A Hyper-Heuristic of Scalarizing Functions (Experimental Study)

Technical Report EVOCINV-01-2017

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April 17, 2017

Table 1: Median and standard deviation of the hypervolume indicator for single heuristics and MOMBI-III on ZDT and DTLZ test suites. The two best values are shown in gray scale, where a darker tone corresponds to the best value. Additionally, the outperformance relation among algorithms is presented, using a confidence level of 99% (for instance, WPO performs significantly better than WN on ZDT1).

Problem	WS (1)	EWC (2)	WPO (3)	WN (4)	CHE (5)	ASF (6)	AASF (7)	MOMBI-III (8)
ZDT1	3.6539e+00 7.26e-4 3,4	3.6620e+00 8.08e-5 1,3,4,5,6,7,8	3.6333e+00 1.60e-2 4	3.0000e+001.78e-4 –	3.6614e+00 4.55e-5 1,3,4	3.6614e+00 4.55e-5 1,3,4	3.6614e+00 9.04e-5 1,3,4	3.6616e+00 7.40e-5 1,3,4,5,6,7
ZDT2	3.0000e+00 1.71e-7 –	3.3286e+00 1.02e-4 1,3,4,5,6,7,8	3.3276e+00 1.20e-3 1,4	3.0000e+004.02e-7 –	3.3281e+00 1.60e-4 1,3,4	3.3281e+00 1.60e-4 1,3,4	3.3281e+00 1.65e-4 1,3,4	3.3283e+00 1.51e-4 1,3,4,5,6,7
ZDT3	4.7070e+00 8.32e-2 4	4.8148e+00 5.20e-5 1,3,4,5,6,7	4.7813e+00 2.41e-1 4	4.0361e+002.52e-4 –	4.8140e+00 1.67e-4 1,3,4	4.8140e+00 1.67e-4 1,3,4	4.8141e+00 1.40e-4 1,3,4	4.8152e+00 4.91e-5 1,2,3,4,5,6,7
ZDT4	3.6521e+00 1.72e-3 3,4	3.6590e+00 1.26e-2 1,3,4	3.5424e+00 1.10e-1 4	2.9967e+002.40e-3 –	3.6584e+00 1.93e-3 1,3,4	3.6584e+00 1.93e-3 1,3,4	3.6580e+00 3.42e-3 1,3,4	3.6573e+00 2.47e-3 1,3,4
ZDT6	2.7741e+00 4.13e-4 4	3.0348e+00 1.25e-3 1,4,5,6,7,8	3.0347e+00 1.18e-3 1,4,5,6,7,8	2.7728e+001.14e-3 –	3.0312e+00 2.59e-3 1,4	3.0312e+00 2.59e-3 1,4	3.0309e+00 2.56e-3 1,4	3.0316e+00 2.20e-3 1,4
DTLZ1	7.8809e+00 1.42e-3 4	7.9637e+00 2.81e-4 1,3,4,6,7	7.8612e+00 7.14e-2 –	7.8807e+002.17e-4 –	7.9691e+00 2.21e-2 1,2,3,4,6,7	7.9317e+00 2.50e-2 1,3,4	7.9364e+00 2.32e-2 1,3,4	7.9745e+00 1.14e-4 1,2,3,4,5,6,7
DTLZ2	7.0000e+00 3.43e-7 –	7.3928e+00 7.36e-4 1,4	7.4174e+00 1.37e-3 1,2,4,5	7.0000e+002.97e-7 –	7.3919e+00 4.29e-3 1,4	7.4190e+00 1.73e-3 1,2,3,4,5	7.4245e+00 3.68e-4 1,2,3,4,5,6,8	7.4236e+00 9.48e-4 1,2,3,4,5,6
DTLZ3	6.2978e+011.30e+0 –	6.3350e+01 1.44e-2 1,3,4	6.2969e+01 1.78e+0 –	6.2979e+012.30e-2 –	6.3384e+01 1.05e-1 1,2,3,4	6.3385e+01 2.47e-2 1,2,3,4	6.3400e+01 1.52e-2 1,2,3,4,5	6.3415e+01 1.36e-2 1,2,3,4,5,6
DTLZ4	7.0000e+00 2.10e-1 –	7.3931e+00 3.01e-1 1,4	7.4186e+00 2.55e-1 1,2,4,5	7.0000e+003.28e-1 –	7.3983e+00 3.00e-1 1,2,4	7.4234e+00 3.49e-1 1,2,3,4,5	7.4252e+00 3.08e-1 1,2,3,4,5,6,8	7.4247e+00 1.84e-1 1,2,3,4,5,6
DTLZ5	5.6716e+00 9.91e-3 –	5.8123e+00 4.24e-2 1,4	6.0001e+00 2.11e-2 1,2,4,5	5.6716e+001.77e-2 –	5.9656e+00 3.00e-2 1,2,4	6.0410e+00 5.87e-3 1,2,3,4,5	6.0412e+00 1.03e-2 1,2,3,4,5	6.1021e+00 1.19e-3 1,2,3,4,5,6,7
DTLZ6	5.4390e+00 9.07e-2 –	5.7566e+00 4.66e-2 1,4	5.7683e+00 8.14e-2 1,4	5.3947e+009.99e-2 –	5.7426e+00 7.22e-2 1,4	5.8099e+00 7.57e-2 1,4,5	5.8163e+00 8.00e-2 1,4,5	5.8509e+00 9.41e-2 1,2,4,5
DTLZ7	1.5994e+01 2.18e-3 4	1.7252e+01 1.08e-1 1,3,4	1.6218e+01 6.09e-2 1,4	1.5868e+013.30e-3 –	1.7230e+01 9.60e-2 1,3,4	1.7475e+01 2.81e-2 1,2,3,4,5	1.7482e+01 3.77e-2 1,2,3,4,5	1.7545e+01 1.02e-2 1,2,3,4,5,6,7

Table 2: Median and standard deviation of the hypervolume indicator for single heuristics and MOMBI-III on WFG test suite.

Problem	WS (1)	EWC (2)	WPO (3)	WN (4)	CHE (5)	ASF (6)	AASF (7)	MOMBI-III (8)
WFG1	5.4395e+01 1.32e+0 2,3,4	4.8913e+01 1.58e+0 3	4.4832e+01 1.80e+0 –	4.5190e+015.64e+0 –	5.2472e+011.63e+0 2,3,4	5.2447e+01 1.63e+0 2,3,4	5.2128e+01 1.75e+0 2,3,4	5.4921e+01 1.67e+0 2,3,4
WFG2	9.9789e+01 3.57e-1 3,4,5	1.0035e+02 1.40e-1 1,3,4,5	9.7306e+01 1.64e-1 4	7.1243e+011.21e+0 –	9.9489e+01 2.85e-1 3,4	1.0031e+02 1.22e-1 1,3,4,5	1.0034e+02 1.36e-1 1,3,4,5	1.0082e+02 1.04e-1 1,2,3,4,5,6,7
WFG3	5.4610e+01 4.01e-1 –	7.3941e+01 5.60e-1 1,3,4	7.2792e+01 8.79e-2 1,4	5.6352e+01 3.26e-1 1	7.4927e+01 2.44e-1 1,2,3,4	7.5219e+01 1.92e-1 1,2,3,4,5	7.5138e+01 2.00e-1 1,2,3,4	7.5220e+01 1.54e-1 1,2,3,4,5
WFG4	5.6991e+01 1.34e-2 –	7.5551e+01 9.92e-2 1,4	7.6980e+01 9.20e-2 1,2,4,5,6,7,8	5.6995e+01 9.27e-3 –	7.5590e+01 1.70e-1 1,4	7.6737e+01 8.43e-2 1,2,4,5,8	7.6743e+01 7.67e-2 1,2,4,5,8	7.6613e+01 8.90e-2 1,2,4,5
WFG5	5.3487e+01 5.78e-6 –	7.2536e+01 9.41e-2 1,4,5	7.3541e+01 4.35e-2 1,2,4,5	5.3487e+01 4.34e-6 –	7.2351e+01 1.53e-1 1,4	7.3688e+01 4.58e-2 1,2,3,4,5	7.3728e+01 6.67e-2 1,2,3,4,5	7.3823e+01 5.24e-2 1,2,3,4,5,6,7
WFG6	5.4326e+01 3.83e-1 –	7.2957e+01 2.84e-1 1,4	7.4509e+01 3.06e-1 1,2,4,5	5.4374e+01 3.99e-1 –	7.2872e+01 3.65e-1 1,4	7.4153e+01 3.93e-1 1,2,4,5	7.4305e+01 3.60e-1 1,2,4,5	7.4245e+01 2.87e-1 1,2,4,5
WFG7	5.7011e+01 4.57e-3 –	7.5688e+01 4.23e-1 1,4	7.6816e+01 5.35e-2 1,2,4,5	5.7012e+01 3.36e-3 –	7.5529e+01 2.59e-1 1,4	7.6852e+01 7.72e-2 1,2,4,5	7.6900e+01 6.40e-2 1,2,3,4,5	7.7048e+01 4.94e-2 1,2,3,4,5,6,7
WFG8	5.2754e+01 7.05e-1 –	7.1874e+01 3.21e-1 1,4	7.2693e+01 3.06e-1 1,2,4,5	5.3782e+01 6.93e-1 –	7.1874e+01 1.88e-1 1,4	7.3041e+01 1.80e-1 1,2,3,4,5	7.2915e+01 1.71e-1 1,2,3,4,5	7.2842e+01 2.28e-1 1,2,4,5
WFG9	5.4922e+01 1.76e+0 –	7.3757e+01 2.56e-1 1,4	7.6333e+01 1.34e+0 1,2,4,5,6,7,8	5.5106e+013.39e+0 –	7.3891e+011.28e+0 1,4	7.5062e+01 1.03e+0 1,2,4,5	7.5058e+01 1.12e+0 1,2,4,5	7.5127e+01 2.18e-1 1,2,4,5

Table 3: Median and standard deviation of the s-energy measure for single heuristics and MOMBI-III on ZDT and DTLZ test suites.

Problem	WS (1)	EWC (2)	WPO (3)	WN (4)	CHE (5)	ASF (6)	AASF (7)	MOMBI-III (8)
ZDT1	1.366e+05 3.34e+04 –	5.741e+04 1.29e+02 1,3,5,6,7	1.275e+05 4.58e+03 –	4.472e+07 1.34e+08 –	6.486e+04 1.03e+01 1,3	6.486e+04 1.03e+01 1,3	6.486e+04 5.02e+01 1,3	5.639e+04 1.34e+02 1,2,3,5,6,7
ZDT2	1.684e+08 9.93e+08 –	5.702e+04 1.13e+04 1,3	6.501e+04 1.05e+02 1	1.414e+00 3.24e+07 1	5.606e+04 2.22e+02 1,2,3	5.606e+04 2.22e+02 1,2,3	5.605e+04 3.14e+03 1,2,3	5.609e+04 1.10e+02 1,2,3
ZDT3	1.931e+06 8.62e+05 –	6.505e+04 1.04e+04 1,3	1.203e+06 1.61e+05 1	5.737e+04 1.08e+08 –	4.938e+04 4.73e+03 1,2,3	4.938e+04 4.73e+03 1,2,3	4.884e+04 5.74e+03 1,2,3	4.404e+04 2.80e+02 1,2,3,5,6,7
ZDT4	1.176e+05 2.11e+04 3,4	5.764e+04 2.25e+03 1,3,4,5,6,7	1.415e+05 6.14e+04 4	8.762e+06 3.16e+08 –	6.486e+04 7.17e+02 1,3,4	6.486e+04 7.17e+02 1,3,4	6.485e+04 5.80e+02 1,3,4	5.629e+04 2.69e+02 1,2,3,4,5,6,7
ZDT6	2.930e+09 1.50e+16 –	7.231e+04 9.64e+03 1,3,4	8.987e+04 1.03e+04 1	9.295e+05 9.01e+10 1	7.012e+04 5.11e+02 1,2,3,4	7.012e+04 5.11e+02 1,2,3,4	7.007e+04 3.52e+03 1,2,3,4	6.936e+04 6.07e+02 1,2,3,4,5,6,7
DTLZ1	1.103e+41 4.69e+60 –	6.064e+06 8.42e+10 1,4	3.319e+13 2.92e+17 1,4	3.302e+44 6.39e+61 –	1.526e+06 4.75e+10 1,2,3,4	8.164e+05 3.23e+14 1,3,4	8.153e+05 1.11e+12 1,2,3,4,5	7.981e+05 8.30e+03 1,2,3,4,5,6,7
DTLZ2	2.792e+23 2.48e+35 –	1.212e+06 2.57e+06 1,4	1.248e+05 1.04e+02 1,2,4,5	9.271e+22 5.04e+43 –	1.771e+07 2.05e+11 1,4	1.238e+05 2.83e+02 1,2,3,4,5	1.206e+05 7.42e+02 1,2,3,4,5,6	1.182e+05 6.26e+02 1,2,3,4,5,6,7
DTLZ3	1.526e+12 5.09e+34 –	6.366e+05 7.94e+08 1,3,4	1.873e+14 3.76e+23 –	2.113e+13 1.77e+33 –	3.949e+05 6.15e+09 1,3,4	1.364e+05 2.88e+04 1,2,3,4,5	1.429e+05 7.87e+04 1,2,3,4,5	1.167e+05 2.13e+03 1,2,3,4,5,6,7
DTLZ4	3.255e+23 8.42e+35 –	1.077e+06 2.91e+24 1,4	1.248e+05 3.72e+15 1,2,4,5	4.021e+25 3.53e+73 –	3.067e+05 1.49e+16 1,4	1.222e+05 3.07e+16 1,2,3,4,5	1.203e+05 1.36e+13 1,2,3,4,5,6	1.176e+05 1.66e+06 1,2,3,4,5,6,7
DTLZ5	5.059e+26 3.88e+62 –	1.433e+20 3.94e+23 1,4	1.037e+18 1.14e+20 1,2,4	1.086e+27 6.09e+57 –	5.090e+16 8.80e+19 1,2,4,7	2.547e+18 1.11e+22 1,2,4	6.107e+19 1.58e+21 1,4	7.102e+06 1.55e+14 1,2,3,4,5,6,7
DTLZ6	2.721e+39 1.45e+44 –	3.706e+07 1.80e+08 1,4	2.703e+07 1.19e+09 1,4	3.910e+27 4.31e+33 1	3.406e+06 9.04e+14 1,4,7	3.433e+07 1.52e+13 1,4	2.982e+09 1.01e+22 1,4	8.296e+05 1.09e+14 1,2,3,4,5,6,7
DTLZ7	2.209e+14 3.57e+18 –	8.873e+05 3.36e+08 1,3,4	7.196e+06 1.02e+08 1,4	1.148e+16 7.92e+17 –	6.833e+05 1.45e+10 1,3,4	1.111e+06 3.45e+06 1,3,4	1.257e+06 3.84e+06 1,3,4	9.824e+04 4.83e+03 1,2,3,4,5,6,7

Table 4: Median and standard deviation of the s-energy measure for single heuristics and MOMBI-III on WFG test suite.

Problem	WS (1)	EWC (2)	WPO (3)	WN (4)	CHE (5)	ASF (6)	AASF (7)	MOMBI-III (8)
WFG1	1.451e+09 1.35e+11 –	1.355e+05 4.40e+06 1,3,4	1.582e+07 8.35e+10 1,4	8.852e+09 6.25e+16 –	1.945e+05 3.17e+05 1,3,4	8.232e+04 1.99e+05 1,2,3,4,5	7.936e+04 1.58e+06 1,2,3,4,5	4.654e+04 6.16e+03 1,2,3,4,5,6,7
WFG2	4.249e+07 8.85e+09 4	6.491e+04 3.35e+05 1,3,4	2.765e+06 1.25e+07 1,4	2.854e+11 9.46e+12 –	1.191e+05 1.24e+07 1,3,4	2.845e+04 8.39e+05 1,2,3,4,5	2.934e+04 8.69e+04 1,2,3,4,5	2.064e+04 1.38e+03 1,2,3,4,5,6,7
WFG3	3.107e+13 2.45e+18 –	1.800e+06 3.20e+09 1,4	2.885e+06 5.79e+09 1,4	3.518e+13 8.16e+15 –	1.830e+05 5.31e+06 1,2,3,4,6,7	9.107e+05 6.35e+06 1,3,4	7.468e+05 8.65e+06 1,3,4	3.911e+04 2.52e+03 1,2,3,4,5,6,7
WFG4	1.312e+10 3.10e+11 –	1.500e+04 7.94e+04 1,4,5	9.368e+03 1.18e+02 1,2,4,5	4.125e+09 2.20e+13 –	3.636e+05 2.38e+06 1,4	9.360e+03 9.61e+02 1,2,4,5	9.438e+03 2.51e+03 1,2,4,5	8.353e+03 1.14e+02 1,2,3,4,5,6,7
WFG5	9.993e+16 2.50e+19 –	1.926e+04 3.78e+04 1,4,5	9.202e+03 3.00e+01 1,2,4,5	2.037e+16 1.47e+21 –	9.692e+06 4.80e+07 1,4	9.111e+03 1.13e+03 1,2,3,4,5	9.047e+03 1.32e+02 1,2,3,4,5	8.419e+03 9.45e+01 1,2,3,4,5,6,7
WFG6	2.161e+10 1.29e+12 –	1.562e+04 3.91e+05 4,5	9.186e+03 6.60e+01 2,4,5	2.731e+10 1.02e+13 –	2.838e+05 3.40e+07 4	9.216e+03 9.11e+02 2,4,5	9.245e+03 9.29e+02 2,4,5	8.354e+03 1.01e+02 2,3,4,5,6,7
WFG7	1.235e+11 2.32e+12 –	2.383e+04 1.06e+06 1,4,5	9.439e+03 4.35e+01 1,2,4,5	3.176e+11 8.53e+13 –	5.113e+05 9.90e+06 1,4	9.324e+03 6.27e+02 1,2,3,4,5	9.301e+03 2.57e+02 1,2,3,4,5	8.415e+03 1.17e+02 1,2,3,4,5,6,7
WFG8	3.879e+09 6.28e+12 –	1.928e+04 1.54e+05 1,3,4,5	2.482e+06 8.80e+06 1,4	9.632e+08 1.51e+11 –	3.408e+05 9.34e+05 1,3,4	3.435e+04 2.19e+05 1,3,4,5	5.806e+04 2.13e+05 1,3,4,5	8.314e+03 1.23e+02 1,2,3,4,5,6,7
WFG9	6.356e+08 5.32e+13 –	2.246e+04 4.22e+04 1,4,5	9.663e+03 2.72e+02 1,2,4,5,6,7	3.228e+09 8.34e+12 –	1.047e+05 1.43e+07 1,4	1.371e+04 5.40e+04 1,4,5	1.196e+04 6.04e+04 1,2,4,5	8.565e+03 9.60e+01 1,2,3,4,5,6,7

Table 5: Median and standard deviation of the Solow-Polasky indicator for single heuristics and MOMBI-III on ZDT and DTLZ test suites.

Problem	WS (1)	EWC (2)	WPO (3)	WN (4)	CHE (5)	ASF (6)	AASF (7)	MOMBI-III (8)
ZDT1	8.24e+00 1.55e-2 3,4	8.36e+00 2.47e-2 1,3,4,5,6,7	7.35e+00 1.30e-1 4	2.00e+00 1.47e-2 –	8.33e+00 3.78e-3 1,3,4	8.33e+00 3.78e-3 1,3,4	8.33e+00 5.34e-3 1,3,4	8.37e+00 1.09e-3 1,2,3,4,5,6,7
ZDT2	2.00e+00 2.85e-3 –	8.37e+00 7.66e-3 1,3,4	8.35e+00 6.87e-3 1,4	2.00e+00 5.22e-3 –	8.37e+00 8.79e-4 1,3,4	8.37e+00 8.79e-4 1,3,4	8.37e+00 3.16e-3 1,3,4	8.37e+00 7.20e-4 1,3,4
ZDT3	5.53e+00 6.97e-1 4	1.12e+01 2.47e-2 1,3,4	6.82e+00 4.89e-1 1,4	2.00e+00 1.49e-3 –	1.15e+01 1.26e-2 1,2,3,4	1.15e+01 1.26e-2 1,2,3,4	1.15e+01 1.11e-2 1,2,3,4	1.15e+01 4.33e-3 1,2,3,4,5,6,7
ZDT4	8.24e+00 2.36e-2 3,4	8.37e+00 1.36e-1 1,3,4,5,6,7	6.69e+00 7.26e-1 4	2.00e+00 9.88e-2 –	8.33e+00 2.56e-2 1,3,4	8.33e+00 2.56e-2 1,3,4	8.33e+00 5.72e-3 1,3,4	8.37e+00 9.37e-3 1,3,4,5,6,7
ZDT6	2.00e+00 8.45e-3 –	7.16e+00 9.58e-2 1,3,4	7.08e+00 8.49e-2 1,4	2.01e+00 1.33e-2 –	7.17e+00 9.85e-2 1,3,4	7.17e+00 9.85e-2 1,3,4	7.14e+00 1.65e-1 1,4	7.18e+00 1.34e-1 1,3,4
DTLZ1	0.00e+003.83e+0 –	8.34e+00 2.79e-2 1,3,4	3.05e+002.35e+0 1,4	0.00e+00 9.61e-1 –	9.44e+00 1.86e+0 1,2,3,4,6,7	8.98e+00 2.33e+0 1,3,4	8.99e+00 1.97e+0 1,2,3,4	9.24e+00 4.25e-1 1,2,3,4,6,7
DTLZ2	0.00e+000.00e+0 –	2.45e+01 1.15e-1 1,4	3.35e+01 1.19e-2 1,2,4,5	0.00e+000.00e+0 –	2.99e+01 4.05e-1 1,2,4	3.37e+01 3.18e-2 1,2,3,4,5	3.41e+01 4.38e-2 1,2,3,4,5,6	3.44e+01 9.22e-2 1,2,3,4,5,6,7
DTLZ3	0.00e+001.54e+0 –	2.56e+01 3.39e-1 1,3,4	3.00e+001.28e+1 1,4	0.00e+001.07e+0 –	3.24e+01 6.53e+0 1,2,3,4	3.39e+01 2.83e-1 1,2,3,4,5	3.40e+01 4.05e-1 1,2,3,4,5	3.47e+01 6.42e-1 1,2,3,4,5,6,7
DTLZ4	0.00e+000.00e+0 –	2.45e+01 7.48e+0 1,4	3.35e+016.28e+0 1,2,4,5	0.00e+000.00e+0 –	3.05e+01 7.70e+0 1,2,4	3.40e+01 9.57e+0 1,2,3,4,5	3.42e+01 7.77e+0 1,2,3,4,5,6	3.45e+01 4.70e+0 1,2,3,4,5,6,7
DTLZ5	0.00e+001.37e+0 –	7.88e+00 1.41e-1 1,4	8.29e+00 3.06e-1 1,2,4	2.23e+001.42e+0 –	8.35e+00 2.04e-1 1,2,4	8.48e+00 1.02e-1 1,2,3,4,5	8.49e+00 1.32e-1 1,2,3,4,5	8.98e+00 4.33e-1 1,2,3,4,5,6,7
DTLZ6	0.00e+000.00e+0 –	1.17e+01 1.03e+0 1,4	1.17e+011.29e+0 1,4	4.33e+001.15e+0 1	1.38e+01 1.35e+0 1,2,3,4,6,7	1.21e+01 1.23e+0 1,4	1.20e+01 1.30e+0 1,4	1.48e+01 2.00e+0 1,2,3,4,6,7
DTLZ7	5.97e+00 4.23e-1 4	3.98e+01 1.51e+01 1,3,4,5,6,7	1.87e+01 8.12e-1 1,4	4.84e+00 9.68e-1 –	3.29e+01 8.95e-1 1,3,4	3.77e+01 8.64e-1 1,3,4,5	3.77e+01 9.35e-1 1,3,4,5	4.53e+01 1.16e+0 1,2,3,4,5,6,7

Table 6: Median and standard deviation of the Solow-Polasky indicator for single heuristics and MOMBI-III on WFG test suite.

Problem	WS (1)	EWC (2)	WPO (3)	WN (4)	CHE (5)	ASF (6)	AASF (7)	MOMBI-III (8)
WFG1	2.15e+01 1.93e+0 4	3.74e+01 3.85e+0 1,3,4	1.99e+01 2.71e+0 4	1.27e+01 3.13e+0 –	5.06e+01 1.87e+0 1,2,3,4	6.04e+01 3.30e+0 1,2,3,4,5	6.07e+01 2.99e+0 1,2,3,4,5	6.80e+01 4.92e+0 1,2,3,4,5,6,7
WFG2	3.55e+01 1.33e+0 3,4	7.81e+01 2.59e+0 1,3,4,5	2.70e+01 3.63e-1 4	7.19e+00 5.35e-1 –	6.95e+01 2.59e+0 1,3,4	9.41e+01 1.83e+0 1,2,3,4,5	9.46e+01 1.62e+0 1,2,3,4,5	1.00e+02 2.50e+0 1,2,3,4,5,6,7
WFG3	4.81e+00 2.70e-1 4	5.53e+01 2.24e+0 1,3,4,5,6,7	3.09e+01 4.90e-1 1,4	4.77e+00 2.83e-2 –	5.20e+01 1.56e+0 1,3,4	5.18e+01 1.45e+0 1,3,4	5.11e+01 1.53e+0 1,3,4	7.51e+01 1.17e+0 1,2,3,4,5,6,7
WFG4	3.47e+00 2.79e-2 –	1.15e+02 8.66e-1 1,4,5	1.22e+02 2.26e-1 1,2,4,5	3.48e+00 3.20e-2 –	1.10e+02 2.03e+0 1,4	1.22e+02 3.21e-1 1,2,4,5	1.22e+02 3.55e-1 1,2,4,5	1.25e+02 4.10e-1 1,2,3,4,5,6,7
WFG5	3.00e+00 3.11e-4 –	1.14e+02 8.23e-1 1,4,5	1.22e+02 6.03e-2 1,2,4,5	3.00e+00 4.05e-4 –	1.06e+02 1.99e+0 1,4	1.23e+02 1.56e-1 1,2,3,4,5	1.23e+02 1.30e-1 1,2,3,4,5,6	1.24e+02 4.00e-1 1,2,3,4,5,6,7
WFG6	3.11e+00 4.81e-2 –	1.14e+02 9.22e-1 1,4,5	1.22e+02 1.90e-1 1,2,4,5,6,7	3.19e+00 2.40e-1 1	1.06e+02 2.34e+0 1,4	1.22e+02 5.71e-1 1,2,4,5	1.22e+02 5.51e-1 1,2,4,5	1.25e+02 4.93e-1 1,2,3,4,5,6,7
WFG7	3.48e+00 3.51e-3 –	1.14e+02 8.71e-1 1,4,5	1.22e+02 7.63e-2 1,2,4,5	3.49e+00 5.64e-2 1	1.05e+02 2.26e+0 1,4	1.22e+02 2.49e-1 1,2,3,4,5	1.22e+02 2.27e-1 1,2,3,4,5	1.24e+02 4.31e-1 1,2,3,4,5,6,7
WFG8	3.28e+00 5.36e-2 –	1.14e+02 9.71e-1 1,3,4,5	7.34e+01 8.76e+0 1,4	3.29e+00 5.09e-2 –	9.93e+01 2.99e+0 1,3,4	1.16e+02 2.05e+0 1,2,3,4,5	1.16e+02 1.86e+0 1,2,3,4,5	1.25e+02 5.11e-1 1,2,3,4,5,6,7
WFG9	5.55e+00 2.58e+0 –	1.14e+02 9.79e-1 1,4,5	1.21e+02 5.79e-1 1,2,4,5,6,7	9.78e+00 1.92e+0 1	1.10e+02 2.15e+0 1,4	1.20e+02 1.15e+0 1,2,4,5	1.19e+02 1.13e+0 1,2,4,5	1.24e+02 3.61e-1 1,2,3,4,5,6,7

Table 7: Median and standard deviation of the hypervolume indicator for the compared MOEAs and MOMBI-III.

Problem	MOEA/D (1)	NSGA-III (2)	MOMBI-II (3)	MOMBI-III (4)
ZDT1	3.660e+00 1.58e-3 –	3.661e+00 1.78e-4 1	3.661e+00 8.04e-5 1	3.662e+00 7.59e-5 1,2,3
ZDT2	3.326e+00 1.28e-3 –	3.328e+00 2.27e-4 1	3.328e+00 1.22e-4 1	3.328e+00 1.14e-4 1,2,3
ZDT3	4.811e+00 2.95e-3 –	4.813e+00 3.36e-4 1	4.814e+00 8.81e-5 1,2	4.815e+00 6.16e-2 1,2,3
ZDT4	3.649e+00 5.11e-3 –	3.658e+00 7.45e-3 1	3.658e+00 4.11e-3 1	3.659e+00 1.64e-3 1
ZDT6	3.036e+00 1.26e-3 2,3,4	3.024e+00 4.58e-3 –	3.031e+00 2.28e-3 2	3.030e+00 2.55e-3 2
DTLZ1	7.975e+00 1.63e-4 3	7.975e+00 5.56e-4 3	7.937e+00 4.78e-3 –	7.975e+00 5.54e-5 3
DTLZ2	7.426e+00 2.34e-5 2,3,4	7.425e+00 4.06e-4 3,4	7.376e+00 7.14e-3 –	7.423e+00 9.85e-4 3
DTLZ3	6.339e+01 1.87e-2 3	6.340e+01 2.83e-2 3	6.336e+01 1.81e-2 –	6.341e+01 6.25e-3 1,2,3
DTLZ4	7.426e+00 1.05e+0 –	7.425e+00 4.37e-1 3	7.407e+00 4.91e-3 –	7.424e+00 1.84e-1 3
DTLZ5	6.050e+00 2.15e-4 2,3	5.954e+00 2.18e-1 –	6.015e+00 3.26e-3 –	6.103e+00 1.09e-4 1,2,3
DTLZ6	5.821e+00 7.95e-2 2,3	5.444e+00 1.15e-1 –	5.748e+00 6.70e-2 2	5.877e+00 7.63e-2 2,3
DTLZ7	9.729e+00 2.61e-2 –	1.739e+01 2.88e-2 1,3	1.736e+01 1.15e-2 1	1.754e+01 1.24e-2 1,2,3
WFG1	5.305e+01 1.45e+0 2	4.907e+01 1.59e+0 –	5.443e+01 1.79e+0 2	5.492e+01 1.67e+0 2
WFG2	9.666e+01 1.16e+0 –	1.003e+02 1.80e-1 1,3	1.001e+02 1.61e-1 1	1.008e+02 1.04e-1 1,2,3
WFG3	7.283e+01 6.74e-1 –	7.408e+01 1.53e-1 1	7.505e+01 1.47e-1 1,2	7.522e+01 1.54e-1 1,2,3
WFG4	7.382e+01 4.23e-1 –	7.656e+01 1.04e-1 1	7.668e+01 9.41e-2 1,2	7.661e+01 8.90e-2 1
WFG5	7.134e+01 5.35e-1 –	7.373e+01 8.87e-2 1,3	7.353e+01 8.09e-2 1	7.383e+01 4.10e-2 1,2,3
WFG6	7.153e+01 6.24e-1 –	7.412e+01 2.69e-1 1	7.401e+01 3.76e-1 1	7.422e+01 3.13e-1 1
WFG7	7.308e+01 8.38e-1 –	7.685e+01 7.76e-2 1	7.682e+01 8.26e-2 1	7.700e+01 5.64e-2 1,2,3
WFG8	6.945e+01 9.24e-1 –	7.285e+01 2.67e-1 1	7.266e+01 2.02e-1 1	7.293e+01 2.29e-1 1,3
WFG9	6.821e+01 1.79e+0 –	7.392e+01 9.19e-1 1	7.489e+01 1.10e+0 1,2	7.513e+01 2.70e-1 1,2

Table 8: Median and standard deviation of the s-energy measure for the compared MOEAs and MOMBI-III.

Problem	MOEA/D (1)	NSGA-III (2)	MOMBI-II (3)	MOMBI-III (4)
ZDT1	6.481e+04 5.26e+01 2,3	6.487e+04 1.45e+02 –	6.486e+04 1.43e+02 –	5.693e+04 1.88e+02 1,2,3
ZDT2	5.606e+04 1.85e+02 4	5.606e+04 3.70e+03 4	5.606e+04 3.45e+02 4	5.612e+04 1.02e+02 –
ZDT3	1.691e+06 5.51e+07 3	1.315e+05 5.69e+04 1,3	2.080e+07 1.28e+07 –	4.401e+04 2.14e+03 1,2,3
ZDT4	6.450e+04 2.29e+02 2,3	6.484e+04 5.55e+03 –	6.489e+04 4.81e+03 –	5.645e+04 3.04e+02 1,2,3
ZDT6	7.073e+04 2.30e+02 –	7.001e+04 1.13e+04 –	7.070e+04 7.50e+05 –	6.936e+04 4.38e+02 1,2,3
DTLZ1	7.820e+05 2.25e+03 2,3,4	7.842e+05 1.12e+08 3,4	8.150e+05 1.43e+03 –	8.011e+05 4.57e+03 3
DTLZ2	1.191e+05 5.54e+00 2,3	1.192e+05 2.14e+02 3	1.250e+05 3.36e+02 –	1.182e+05 5.05e+02 1,2,3
DTLZ3	1.163e+05 3.65e+07 2,3	1.355e+05 9.25e+06 –	1.391e+05 3.62e+10 –	1.164e+05 1.51e+03 2,3
DTLZ4	1.191e+05 - –	1.192e+05 9.79e+12 3	1.236e+05 7.82e+02 –	1.176e+05 1.68e+06 1,2,3
DTLZ5	6.376e+15 8.00e+15 3	8.663e+14 1.89e+37 –	6.450e+16 1.41e+18 –	3.041e+06 1.52e+05 1,2,3
DTLZ6	1.214e+09 5.23e+10 2,3	3.282e+11 1.64e+21 –	4.892e+13 4.33e+15 –	4.489e+05 1.78e+05 1,2,3
DTLZ7	3.073e+10 1.73e+18 –	1.731e+06 1.01e+07 1,3	5.544e+11 1.30e+12 –	5.075e+04 2.22e+03 1,2,3
WFG1	8.868e+09 7.33e+11 –	1.566e+05 5.71e+06 1,3	3.479e+08 6.75e+10 1	4.654e+04 6.16e+03 1,2,3
WFG2	2.496e+04 1.21e+08 3	2.004e+04 8.16e+04 1,3	9.639e+08 3.12e+11 –	2.064e+04 1.38e+03 1,3
WFG3	2.202e+09 1.26e+10 –	2.662e+06 2.47e+08 1,3	8.204e+08 1.89e+11 –	3.911e+04 2.52e+03 1,2,3
WFG4	8.175e+03 1.28e+02 2,3,4	8.931e+03 4.38e+01 3	9.313e+03 1.00e+05 –	8.353e+03 1.14e+02 2,3
WFG5	1.659e+05 8.21e+07 –	8.852e+03 2.70e+01 1,3	9.156e+03 3.99e+07 1	8.398e+03 1.25e+02 1,2,3
WFG6	7.663e+03 1.41e+02 2,3,4	8.937e+03 7.02e+04 3	1.010e+04 6.12e+05 –	8.397e+03 1.29e+02 2,3
WFG7	7.665e+03 1.40e+02 2,3,4	8.858e+03 1.20e+03 3	9.173e+03 1.64e+06 –	8.447e+03 1.18e+02 2,3
WFG8	8.032e+03 2.16e+02 2,3,4	2.182e+04 5.33e+05 3	3.886e+08 2.05e+10 –	8.293e+03 1.42e+02 2,3
WFG9	2.931e+05 3.65e+15 –	1.083e+04 3.91e+04 1,3	5.015e+05 1.12e+07 –	4.314e+03 4.83e+01 1,2,3

Table 9: Median and standard deviation of the Solow-Polasky indicator for the compared MOEAs and MOMBI-III.

Problem	MOEA/D (1)	NSGA-III (2)	MOMBI-II (3)	MOMBI-III (4)
ZDT1	8.37e+00 1.48e-1 2,3	8.33e+00 3.67e-2 –	8.33e+00 4.82e-3 –	8.37e+00 1.27e-2 2,3
ZDT2	8.36e+00 2.05e-1 –	8.37e+00 1.60e-3 –	8.37e+00 7.66e-4 –	8.37e+00 6.71e-4 –
ZDT3	1.14e+01 1.24e-1 –	1.16e+01 6.73e-2 1,3,4	1.15e+01 1.55e-2 –	1.15e+01 2.46e-1 1,3
ZDT4	8.89e+00 2.60e-1 2,3	8.34e+00 7.78e-2 –	8.34e+00 3.58e-2 –	8.37e+00 1.08e-2 2,3
ZDT6	6.90e+00 7.57e-2 –	7.16e+00 4.72e-2 1,3	6.91e+00 5.86e-2 –	7.15e+00 1.01e-1 1,3
DTLZ1	9.31e+00 1.74e-2 3,4	9.30e+00 2.11e-1 3,4	8.99e+00 1.04e-2 –	9.23e+00 1.89e-1 3
DTLZ2	3.44e+01 8.89e-4 3	3.44e+01 1.09e-2 1,3	3.36e+01 3.70e-2 –	3.44e+01 5.67e-2 1,2,3
DTLZ3	3.50e+01 4.97e-1 2,3	3.46e+01 1.71e+0 3	3.40e+01 2.57e-1 –	3.47e+01 5.66e-1 3
DTLZ4	3.44e+01 1.68e+1 –	3.44e+01 1.17e+1 1,3	3.38e+01 6.42e-2 –	3.45e+01 4.70e+0 1,2,3
DTLZ5	4.45e+01 2.62e-1 2,3,4	8.00e+00 8.18e+0 –	8.45e+00 1.22e-2 2	8.81e+00 7.25e-4 2,3
DTLZ6	5.12e+01 1.08e+0 2,3,4	2.42e+01 3.51e+0 3,4	1.26e+01 1.06e+0 –	1.30e+01 1.13e+0 –
DTLZ7	4.63e+00 1.15e-1 –	4.33e+01 1.16e+0 1,3	3.85e+01 3.88e-1 1	4.39e+01 8.81e-1 1,3
WFG1	5.87e+01 9.99e+0 –	5.68e+01 3.75e+0 –	5.73e+01 2.49e+0 –	6.80e+01 4.92e+0 2,3
WFG2	9.56e+01 1.19e+0 3	1.04e+02 7.02e-1 1,3,4	8.80e+01 2.02e+0 –	1.00e+02 2.50e+0 1,3
WFG3	8.80e+01 5.47e-1 2,3,4	6.13e+01 4.23e+0 3	4.64e+01 9.63e-1 –	7.51e+01 1.17e+0 2,3
WFG4	1.25e+02 3.68e-1 2,3,4	1.23e+02 1.18e-1 3	1.22e+02 3.96e-1 –	1.25e+02 4.10e-1 2,3
WFG5	1.23e+02 1.00e+0 3	1.23e+02 7.31e-2 3	1.23e+02 1.93e-1 –	1.25e+02 4.72e-1 1,2,3
WFG6	1.26e+02 4.23e-1 2,3,4	1.23e+02 1.87e-1 3	1.21e+02 5.10e-1 –	1.25e+02 5.09e-1 2,3
WFG7	1.26e+02 3.53e-1 2,3,4	1.23e+02 1.53e-1 3	1.22e+02 1.78e-1 –	1.24e+02 4.63e-1 2,3
WFG8	1.25e+02 3.94e-1 2,3	1.22e+02 8.19e-1 3	1.04e+02 2.46e+0 –	1.25e+02 5.22e-1 2,3
WFG9	1.22e+02 1.00e+0 2,3	1.21e+02 8.99e-1 3	1.15e+02 1.35e+0 –	1.24e+02 3.63e-1 1,2,3

Table 10: Median and standard deviation of the hypervolume indicator on many-objective instances of DTLZ1.

m	MOEA/D (1)	NSGA-III (2)	MOMBI-II (3)	MOMBI-III (4)
4	1.5995e+01 1.23e-4 3	1.5995e+01 8.66e-5 1,3	1.5945e+01 8.54e-3 –	1.5995e+01 4.35e-5 1,3
5	3.1999e+01 9.18e-5 3	3.1999e+01 6.97e-5 1,3	3.1930e+01 1.68e-2 –	3.1999e+01 3.65e-5 1,2,3
6	6.3998e+01 4.27e-4 3	6.4000e+01 9.66e-5 1,3	6.3922e+01 2.08e-2 –	6.4000e+01 7.23e-5 1,2,3
7	1.2800e+02 8.25e-4 3	1.2800e+02 1.36e-2 1,3	1.2784e+02 5.71e-2 –	1.2800e+02 1.17e-4 1,3
8	2.5599e+02 4.47e-3 3	2.5600e+02 2.00e-2 1,3	2.5577e+02 8.23e-2 –	2.5600e+02 8.56e-4 1,3
9	5.1196e+02 2.69e-2 3	5.1199e+02 7.76e-2 1,3	5.1164e+02 1.78e-1 –	5.1200e+02 1.36e-3 1,2,3
10	1.0238e+03 7.78e-2 3	1.0240e+03 3.41e-2 1,3	1.0232e+03 2.63e-1 –	1.0240e+03 8.59e-4 1,2,3