Instance	V1-MN	V2-MN	V3-MN	V4-MN	V5-MN	V6-MN
DTLZ1	0.58	0.57	0.44	0.56	0.46	0.61
DTLZ2	0.99	0.99	0.99	0.99	0.99	0.99
DTLZ3	0.47	0.44	0.36	0.44	0.36	0.52
DTLZ4	0.98	0.97	0.98	0.98	0.98	0.96
DTLZ5	1	1	1	1	1	1
DTLZ6	0.85	0.73	0.69	0.73	0.73	0.83
DTLZ7	1	1	1	1	1	1
DTLZ8	0.13	0.08	0.06	0.07	0.05	0.18
DTLZ9	0.44	0.47	0.44	0.44	0.45	0.47

Vx and MN denote Var*x* and NUMS, respectively

c) Average Effect Size Score

AVERAGE EFECT SIZE ON THE MIN-EUCLIDEAN							
	MN	V1	V2	V3	V4	V5	V6
MN		0.28	0.30	0.33	0.31	0.33	0.27
V1	0.72		0.54	0.54	0.52	0.54	0.52
V2	0.70	0.46		0.51	0.48	0.50	0.49
V3	0.67	0.46	0.49		0.48	0.49	0.48
V4	0.69	0.48	0.52	0.52		0.52	0.50
V5	0.67	0.46	0.50	0.51	0.48		0.48
V6	0.73	0.48	0.51	0.52	0.50	0.52	

Vx and MN denote Var*x* and NUMS, respectively

d) Best approaches vs. MOEA/D-NUMS with five objective functions for minimum distances

MIN-EUCLIDEAN			MIN-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems	**APPROACH**	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems
VAR6	1,2,3,4,5,6,7	8	VAR6	4,7,8,9	1,2,3,6
VAR1	1,2,4,5,6,7	3,8,9	VAR2	4,7,8	1,2,3,6,9

e) Best approaches vs. MOEA/D-NUMS with five objective functions for average distances

AVE-EUCLIDEAN			AVE-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems	**APPROACH**	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems
VAR6	2,4,5,6,7	1,3,8	VAR6	4,7,8,9	1,2,3,5,6
VAR1	2,4,5,6,7	1,3,8,9	VAR2	4,7,8,9	1,2,3,5,6

5.2.3 With ten-objective functions

Table 14 provides some results related to the distance to the RoI in problems with ten objective functions. The Borda count aggregates the rank orders from all the problems. The resulting ranking suggests the most promising variants to use to integrate preferences, which are shown in Table 14 according to each indicator and compared with MOEA/D-NUMS. Similar results about the most promising variants are obtained from the effect size information shown in the same table.

Table 14. Summary of results from comparison with MOEA/D-NUMS for ten objectives.

a) Borda score and aggregated ranking

MIN-EUCLIDEAN		
VAR1	**29.5**	**1**
VAR6	30.5	2
VAR2	31.5	3
VAR4	38.0	4
VAR3	40.5	5.5
NUMS	40.5	5.5
VAR5	41.5	7

AVG-EUCLIDEAN		
VAR2	**32.5**	**1**
NUMS	35.0	2
VAR1	36.0	3
VAR6	36.5	4
VAR3	37.0	5.5
VAR4	37.0	5.5
VAR5	38.0	7

MIN-TCHEBYCHEV		
VAR6	**27.0**	**1**
VAR2	28.0	2
VAR1	32.5	3
VAR5	38.5	4
VAR3	41.5	5.5
NUMS	41.5	5.5
VAR4	43.0	7

AVG-TCHEBYCHEV		
NUMS	**27.0**	**1**
VAR2	33.5	2
VAR6	36.0	3
VAR1	36.5	4
VAR5	38.0	5
VAR3	40.5	6.5
VAR4	40.5	6.5

b) Effect Size Score

EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE						
Instance	V1-MN	V2-MN	V3-MN	V4-MN	V5-MN	V6-MN
DTLZ1	0.63	0.75	0.58	0.62	0.56	0.74
DTLZ2	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ3	0.49	0.52	0.47	0.48	0.47	0.53
DTLZ4	0.94	0.95	0.98	0.96	0.99	0.91
DTLZ5	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ6	0.27	0.07	0.07	0.09	0.08	0.23
DTLZ7	1.00	1.00	1.00	1.00	1.00	1.00
DTLZ8	0.47	0.42	0.45	0.47	0.45	0.42
DTLZ9	0.01	0.01	0.00	0.01	0.00	0.01

V*x* and MN denote Var*x* and NUMS, respectively

c) Average Effect Size Score

AVERAGE EFECT SIZE ON THE MIN-EUCLIDEAN DISTANCE							
	MN	V1	V2	V3	V4	V5	V6
MN		0.35	0.36	0.38	0.37	0.38	0.35
V1	0.65		0.50	0.56	0.53	0.56	0.48
V2	0.64	0.50		0.56	0.53	0.56	0.48
V3	0.62	0.44	0.44		0.47	0.51	0.42
V4	0.63	0.47	0.47	0.53		0.54	0.45
V5	0.62	0.44	0.44	0.49	0.46		0.42
V6	0.65	0.52	0.52	0.58	0.55	0.58	

V*x* and MN denote Var*x* and NUMS, respectively

d) Best approaches vs. MOEA/D-NUMS with ten objective functions for minimum distances

MIN-EUCLIDEAN			MIN-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems	**APPROACH**	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems
VAR1	1,2,4,5,7	6,8,9	**VAR6**	1,2,3,4,5,7	6,8
VAR6	1,2,3,4,5,7	6,8,9	**VAR2**	1,2,3,4,5,7,9	6,8
VAR2	1,2,3,4,5,7	6,8,9	----	----	----

e) Best approaches vs. MOEA/D-NUMS with ten objective functions for average distances

AVE-EUCLIDEAN			AVE-TCHEBYCHEV		
APPROACH	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems	**APPROACH**	Outperforms MOEA/D-NUMS in problems	Outperformed by MOEA/D-NUMS in problems
VAR1	2,4,5,7	1,3,6,8,9	**VAR6**	2,7	1,4,5,6,8,9
VAR6	2,4,5,7	6,8,9	**VAR2**	2,7,9	1,4,5,6,8
VAR2	2,4,5,7	6,8,9	----	----	----

5.3 Discussion

After a careful analysis of the previous results, we emphasize the following main remarks:

1. Incorporating preferences in the MOEA/D update phase allows us to obtain better approximations of the region of interest, especially using Euclidean distances and/or in problems with many objective functions; this was proved by our experiments with six different variants of preference incorporation. The effectiveness of particular approaches depends on the problem, the distance indicator, the number of objective functions, and even the specific setting of the outranking model parameters. We could not find a single approach that outperforms all the other approaches for all the problems, indicators, and numbers of objective functions.
2. MOEA/D tends to perform better with the Tchebychev indicator than with the Euclidean distance. This is natural given the way solutions are updated in MOEA/D.
3. In problems with three objective functions, and in terms of the Tchebychev distance, MOEA/D performs better than the approaches in which preferences are incorporated. However, its performance according to all the indicators is degraded when the number of objectives increases. Nevertheless, MOEA/D outperforms all the other approaches in terms of the four distance indicators on DTLZ6 with ten objective functions, although there is a specific set of preference parameters for which VAR1 performs better than MOEA/D.
4. With three objective functions and according to Euclidean indicators, VAR3 and VAR5 are the most promising methods of incorporating preferences.
5. As the number of objective functions increases, VAR1, VAR2, and VAR6 improve their relative performance concerning MOEA/D and the other variants of preference incorporation. With five objectives, MOEA/D is competitive in terms of the minimum Tchebychev distance with VAR6 and VAR2 but is clearly outperformed by these variants of preference incorporation according to the average Tchebychev distance and the Euclidean indicators. With ten objectives, MOEA/D is outperformed in most problems by VAR2, VAR6, and VAR1 according to the four distance indicators; MOEA/D-NUMS is outperformed by VAR2 and VAR6 according to both minimum distance indicators.
6. Compared with the approaches that incorporate preferences, the performance of MOEA/D is degraded as the complexity of problems increases.
7. In problems with five objective functions, VAR1 and VAR6 have the best performance in terms of the minimum Euclidean distance. VAR6 is the best according to the minimum Tchebychev distance, although MOEA/D-NUMS is competitive according to this indicator. In terms of average distances, VAR2 and VAR6 exhibit the best performances, and MOEA/D-NUMS is competitive according to the Tchebychev average distance.
8. In problems with ten objectives, VAR2 and VAR6 are the best variants in terms of the minimum Tchebychev norm. Along with VAR1, they are also the best variants according to the minimum Euclidean distance. MOEA/D-NUMS

performs very well according to the average Tchebychev distance.

9. MOEA/D-NUMS is the best method for solving some instances of DTLZ6, DTLZ8, and DTLZ9, but it is usually outperformed by several variants of our proposal for the remaining problems.
10. Our proposal consistently performs better than MOEA/D and MOEA/D-NUMS for perhaps the most difficult problems, DTLZ4 and DTLZ7.

6. Conclusions

This paper has explored the use of the interval outranking approach to articulate preferences in the MOEA/D update phase. Interval outranking is an easy way to model imprecise preferences and criteria scores, even for ill-defined preferences from a collective entity that is in charge of the decision-making process. In such a case, if the DMs are compatible with outranking models, after some discussions and exchanges of opinions, the group members can set preference parameters (weights and various thresholds) as interval numbers, thus aggregating diverse judgments.

Our results show that the convergence of MOEA/D to the region of interest can be improved if, in the update phase, the updating criterion based on the Tchebychev distance is complemented with a criterion based on a binary preference derived from the degree of credibility of the outranking. This assertion is truer as the number of objective functions and the complexity of the problem increase, and it is true according to the Euclidean distance to the RoI.

There are several ways to define the preference relation derived from the outranking information. Generally speaking, all these methods are able to outperform MOEA/D in terms of the Euclidean distance to the RoI. However, their effectiveness depends on the problem, the distance indicator, the number of objective functions, and even the particular DM's preferences. We could not find a single method that outperforms all the other methods for all of the problems, indicators, and numbers of objective functions. With respect to the ways to incorporate preferences, MOEA/D tends to perform better with the Tchebychev indicator than with the Euclidean distance. Nevertheless, as the number of objectives grows, there are some variants of articulating preferences that consistently perform better than MOEA/D regarding both indicators in most DTLZ problems, although MOEA/D is still competitive in some problems.

When the number of objectives increases, MOEA/D-NUMS performs better in terms of average distances, particularly with the Tchebychev norm. With respect to minimum distances, MOEA/D-NUMS is generally outperformed by several of our preference incorporation proposals, although MOEA/D-NUMS remains competitive in some DTLZ problems.

Thus, choosing the most appropriate approach is itself a multi-criteria decision problem. It depends on the optimization problem, the number of objective functions, the distance indicator that is considered most relevant by the DM, and even the preferences of specific DMs. If the optimization problem faced by the DM exhibits similar characteristics to a particular DTLZ problem, the results of this work can recommend the most appropriate approach according to the results and discussion in Section 5 and Appendices 1 and 2. Otherwise, even when no similarity with DTLZ problems is detected, the discussion in Section 5 is useful for identifying the most promising approach. However, it should be underlined that when facing a particular problem, the approach with the best performance could depend on the DM's preferences. More research is needed to explain this fact and the dependence on the problem and the number of objective functions. Likely, the selective pressure generated by incorporating the preferences could become excessive under certain conditions. This should be explored in a future paper.

Concerning the threats to the validity of this work, the research has increased its external validity by handling a wide variety of features: i) the broad range of objective functions (from 3 to 10); ii) the number and randomness of DMs; iii) the broad set of benchmark problems, which cope with very different and complex Pareto fronts (e.g., separable, unimodal, multimodal, deceptive, non-deceptive, linear, concave, continuous, disconnected); and iv) two kinds of distance. The external validity can be further extended by new studies that increase the size of the DM's random sample. Let us point out that experimenting with such features leads to the conclusion that the achieved results would not be degraded by using other parameter settings or solving new optimization problems, perhaps with other distances . Concerning the internal validity, like all metaheuristics, the randomness of MOEA/D/O is still an imminent internal threat if the algorithm is only run once. Hence, the main internal threat to MOEAD/O is its stochastic nature. This work takes care of this by using 300 executions per case and by validating the quality of the achieved solutions through tests for statistical significance and effect size. Last, the primary construct threat associated with this paper is the definition of the region of interest. In this definition, we have assumed that the DMs are compatible with the outranking model of preferences. Under this premise, the RoI should be composed of non-strictly outranked solutions, as defined by [1] and [20]. This is fully consistent with the discussion in subsection 4.2.

Other interesting avenues of future research are the following: i) the use of the proposed approaches to handle preferences combined with state-of the art many-objective evolutionary algorithms such as [11] and [40]; and ii) the combination of our results with the proposals of Fernández et al. [24] and [2] in the context of group evolutionary multi-objective optimization.

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